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Optimization of turbine cascades in tidal flows

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Abstract: This paper analyses the design and operation of a turbine cascade in a tidal flow. The parameter of interest of the investigation is the overall energy output of the cascade, i.e. the harvest factor expressing the proportion of available energy that can be converted into usable energy. Optimizing the operation strategy means not operating each turbine in its power optimum which would result in high wake mixing losses and thus leaving more energy for its downstream turbines to maximize the overall output.

1 Introduction

Low head hydro-power such as tidal power offers a promising contributing to the world's future renewable energy mix. Thus presumably, tidal turbines will be installed in tidal channels at several sites on earth in the future to harvest tidal energy. As the maximum power output of a single turbine is limited due to physical and mechanical limitations, the turbines will be combined to large turbine arrays (cf. figure 1).

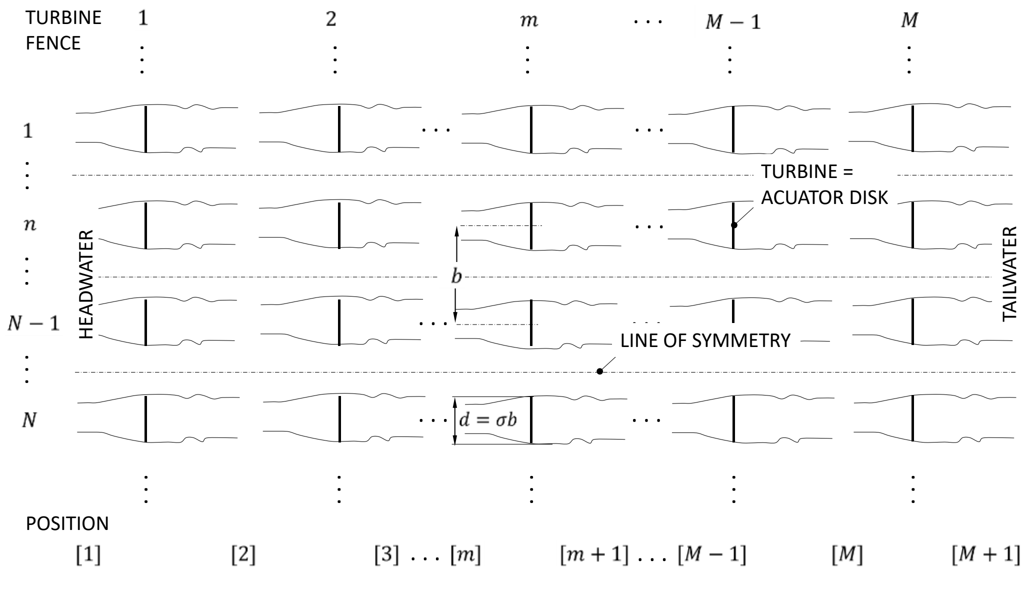

Fig. 1: Regular turbine array

Since the disturbance of each turbine propagates both downstream and upstream [1], the turbines in an array influence each other. Furthermore, the turbine field influences the flow state of the whole tidal system as considered by Schmitz & Pelz [2]. I.e. the volume flow reduces with increasing overall thrust and thus with the number and size of the turbines.

These interactions must be considered when planning turbine fields to achieve optimal power output. This raises the question of an optimal array design (number, arrangement) and operating strategy (distribution of the entire power generation on the individual turbines).

As discussed by Pelz et. al. [1] the turbine array can be treated as a nonlinear resistance showing negligible inductance and capacity, i.e. the flow through the turbine array can be treated quasi-stationary. This is true as long as the time $l/\sqrt{gh}$ needed for a wave to travel through the channel (which is $< 10^3 s$ for a channel of maximum $10\,\mathrm{km}$ length and $10\,\mathrm{m}$ depth) is much smaller than the tides' cycle time $T \approx 4.5 \times 10^4\ s$.

As multiple parallel cascades of an array can be modelled by means of symmetry, it is sufficient to analyse the design and operation of a single turbine cascade. The investigated cascade consists of M equal tidal turbines of width d. Each turbine is operated at its operation point $H_{T,m} := P_{T,m}/(\varrho g Q_{T,m})$ with $P_{T,m}$ being the shaft power of the mth turbine and $Q_{T,m}$ the volume flow rate through the mth turbine area. ϱ is the density of water and g the gravity body force.

As, due to the symmetry of the array, each turbine of the cascade can be linked with a channel width b, i.e. the spacing of the turbines, the turbine width can be measured by means of the dimensionless blockage ratio $\sigma := d/b$ (cf. figure 1).

The overall power output of the cascade is measured by the harvest factor $C_P := \sum P_T / P_{avail}$ which expresses the proportion of available energy P_{avail} that can be converted into usable energy $\sum P_T$ by the turbines. It is to be distinguished from the turbines' efficiencies η_T which describe the internal losses of each turbine.

When operating a turbine cascade, the operator aims to maximize the gained mechanical work, i.e. to solve the optimization problem

$$\max\left(C_P\left(H_{T,m}/H_1\right): H_{T,m}/H_1 \in [0,1] \text{ für } M \in \mathbb{N}, \sigma \in [0,1], Fr_2 \in [0,1]\right)$$

with the energy height at the entry of the array H_1 and the Froude number after the last turbine Fr_2 as boundary conditions describing the flow condition of the tidal channel.

In this contribution the optimization problem is solved in order to answer two principle questions:

What is the maximal harvest factor to be achieved with a cascade of M turbines?

And secondly,

how is the turbine cascade to be operated to gain this maximal harvest factor?

2 Available Power and harvest factor of an ideal turbine

Before going into the optimization, it is worth looking on the available power P_{avail} and the maximum reachable harvest factor, i.e. of a system with full blockage $\sigma = 1$.

Betz [3] describes the available power of a wind turbine by an optimal actuator disc extracting all headflow energy, i.e. a machine with no tailflow. For an open channel such an optimal disc is given by a downstream moving vertical blade, e.g. a moving dam., as described by Pelz [4]. The optimal shaft power of that hypothetical reference machine and thus the available power of an open channel flow with an effective energy head $H_{eff} \coloneqq h_1 + u_1^2/2g + \Delta z$ (with initial water height h_1, initial velocity u_1 and total bed drop Δz) is given by

$$P_{avail} = 2\varrho b g^{3/2} \left(\frac{2}{5} H_{eff}\right)^{5/2}$$

(c.f. Pelz [4]). Even for a perfect turbine with full blockage $\sigma = 1$ and thus no wake losses and no internal losses ($\eta_T = 1$) however, the extractable power is lower as part of that energy is inherently transported away in the tailwater which results in $C_P < 1$. At the optimal operation point $Fr_{2,opt} = 1$ and $H_{T,opt} = 0.4\, H_1$ the usable shaft power is

$$P_{T,\sigma=1} = \varrho b g^{3/2} \left(2/5\, H_{eff}\right)^{5/2}$$

(c.f. Pelz [4]) which means a harvest factor of

$$C_{P,\sigma=1} = 0.5.$$

If the blockage ratio is smaller, as is for usual tidal turbines, the extractable power is even lower due to mixing losses $P_{D,mix}$ in the turbine wake. These losses, measured by the mixing inefficiency $\varepsilon \coloneqq P_{D,mix}/P_{avail}$, are the higher the lower the blockage ratio and the higher the turbine head of the turbine. Higher turbine heads are however necessary to reach higher power extraction (cf. figure 2, according to the turbine model of Pelz. et. al. [1] for an exemplary value of $\sigma = 0.4$).

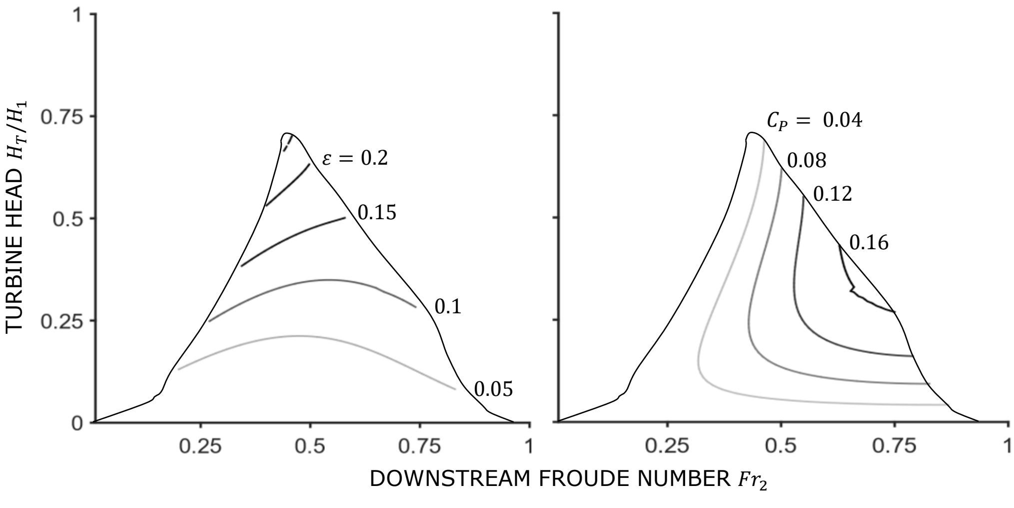

Fig. 2: mixing inefficiency ε and harvest factor C_P as a function of turbine head H_T and downstream Froude Number Fr_2 for a single turbine with exemplary blockage of $\sigma = 0.4$.

3 Minimal Example: Cascade of two turbines ($M = 2$)

For given downstream condition, using more turbines enables to operate each turbine with lower turbine head, thus reducing the mixing losses and hence reaching higher power outputs. Figure 3 shows the possible power gain $C_{P(M=2)}/C_{P(M=1)}$ that can be achieved by adding a second turbine into the channel as a function of the downstream Froude number Fr_2 and the blockage ratio σ of the turbines.

It can be seen, that the power gain is bigger for smaller blockage ratios and downstream Froude numbers. This effect is easily explained: As the energy extraction of the first turbine $P_{D,mix,1} + P_{T,1}/\eta_{T,1}$ is lower for lower Froude numbers (cf. figure 2) and lower blockage ratios, more energy remains in the downstream flow in order to be utilized by the second.

For quite slow velocities, i.e. $Fr_2 \to 0$, the power extraction tends towards zero. Thus, the flow conditions at both turbines are basically identical which is why the second turbine harvests as much energy as the first one: $C_{P(M=2)}/C_{P(M=1)} = 2$. However, this is only a thought experiment as this harvest is zero.

The same holds for vanishing blockage $\sigma \to 0$. For the opposite case of a fully blocked channel ($\sigma \to 1$) however, a second turbine would not yield any power gain, i.e. $C_{P(M=2)}/C_{P(M=1)} = 1$ as a single turbine can already extract all the - for the given downstream condition - extractable energy.

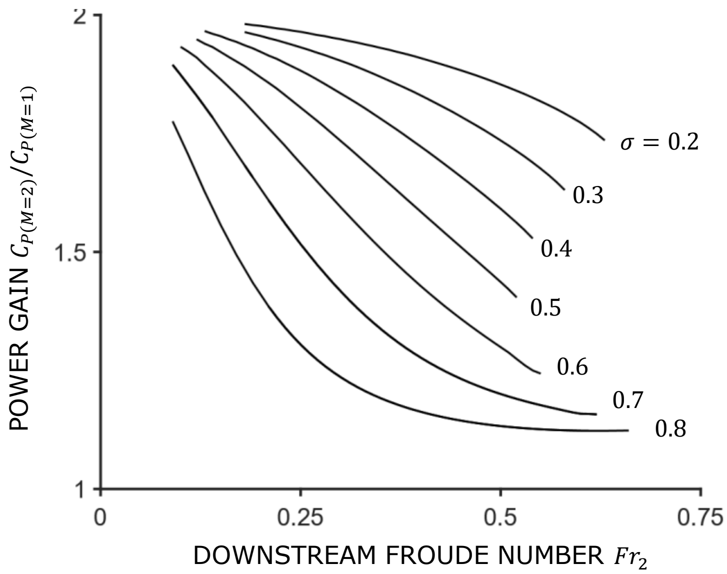

Fig. 3: Possible power gain by a second turbine as a function of downstream Froude number and blockage ratio

In order to reach the optimal power gains depicted in figure 3, the overall power output has to be split on the two turbines intelligently as shown in figure 4. For small blockage ratios $\sigma \leq 0.5$ the power extraction is more and more shifted to the second turbine with higher downstream Froude numbers.
For higher blockages $\sigma > 0.5$ however this effect turns and the energy extraction tends back to the first turbine for higher Froude numbers.
As the flow conditions at both turbines are identical for $Fr_2 \to 0$, both turbines are both optimally operated at the same operation point yielding $H_{T,1}/H_{T,2} = 1$.

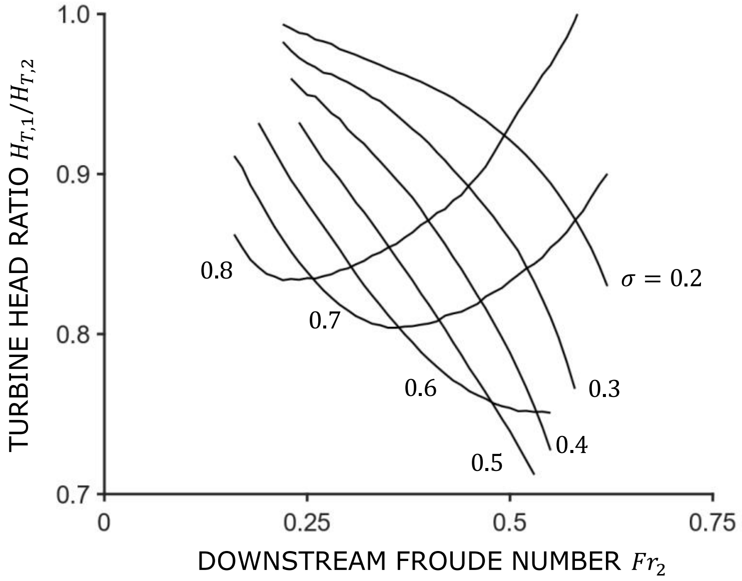

Fig. 4: Optimal turbine head ratio of a minimal cascade of two tidal turbines as a function of downstream Froude number Fr_2 and blockage ratio σ

4 Cascade of multiple turbines

By adding more turbines, even higher power gains can be achieved. Although, the relative gain becomes smaller the more turbines are added as can be seen in figure 5, which shows the harvest factors for turbine cascades of up to seven turbines an exemplary Froude number of $Fr_2 = 0.5$.

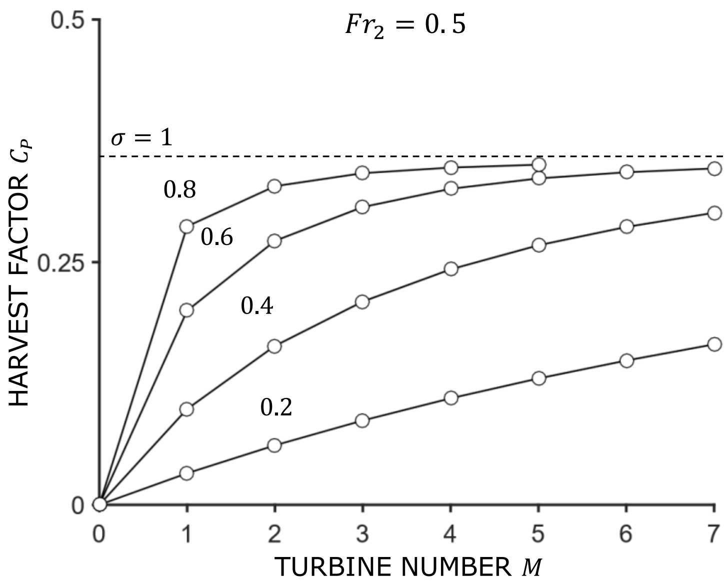

Fig. 5: Harvest factors for turbine cascades of up to seven turbines for an exemplary Froude number of $Fr_2 = 0.5$.

Independent from the blockage ratio, the harvest factor tends towards the limit given by the case of full blockage $\sigma = 1$ for the according Froude number, i.e. no mixing losses, when a large number of turbines $M \rightarrow \infty$ is applied. For higher blockage ratios this limit is reached earlier, thus less turbines have to be used in order to gain the maximum possible power output. The upper limit of $C_{P,\sigma=1} = 0.5$ is however not reachable for blockages $\sigma < 1$, as transition to supercritical flow occurs before the optimal Froude number $Fr_2 = 1$ is reached (c.f. [1]).

The according turbine heads to achieve the harvest factors shown in figure 5 are depicted in figure 6. It can be seen, that the turbine heads are getting higher the more downstream a turbine lies, i.e. with higher turbine index m. Thus, the turbine head ratios are smaller one as already derived in the minimal example in chapter 3.

With higher turbine number M the turbine heads are smaller, as the overall power extraction is split on more turbines. As already stated earlier, the reduction of turbine heads and thus of the mixing losses results in higher harvest factors which can be seen in figure 5.

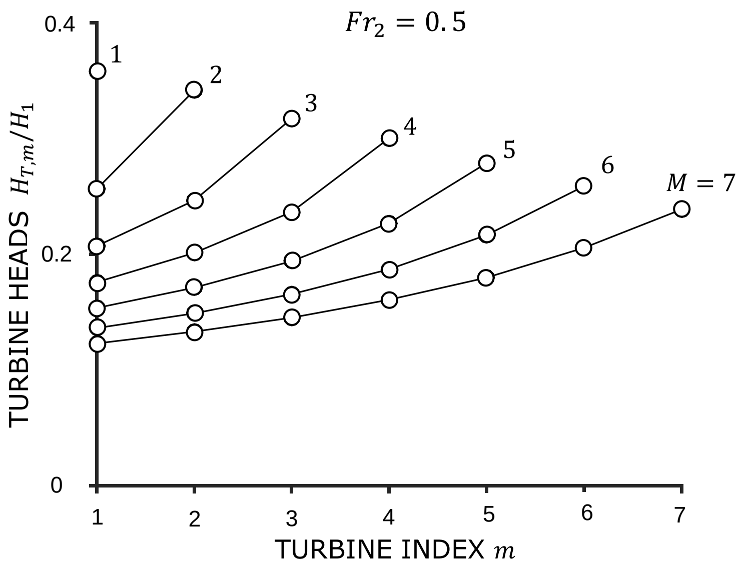

Fig. 6: Harvest Factors for turbine cascades of up to seven turbines for an exemplary Froude number of $Fr_2 = 0.5$.

5 Summary

To conclude, the two questions posed in the beginning can now be answered:

The maximal reachable harvest factor of a cascade of M turbines is those of a power station with full blockage. This harvest factor is reached with fewer turbines, the higher the blockage ratio of each turbine is.

It is optimal to operate downstream turbines with slightly higher turbine heads than their upstream turbines. The optimal turbine head ratio is however depended on the downstream Froude number and the blockage ratio.

By contrast to the simple approach of a maximum power control in each turbine, i.e. “egoistical” turbines, the optimal control strategy for a turbine cascade considers the interactions of the turbines. This means not operating each turbine in its power optimum which would result in high wake mixing losses and thus leaving more energy for its downstream turbines to maximize the overall power output.

References

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