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Multipole optimization of turbine arrays in tidal flows

C Schmitz[12], D Krämer[2], P Pelz[2]

[1] Industrial Science GmbH, Alexanderstraße 25, 64283 Darmstadt, Germany
[2] Technische Universität Darmstadt, Chair of Fluid Systems, Otto-Berndt-Straße 2, 64287 Darmstadt, Germany,

[1] christian.schmitz@industrial-science.de

Abstract. This paper proposes a multipole optimization as introduced by Holl et al. (2016) for the requirement-based dimensioning of a turbine fence considering technical and economical scaling functions. The considered fence of N stream generators with characteristic size d_T is situated in a generic tidal channel with typical flow state. The system is formulated by the energy, mass and cash flows through which the components (turbine, generator, tower/ foundation and grid connection) communicate with each other and the tidal system. It is shown, that the levelized cost of energy (LCOE) can be reduced above all by increasing the channel blockage, i.e. by using more or larger turbines.

1. Introduction

Low head hydro-power such as tidal power offers a promising contributing to the world's future renewable energy mix. Thus presumably, tidal turbines will be installed in tidal channels at several sites on earth in the future to harvest tidal energy. As the maximum power output of a single turbine is limited due to physical and mechanical limitations, the turbines will be combined to large arrays (cf. figure 1).

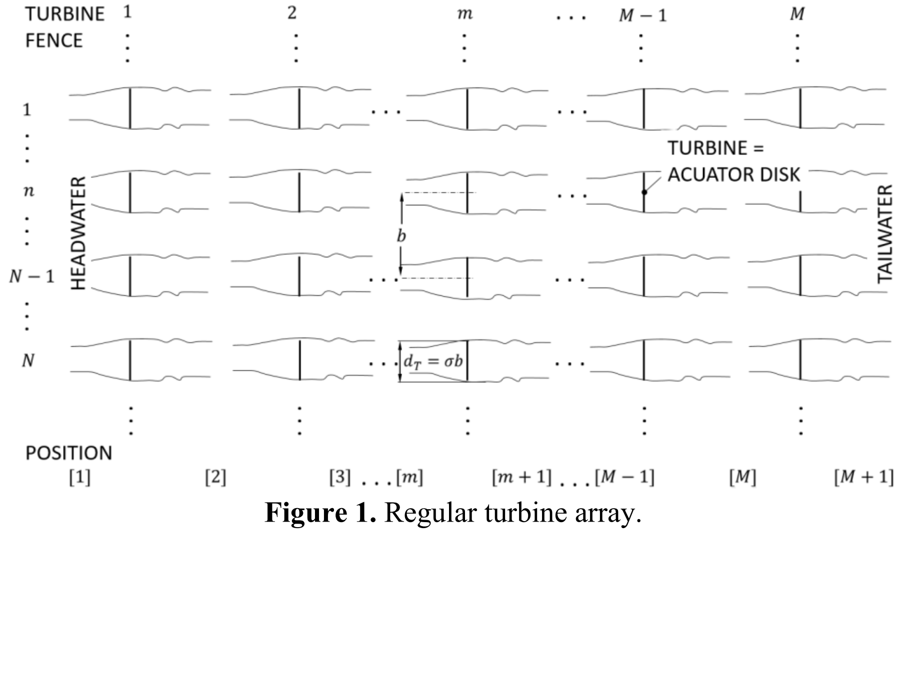

Figure 1. Regular turbine array.

Since the disturbance of each turbine propagates both downstream and upstream [1], the turbines in an array influence each other. Furthermore, the turbine field influences the flow state of the whole tidal system as considered by Schmitz & Pelz [2]. I.e. the volume flow reduces with increasing overall thrust and thus with the number and size of the turbines. These interactions must be considered when planning turbine fields to achieve optimal power output.

For a sustainable system design however, the array design must be characterized not only by means of energetic efficiency but also by a low environmental impact while at the same time accomplishing political restrictions and maintaining economic profitability. The interrelations are however often in conflict with each other. Thus, the optimal solution can only represent a trade-off between these stress criteria.

This paper introduces a multipole optimization as introduced by Holl et al. [3] for the requirement-based dimensioning and positioning of a turbine field in compliance with economic considerations for a generic tidal channel with typical flow state.

For reasons of simplicity only one single fence (channel cross direction) is considered in this paper (An expansion of the multipole model to a whole turbine array is planned). The considered fence of N stream generators with characteristic size d is situated in a generic tidal channel. The mathematical representation is formulated by the energy and cash flows through which the components (turbine, generator, inverter, tower/ foundation and grid connection) communicate with each other and the tidal system.

Energy and cash flows are connected via the price of electricity f_{el}. Optimizing the levelized cost of energy (LCOE), i.e. the price of electricity f_{el} for cost-covering conditions $G_E = 0$, - instead of the energy extraction - yields the techno-economical optimal fence design (number and size of the stream turbines) for the considered channel. Since the modelling process is associated with uncertainties (simplifications, data subject to fluctuations, etc.), a probabilistic approach is used. Hence, not only the optimal system design, but also the associated uncertainty is depicted.

2. Method

The applied method (Multi-Pole System Analysis - MPSA) consists of the four steps system modelling, system analysis, optimization and sensitivity analysis as depicted in figure 2 [3]. In the first step the system architecture is constructed by defining the interaction of the considered fluxes, i.e. energy and cash, for each and between the components. This leads to a compact mathematical representation of the considered system. In the second step the defined interactions are described by analytical and empirical scaling laws forming the basis for an optimization performed in step three. In this paper the energy production costs are used as objective function. The correlations defined in step two work as constraints for the optimization algorithm. To evaluate the robustness of the derived system regarding the model uncertainties a sensitivity analysis is performed in step four. If necessary, the model granularity or the underlaying data are refined until the aimed robustness is achieved.

In the following chapters, the MPSA method will be applied to the considered turbine fence.

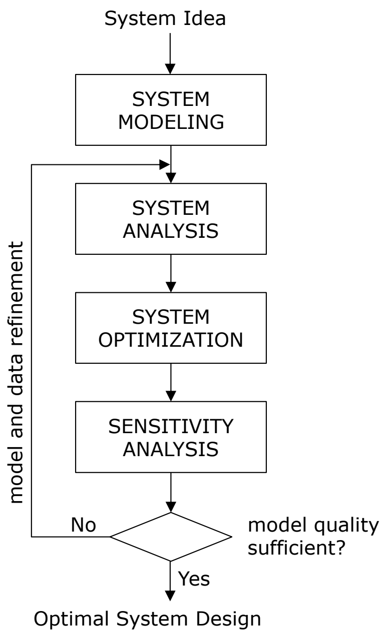

Figure 2. MPSA method.

3. System Modeling

3.1. Single turbine

A single tidal turbine is modeled by its components tower/ foundation, turbine, generator, inverter and grid connection as depicted in the multipole model in figure 3. Three fluxes, i.e. two energy fluxes (black) and one cash flux (grey) is considered: First the energy flux through the tidal channel, i.e. the available power P_{avail}, which is partly harvested by the turbine under the cost of mixture losses P_{loss} in the wake. The harvested turbine power P_T is then transformed by the downstream components generator and inverter to electrical energy which is transported to land via the grid connection. All the components are associated with energy losses as well. The available power after the turbine however simply goes through the other components without any further manipulation and is thus available for eventual downstream turbines. When sold, the electrical energy P_E yields an income flux $\dot{R}$ which adds to the cash flux. The cash flux is composed of the investment costs I_0 (the sum of the investment costs of each component) which is transformed into a flux by the capital recovery factor CRF and by constant costs (e.g. maintenance), which are considered as a fraction λ of the initial investment costs. In this paper λ is estimated with 0.07 ± 0.04 according to the data of Denny [4] and Boronowski et. al. [5]. Thus, the necessary invest enters the whole system as an initial cost flux. Ideally, the income fluxes turn the cost flux to a profit flux, i.e. a negative cost flux.

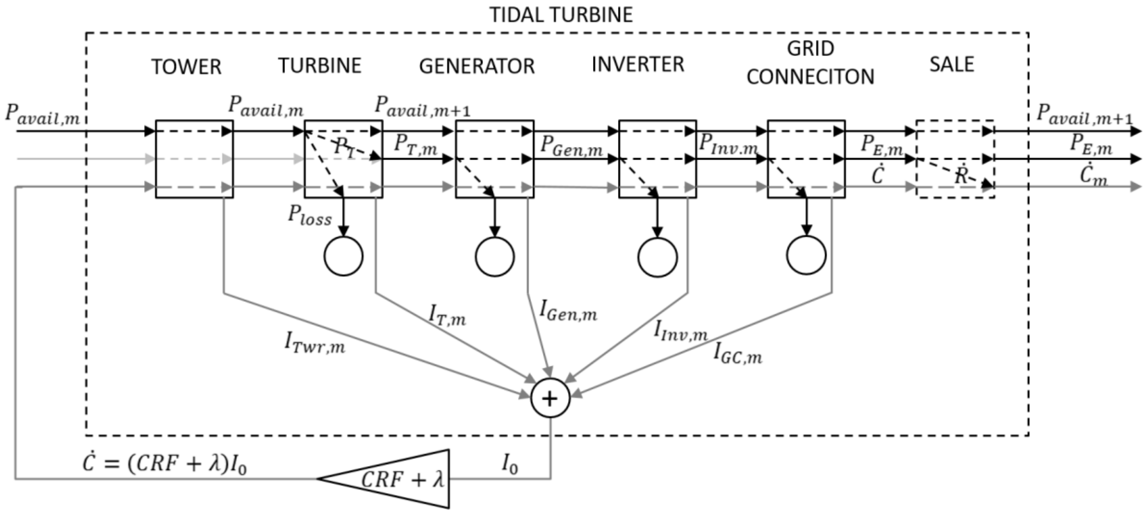

Figure 3. Multi-pole representation of a single tidal turbine.

Mathematically each block is represented by a component matrix (see set of equations (1)) transforming the input fluxes into output fluxes. Interactions between the fluxes are nondiagonal elements. If an input flux is not affected by the component, e.g. the available power by the components downstream of the turbine, the according diagonal element is equal to 1.

The tower does not change any of the fluxes - its component matrix is the identity matrix - it does however influence the initial cost flux by its initial investment cost. The turbine harvests a fraction C_P from the available power. The remaining power is dissipated C_{loss} or by the fraction $C_{through}$ remains in the downstream flow ($C_P + C_{through} + C_{loss} = 1$). Generator, inverter and grid connection are modelled by its efficiencies η_i and the sale by the electricity price f_{el}.

$$
\begin{aligned}
A_{Twr} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad
A_T = \begin{pmatrix} 1/C_{through} & 0 & 0 \\ 1/C_P & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad
A_{Gen} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\eta_{Gen} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
A_{inv} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\eta_{Inv} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad
A_{GC} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\eta_{GC} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad
A_{Sale} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -f_{el} & 1 \end{pmatrix}
\end{aligned}
\tag{1}
$$

As all components are series connected, all component matrices multiplied yield the subsystem matrix of one tidal turbine:

$$A_{TT} = \begin{pmatrix} \frac{1}{C_{through}} & 0 & 0 \\ \frac{1}{C_P \eta_{Gen} \eta_{inv} \eta_{GC}} & 1 & 0 \\ 0 & -f_{el} & 1 \end{pmatrix} \tag{2}$$

3.2. *Turbine field*
The multi-pole representation of the turbine fence is now a combination of multiple tidal turbine subsystems (see figure 4 for $N = 3$). For better readability the feedback from the initial invest to the cost flow has been removed. However, the initial cost flux $\dot{C}$ is again calculated by the total invest as described for the single turbine and depicted in figure 3. For $N \neq 1$, the total available power and invest (the initial generated power is obviously equal to zero and thus depicted in light grey) are split into N branches, which are again connected at the end by a summation.

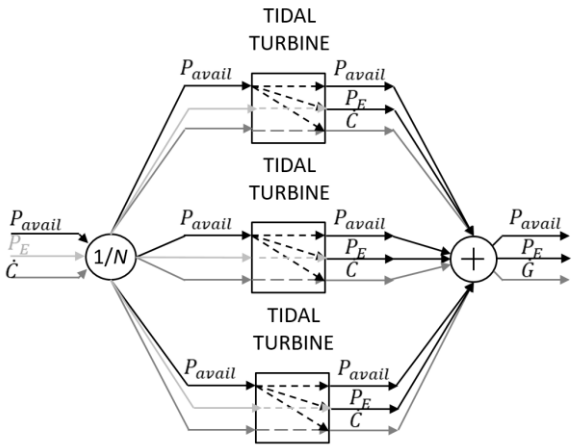

Figure 4. Representation of a tidal fence.

For a regular turbine fence as considered in this paper the according mathematical description is simply given by

$$A_{TF} = N A_{TT} \tag{3}$$

4. System analysis
After the general architecture of the system is defined, the interactions have to be described by analytical or empirical scaling functions as presented in the following for the turbine:

4.1. *Turbine*
The technical description of the turbine is done as described in Pelz et. al. (under consideration for publication in JFM) by a generic disc actuator model of relative with $\sigma = d_T/b$ (see figure 1). Pelz et. al. solve the energy, momentum and mass balances for five representative control volumes, i.e. the turbine itself, the turbine stream tube up- and downstream of the turbine, the bypass flow and the mixing zone.
This yields the extracted and dissipated power fractions C_P, C_{loss} as a function of the inlet and outlet condition as well as the turbine operation point. It is thus suitable as a built-in model for the model architecture presented in chapter 3.

The turbine efficiency η_T describing the internal turbine losses is set to a fixed value of 0.9. Considering the fact, that a tidal turbine does not operate at its maximum power output all the time due to the tidal cycle a capacity factor of 30% is applied. (compare e.g. Allan et. al. [6])

To describe the necessary invest for a turbine of a certain size d_T, a literature and market survey has been performed in order to build an empirical scaling law. As shown in figure 5.1 the turbine cost, i.e. the can be approximated by an exponential dependency of the turbine diameter d_T.

The grey lines show the associated uncertainties of the approximation. To handle the impact of these uncertainties on the result a statistic analysis is applied (see chapter 5).

4.2. Generator

The generator is considered to be a synchronous generator operated without gearbox. The generators efficiency η_{Gen} is modeled by the market survey based empirical scaling function depicted in figure 5.2. For the economical assessment (see figure 5.3) only information of three generators had been available, which leads to quite high uncertainties for extrapolation. An enhancement of the data foundation is currently in progress.

4.3. Inverter

The generated power is transformed for transport by the inverter with an efficiency η_{Inv}, which is modelled by a fixed value of 0.97. The inverter cost is due to market information considered to scale with its input power with $I_{inv} = P_{Gen}$ 32 €/kW.

4.4. Grid Connection

The transport to shore is as well associated with energy dissipation and invest. According to market information, the energy dissipation is scaling with the transported power and distance as depicted in figure 5.4

The associated investment costs $I_{GC} = I_{GC1} + I_{GC2} + I_{GC3}$ is modelled by $I_{GC1} = P_{inv}$ 85 €/kW for the internal medium voltage park connection, $I_{GC2} = P_{inv}$ 48.5 €/kW for the park substation and $I_{GC3} = P_{inv}\, l$ 0.6 €/kWkm for the transportation to shore over distance l. [7],[8]

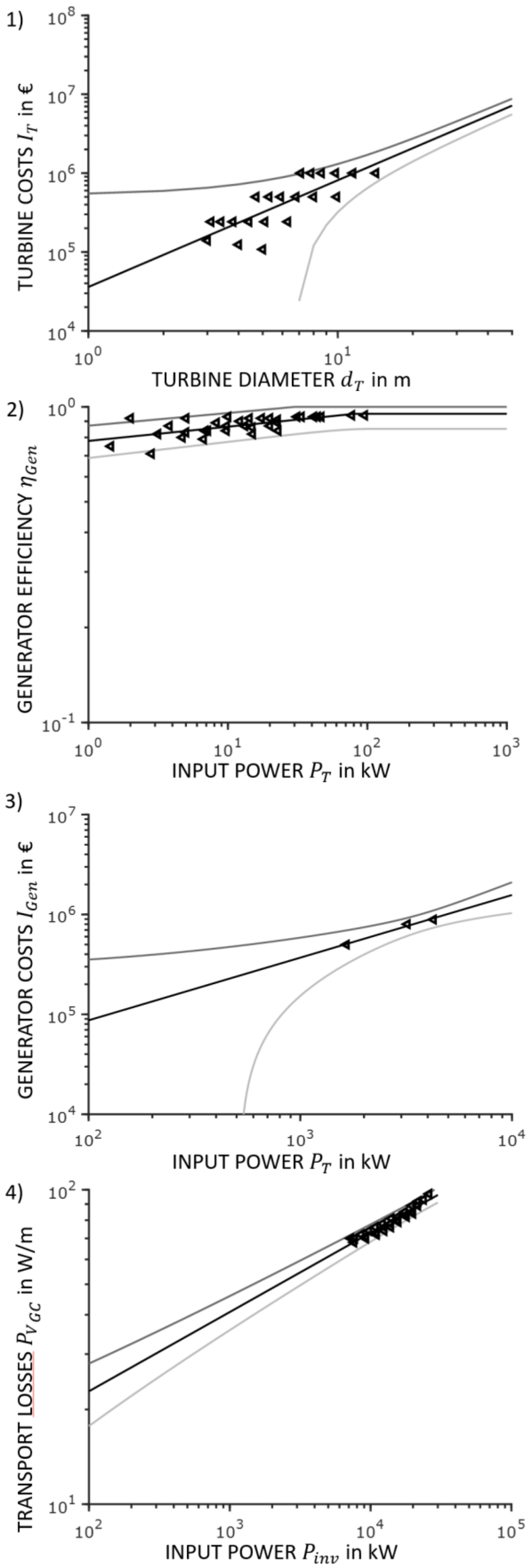

Figure 5. Empirical scaling functions.

4.5. Tower

The tower does not affect the energy flux but is associated with a certain invest I_{twr} which is scaling with the tower mass according to Engel [7] with $I_{twr} = m_{twr}$ 4.4 €/kg. The tower mass is estimating by calculating the necessary tower diameter for a steel tower with height $h \approx d_T$ considering the mechanical load due to flow resistance of tower and turbine.

5. System Optimization and Sensitivity Analysis

With all components of the turbine described by their energetic end economical share on the system fluxes, the turbine fences performance, measured by its LCOE, can be calculated by equations (2) and (3) for different turbine numbers N and diameters d_T, which yields a 2D-performance map.

In order to consider the uncertainties of the underlying data (depicted by the grey lines in figures 5.1 to 5.4) a Monte-Carlo Simulation (see e.g. Ulam & Neumann [8]) is performed, calculating the LCOE-map for multiple sets of input functions according to their probability distributions within the uncertainty ranges. A stochastic analysis of the gained multiple results then yields the mean LCOE-map and its associated uncertainties.

Figure 6 shows the LCOE-map for a generic channel of with $B = 500$ m, height $H = 30$ m and a flow velocity, in terms of the related Froude number of $Fr = 0.2$ (the black line indicates the mean value, whereas the grey lines show the resulting uncertainties). The turbine number N is limited by full blockage of the channel, i.e. $N = B/d_T$ and the turbine diameters d_T when reaching the water height H.

The - for tidal channels - quite high flow velocity was necessary due to numerical instabilities of the turbine model for small Froude numbers. The same holds for small blockages, i.e. few turbines with small diameters, so that there is no data available in that region. (An improvement of the models stability is in work.)

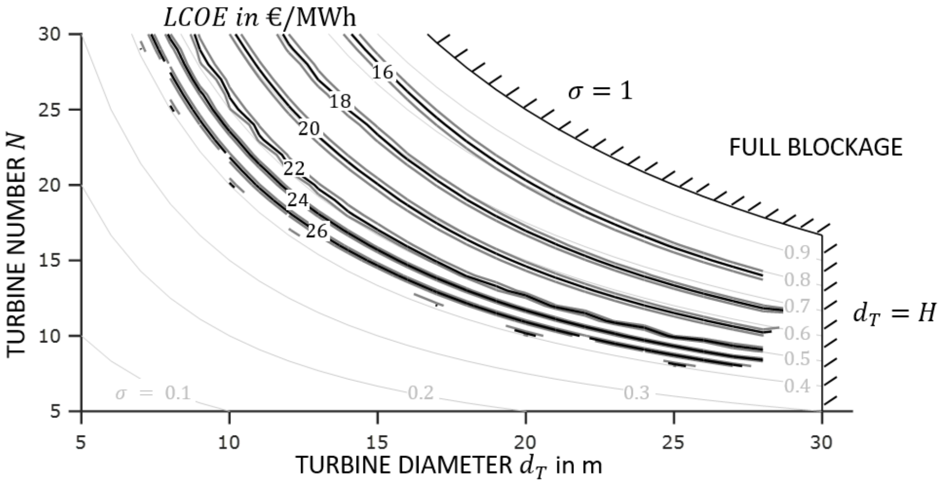

Figure 6. Levelized cost of energy (LCOE) as a function of turbine number N and diameter d_T for a generic channel with $B = 500\ m$, height $H = 30\ m$ and Froude number $Fr = 0.2$.

The estimated energy production costs (LCOE) decrease with higher blockage ratios $\sigma = Nd_T/B$. This can be achieved by either a lot of turbines or by large turbine diameters. It can however be seen, that the gained cost reduction gets smaller with higher blockage.

The estimated production costs are around 20€/MWh, which is in the magnitude of other research results (see e.g. Allan et. al. [6]). They are however lower due to the assumed high flow velocity over the whole channel and the high blockage ratios $\sigma > 0.4$ as energy production costs are rising with lower blockages and available energy.

6. Summary and Conclusion

The MPSA method as introduced by Holl et al. [3] is applied to a regular tidal fence to determine the techno-economical optimal system design. According to the four steps of the MPSA method, a system model of *N* tidal turbines of size d in a generic tidal channel is derived considering the interactions between energetic and economical aspects. A detailed description of the turbine fence model is given using analytical or empirical scaling laws.

The model is evaluated for a generic channel yielding a 2D-map of the energy production costs (LCOE) as a function of the turbine number N and diameter d_T. The energy production is around $20€/\mathrm{MWh}$ where lower costs can be achieved with high blockage ratios, i.e. a high number of turbines or large turbines.

The proposed Multipole System Analysis (MPSA) thus offers a quick possibility to study the influence of fence design (number and diameter of the turbines) and boundary conditions (width, height and flow velocity of the tidal channel) on energy extraction an investment costs. It is thus a useful tool for the evaluation and predimensioning of tidal fields.

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