Optimizing water supply networks for slums in urban areas

TECHNISCHE UNIVERSITÄT DARMSTADT

FLUID SYSTEM TECHNIK

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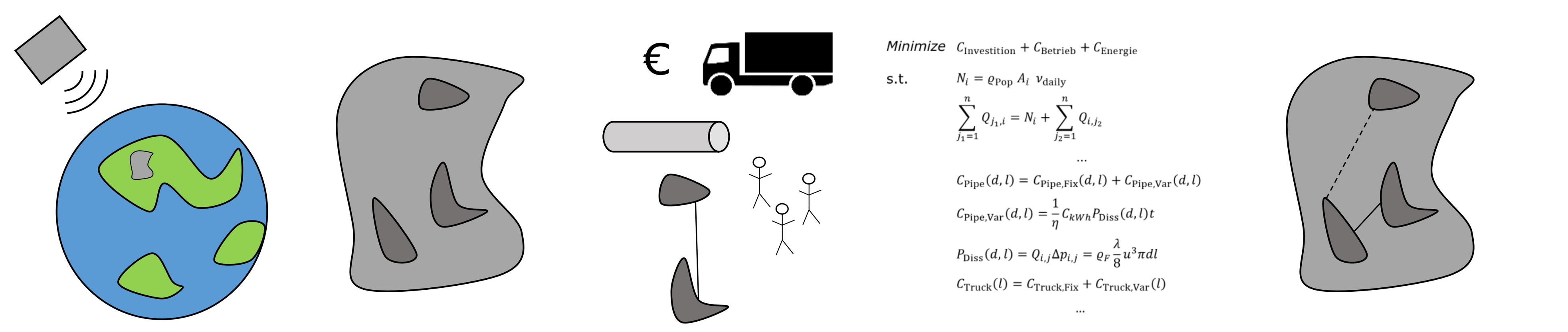
Developed Framework

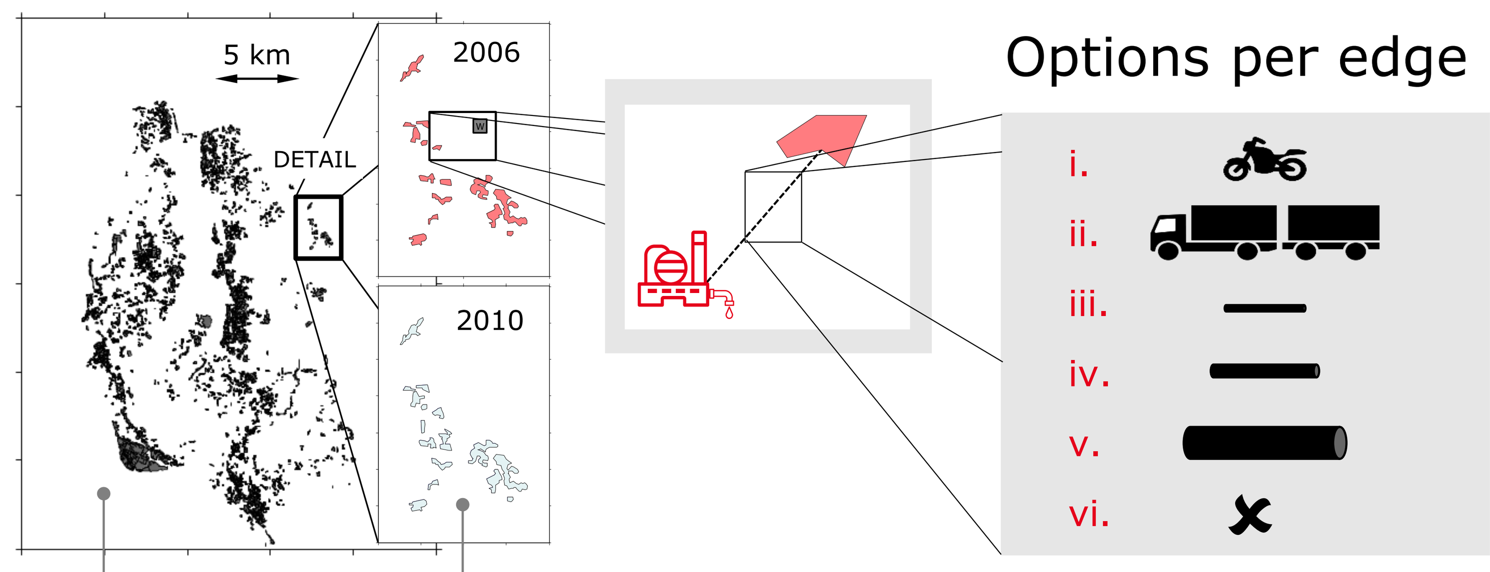
REMOTE SENSING | CLASSIFI-CATION | COST MODEL | OPTIMIZATION MODEL | RESULT APPLICATION



Interdisciplinary approach to plan optimal water supply networks for slums in a large urban area

- Using remote sensing data to classify slums for demand estimate
- Combining different ways of transportation (e.g. pipes, vehicles)
- Minimizing total costs and CO_2-Emissions

Addressed Complication



15 Slums → 10^{432} topologies

2 370 Slums → $10^{10\,808\,484}$ topologies

High number of possible network topologies makes brute-force impossible

➢ Mathematical optimization (mixed integer problem)

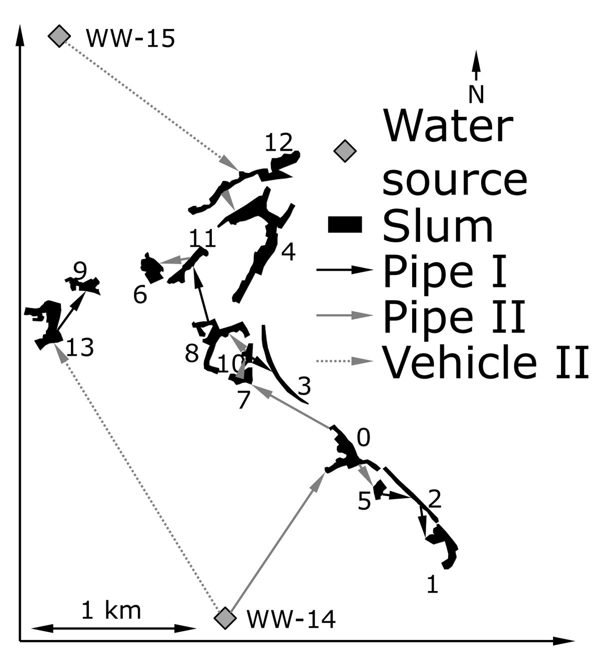
Generated Results

Optimization model formulated and solved as Mixed Integer Linear Problem (MILP). Further model extensions include

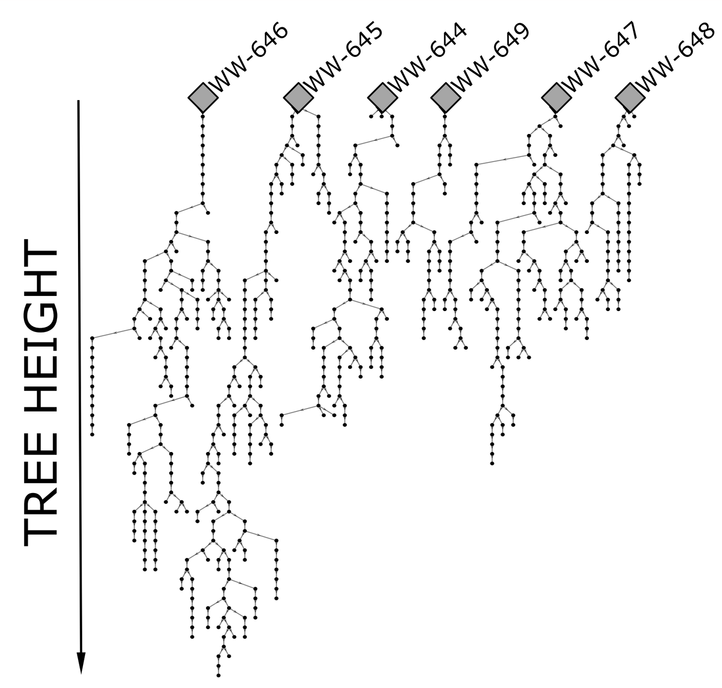
- Grey water network planning
- CO_2-Emission minimization

Developed solution methods address runtime issues with standard MILP solvers

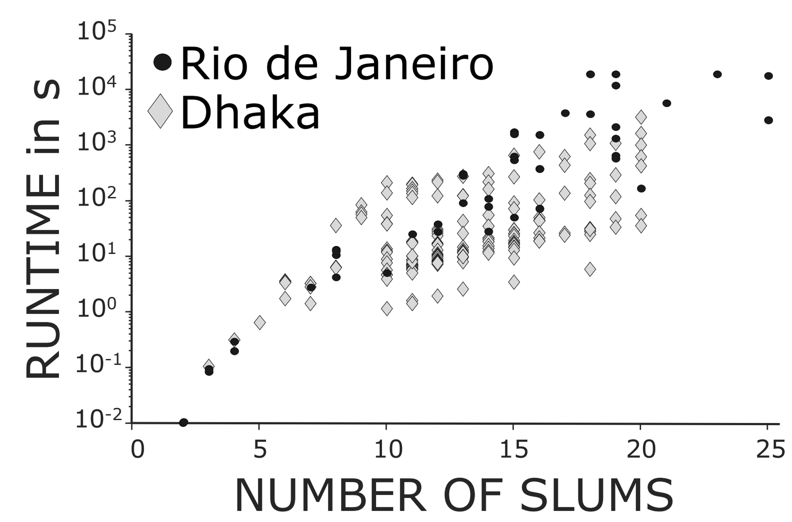
i. Clustering of slums in city into subproblems
ii. Heuristic based on minimal spanning tree and greedy algorithm
iii. Start solution for MILP solver based on minimal spanning tree



Solution of MILP for slum selection



Heuristic solution for Rio de Janeiro (as tree)



Run time analysis for clustering method

Modeled Mixed Integer Linear Problem

Objective

minimize Truck costs + Fix pipe costs + Variable pipe costs + Tank costs

$$\sum_{i\in N_w}\sum_{j\in N}\Big(\sum_{k\in K_{\text{truck}}} x_{\text{truck}}(i,j,k)\cdot \text{Cost}_{\text{truck}}(i,j,k) + \sum_{k\in K_{\text{pipe}}}\big(x_{\text{pipe}}(i,j,k)\cdot \text{Cost}_{\text{pipe,fix}}(i,j,k) + x^Q_{\exp}(i,j)\cdot \text{Cost}_{\text{pipe,var}}(i,j,k)\big)\Big) + \sum_{i\in N}\sum_{k\in K_{\text{tank}}} x_{\text{tank}}(i,k)\cdot \text{Cost}_{\text{tank}}(k)$$

Constraints

Flow condition

$$\text{s.t.}\ \sum_{j\in N_w} x^Q(j,i) = \sum_{j\in N} x^Q(i,j) + Q_{\text{daily}}(i) \qquad \forall\, i\in N_w$$

$$x^Q_p(i,j) + x^Q_t(i,j) = x^Q(i,j)$$

Capacity

$$\sum_{k\in K_{\text{truck}}} x_{\text{truck}}(i,j,k)\cdot \text{Capa}_{\text{truck}}(i,j,k) \ge x^Q_t(i,j) \qquad \forall\, i\in N_w,\ j\in N$$

$$\sum_{k\in K_{\text{pipe}}} x_{\text{pipe}}(i,j,k)\cdot \text{Capa}_{\text{pipe}}(i,j,k) \ge x^Q_p(i,j)$$

$$\sum_{j\in N} x^Q(w,j) \le \text{Capa}_W(w) \qquad \forall\, w\in W$$

Selection relation

$$\sum_{k\in K_{\text{truck}}} x_{\text{truck}}(i,j,k) + \sum_{k\in K_{\text{pipe}}} x_{pipe}(i,j,k) \ge x_{\text{use}}(i,j) \qquad \forall\, i\in N_w,\ j\in N$$

$$\sum_{k\in K_{\text{pipe}}} x_{\text{pipe}}(i,j,k) \le x_{\text{use}}(i,j)$$

$$\sum_{k\in K_{\text{truck}}} x_{\text{truck}}(i,j,k) \le x_{\text{use}}(i,j)\cdot N^{max}_{\text{truck}}$$

No loops

$$x_{\text{pipe}}(i,i) = 0,\quad x_{\text{use}}(i,i) = 0,\quad x_{\text{truck}}(i,i) = 0 \qquad \forall\, i\in N_w$$

Volume flow linearization

$$x^Q_p(i,j) = Q_{\text{lin}}(1) + \sum_{m=2}^{|N_{Q^3}|}\big(Q_{\text{lin}}(m) - Q_{\text{lin}}(m-1)\big)\lambda_{Q_{\text{lin}}}(i,j,k,m)$$

$$x^Q_{\exp}(i,j) = Q^3_{\text{lin}}(1) + \sum_{m=2}^{|N_{Q^3}|}\big(Q^3_{\text{lin}}(m) - Q^3_{\text{lin}}(m-1)\big)\lambda_{Q_{\text{lin}}}(i,j,k,m) \qquad \forall\, i\in N_w,\ j\in N,\ k\in K_{\text{pipe}}$$

$$\lambda_{Q_{\text{lin}}}(i,j,k,m) \le x_{\text{pipe}}(i,j,k)$$

$$\lambda_{Q_{\text{lin}}}(i,j,k,m) \ge z_{Q_{\text{lin}}}(i,j,k,m),\ \lambda_{Q_{\text{lin}}}(i,j,k,m) \le z_{Q_{\text{lin}}}(i,j,k,m-1) \qquad \forall\, i\in N_w,\ j\in N,\ k\in K_{\text{pipe}},\ m\in N_{Q^3}$$

Tank capacity

$$\sum_{k\in K_{\text{tank}}} x_{\text{tank}}(i,k)\cdot \text{Capa}_{\text{tank}}(k) \ge x^Q_{\text{T}_{\text{out}}}(i) + x^Q_{\text{T}_{\text{in}}\text{P}_{\text{out}}}(i) + x^Q_{\text{T}_{\text{in}}\text{daily}}(i)$$

$$x^Q_{P_{\text{in}}!P_{\text{out}}}(i) \ge \sum_{j_1\in N} x^Q_p(j_1,i) - \sum_{j_2\in N} x^Q_p(i,j_2)$$

$$x^Q_{P_{\text{in}}!P_{\text{out}}}(i) \le \sum_{j_1\in N} x^Q_p(j_1,i) - \sum_{j_2\in N} x^Q_p(i,j_2) + \mathcal{M}\cdot\lambda_{P_{\text{in}}!P_{\text{out}}}(i)$$

$$x^Q_{P_{\text{in}}!P_{\text{out}}}(i) \le \mathcal{M}\cdot\big(1-\lambda_{P_{\text{in}}!P_{\text{out}}}(i)\big) \qquad \forall\, i\in N$$

$$x^Q_{\text{T}_{\text{in}}\text{daily}}(i) \ge Q_{\text{daily}}(i) - x^Q_{P_{\text{in}}!P_{\text{out}}}(i)$$

$$x^Q_{\text{T}_{\text{in}}\text{P}_{\text{out}}}(i) \ge \sum_{j_1\in N} x^Q_p(i,j_1) - \sum_{j_2\in N} x^Q_p(j_2,i)$$

$$x^Q_{\text{T}_{\text{out}}}(i) \ge \sum_{j\in N_w} x^Q_t(i,j)$$