

A Similarity based Efficiency Model of Spindle Screw Pumps

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Abstract. Spindle screw pumps are used in numerous different applications and industrial sectors. When selecting a suitable spindle screw pump for a specified application, manufacturers are often confronted with a lack of comparable measurement data for the desired combination of operating conditions and pumping fluid. Consequently, the volumetric and mechanical-hydraulic efficiency of a pump under the operating conditions of the application need to be estimated. In this context, this paper discusses the application of similarity for three-spindle screw pumps and introduces a similarity based efficiency model. The model is validated by means of a measured pump characteristic at different operating conditions varying pressure, rotational speed and the viscosity of the pumping fluid. The validation results prove that a pump characteristic can be represented over a wide operating range based on similarity. An estimation of the volumetric and the mechanical-hydraulic efficiency at a changed viscosity is achieved with high accuracy. Furthermore, a new approach to monitor and to evaluate manufacturing uncertainty based on the model parameter relative gap is presented. Finally, the paper gives an outlook on future experimental investigations at TU Darmstadt on model series of three-spindle screw pumps containing pumps of different displacement volume.

1. Introduction

Spindle screw pumps are characterized by their wide operating range, low-pulsation performance and high volume specific delivery capacity. This motivates their utilization in numerous different applications and industrial sectors, e.g. coolant supply in machine tools, fuel injection in marine diesel engines or lubricant supply in technical facilities.

When selecting a spindle screw pump for a specified application of a customer, manufacturers are often confronted with a lack of comparable measurement data for the desired combination of operating conditions and pumping fluid. Especially varying viscosities of the pumping fluid immensely affects the efficiency of spindle screw pumps. However, highly precise calculations based on numerical simulations or experimental tests usually are too costly and time consuming and are therefore neglected. This makes it necessary to estimate the volumetric efficiency and mechanical-hydraulic efficiency of a pump under the operating conditions of the application. A useful estimation needs to be easy to apply, physically based and sufficiently accurate. Furthermore a reduction of the complexity is advantageous and requires to focus on the major influence parameters.

Against this background, similarity of spindle screw pumps promises to be a simple and effective approach to estimate the volumetric and mechanical-hydraulic efficiency. On this basis, the authors introduced a dimensionless and type-independent efficiency model of positive displacement pumps that meet the above conditions [1]. Furthermore they discuss the application of this model in the

context of legislative efficiency regulations [2] and they apply similarity on external gear pumps with promising results [3]. The model considers the operating conditions, i.e. pressure difference Δp and rotational speed n , the fluid properties, i.e. kinematic viscosity ν , density ρ and compressibility κ , and the machine parameters, i.e. displacement volume V and average gap height \bar{s} .

As a result, four dimensionless variables are identified that describe the efficiency behavior: (i) specific pressure Δp^+ , (ii) Reynolds number Re , (iii) specific compressibility κ^+ , and (iv) relative gap ψ :

$$\begin{aligned}\Delta p^+ &:= \frac{\Delta p}{\nu^2 \rho V^{-2/3}}, & Re &:= \frac{nV^{2/3}}{\nu}, \\ \kappa^+ &:= \kappa \Delta p, & \psi &:= \frac{\bar{s}}{V^{1/3}}.\end{aligned}\tag{1}$$

A fundamental basis of the previous work [1,2,3] were experimental investigations on spindle screw pumps conducted by Corneli et al. [4] that have not been published until now. These experimental investigations on identical screw pumps with modified gaps are used to validate the efficiency model of Pelz et al. [1] in terms of spindle screw pumps and the results are presented in this paper. Furthermore, the approach of another study that will be carried out in the second half of 2018 is introduced. This future study will focus on model series of spindle screw pumps and will examine the scalability of spindle screw pump characteristics based on the displacement volume.

This paper is structured as follows: The paper begins with a brief literature overview on efficiency models of positive displacement pumps. In the second section, similarity of spindle screw pumps is discussed. Subsequently, the experimental setup of Corneli et al. [4] that was used for the experimental investigations on identical spindle screw pumps with modified gaps is described. In the ‘result’ section, three research questions are discussed based on the experimental investigations of Corneli et al. [4]: (i) How accurate can a spindle screw pump characteristic be represented based on similarity? (ii) How accurate is an estimation of the volumetric and mechanical-hydraulic efficiency based on similarity? (iii) Is a representation of manufacturing tolerances based on the model parameter relative gap ψ useful? The third research question is discussed on the dimensionless pump characteristics of identical screw pumps with modified gaps. The paper closes with conclusions and an outlook on future work.

2. Literature

The literature gives numerous studies on the efficiency of positive displacement pumps. The most comprehensive overviews give Ivantsyn and Ivantysynova [5] and Kohmäscher et. al [6]. The authors of this paper already summarized these overviews in [1,33]. Nonetheless, for reasons of completeness and readability, this paper also gives this brief summary of the most important studies on efficiency of positive displacement pumps.

Ivantsyn and Ivantysynova [5] and Kohmäscher et. al [6] differentiate efficiency models into physical, empirical and data-driven models.

The physical models describe the volumetric and mechanical-hydraulic losses in positive displacement pumps. On this basis, Wilson [7] developed an efficiency model in the 1940s that assumes the leakage to be a laminar flow and describes the mechanical-hydraulic losses as dependent on viscous friction only. Schlösser and Hilbrands [8,9,10,11] extend the leakage model with a turbulent flow component. The mechanical-hydraulic losses are complemented with a pressure-related and an inertia-related loss. Thoma [12] und Bravendik [13] further develop these models focusing on pumps with adjustable displacement volumes.

All these physical models rely on dimensionless loss coefficients, comparable with pressure loss coefficients. These loss coefficients need to be determined empirically and are assumed to be constant. However, Zarotti and Nervegna [14], Rydberg [15] as well as McCandlish and Dorey [16] conclude from their studies that this assumption can be violated, e.g. by changing gap height due to varying operating conditions. Consequently, this may lead to inaccuracy and model uncertainty. That is why

they complete the physical description with empirical formulas based on experimental findings and increase the complexity of the models according to Ivantsyn und Ivantysynova [5].

Following Kohmäscher et. al. [6], other model based approaches are the numerical or data-driven models of Ivantsyn and Ivantysynova [5], Huhtala [17] and Baum [18]. These models require a large amount of measurement data that is approximated by different numerical methods, e.g. non-linear polynomial approximation or neural networks.

In summary, a lot of research on the efficiency of positive displacement pumps was carried out in the past decades, leading to more complex and detailed models.

However, regarding the requirements for an estimation, i.e. being easy to apply, physically based and sufficiently accurate, physical models are still of high value. In this context, Pelz et. al. [1] introduces a semi-analytical efficiency model based on dimensional analysis [19] that reduces the complexity due to a variety of different gap geometries. In the following sections the similarity of spindle screw pumps is discussed and the experimental validation is presented.

3. Similarity of three-spindle screw pumps

This section introduces the physical model approach and the application of similarity on spindle screw pumps. The authors validate this approach for external gear pumps based on experimental data and prove its beneficial use [3].

Of major importance is the isentropic efficiency η that represents a measure of the energetic quality of a pump. Based on the first law of thermodynamics, for a time averaged stationary and thermally isolated machine the isentropic efficiency is defined as the hydraulic power divided by the shaft power P_S . The hydraulic power is obtained as the product of the volume flow Q_1 at the inlet of the pump, the discharge pressure Δp and a correction factor that depends on the compressibility κ . Considering the shaft power is the product of the shaft torque M_S and the rotational speed n , one obtains the well-known definition of the efficiency η as (cf. [1])

$$\eta := \frac{Q_1 \Delta p}{2\pi M_S n} \left(1 - \frac{\kappa \Delta p}{2}\right). \quad (2)$$

In the following, the influence of the compressibility is neglected due to low discharge pressures, i.e. $\kappa \Delta p \ll 1$. Consequently, the volume flow rate $Q_1 = Q$ is assumed to be constant for $\kappa \Delta p \ll 1$. Extending equation (2) with the displacement volume V , the efficiency can be written as the product of the volumetric efficiency η_{vol} and the mechanical-hydraulic efficiency η_{mh}

$$\eta = \eta_{vol} \eta_{mh}, \quad \eta_{vol} := \frac{Q}{nV}, \quad \eta_{mh} := \frac{\Delta p V}{2\pi M_S}. \quad (3)$$

Both partial efficiencies can be represented as a function of the respectively responsible loss: for the volumetric efficiency, this is the leakage Q_L . Taking the theoretical volume flow rate $Q_{th} = nV = Q + Q_L$ into account, the volumetric efficiency η_{vol} can be written as

$$\eta_{vol} := \frac{Q}{nV} = 1 - \frac{Q_L}{nV}. \quad (4)$$

The friction torque M_{mh} represents the mechanical-hydraulic losses. Considering the shaft torque is the sum of the hydraulic torque $M_{hyd} = \Delta p V / 2\pi$ and the friction torque M_{mh} , gives the mechanical-hydraulic efficiency η_{mh}

$$\eta_{mh} := \frac{\Delta p V}{2\pi M_S} = \frac{1}{1 + 2\pi \frac{M_{mh}}{\Delta p V}}. \quad (5)$$

Following the above approach, a description of the losses, i.e. the leakage Q_L and the friction torque M_{mh} , results in a description of the volumetric and mechanical-hydraulic as well as total efficiency.

On this basis, we continue performing a dimensional analysis, the basis of our modeling and of similarity. The procedure is as follows:

firstly, all major influencing variables on the losses are determined. The following six influencing variables are considered: the operational parameters discharge pressure Δp and rotational speed n , the properties of the pumping medium density ρ and kinematic viscosity ν , and the geometric parameters displacement volume V and average gap height \bar{s} of the pump. The average gap height \bar{s} is a newly introduced size representing an average height all gap heights of the several different gaps of a screw pump. It is motivated by the analogy of a hydrodynamic journal bearing with the average height of the lubrication gap \bar{h} , i.e. \bar{s} is interpreted as \bar{h} . The characteristic length of the pump is $V^{1/3}$.

Secondly, performing a dimensional analysis reduces the number of model variables and, thus, simplifies the model while maintaining the physical significance [19]. This yields five dimensionless variables characterizing the operating state of a pump: The specific pressure Δp^+ , Reynolds number Re , and relative gap size ψ are the independent dimensionless variables and are defined as

$$\Delta p^+ := \frac{\Delta p}{\nu^2 \rho V^{-2/3}}, \quad Re := \frac{n V^{2/3}}{\nu}, \quad \psi := \frac{\bar{s}}{V^{1/3}}. \quad (6)$$

Furthermore, both the leakage Q_L and the friction torque M_{mh} are also represented by dimensionless variables, the specific leakage Q_L^+ and the specific friction torque M_{mh}^+ . These dependent dimensionless variables are defined as

$$Q_L^+ := \frac{Q_L}{\nu V^{1/3}}, \quad M_{mh}^+ := \frac{M_{mh}}{\Delta p V}. \quad (7)$$

Finally, the specific leakage $Q_L^+ = Q_L^+(\Delta p^+, Re, \psi)$ and the specific friction torque $M_{mh}^+(\Delta p^+, Re, \psi)$ are functions of the specific pressure, Reynolds number and relative gap, which need to be determined. This leads directly to the descriptions of the volumetric, of the hydraulic-mechanical, and of the total efficiency:

$$\begin{aligned} \eta_{vol} &= 1 - \frac{1}{Re} Q_L^+(\Delta p^+, \psi), \\ \eta_{mh} &= \frac{1}{1 + \frac{2\pi}{1 - \kappa \Delta p / 2} M_{mh}^+(\Delta p^+, Re, \psi)}, \\ \eta &= \frac{1 - \frac{1}{Re} Q_L^+(\Delta p^+, \psi)}{1 + \frac{2\pi}{1 - \kappa \Delta p / 2} M_{mh}^+(\Delta p^+, Re, \psi)}. \end{aligned} \quad (8)$$

On the basis of measurement data provided by pump manufacturers, Pelz et. al. [1] illustrate that the specific leakage Q_L^+ can be approximated by a semi empirical model in terms of a power law.

$$Q_L^+ = L * (\Delta p^+ \psi^3)^m. \quad (9)$$

The dimensionless leakage coefficient L and the exponent m are the only model parameters. The leakage coefficient includes the ratios of leakage specific geometric parameters of the pump, e.g. gap length and gap width, to the characteristic length of the pump $V^{1/3}$. At the same time, it was shown, that the influence of the Reynolds number is negligible for screw pumps. However, the investigations of Schänzle et al. [3] on external gear pumps at low operating pressures show a clear dependency of the specific leakage on the Reynolds number. The influence of the Reynolds number represents the drag flow of the leakage caused by the rotation of gears. In screw pumps the drag flow is negligible in comparison to the pressure driven flow which is represented by the specific pressure.

On the other hand, Schlösser and Hilbrands [10] introduced a physically based approach for the estimation of the friction torque M_{mh} that represents a linear combination of a pressure-related loss, a viscous friction-related loss and inertia-related loss:

$$M_{mh} = C\Delta pV + R_\mu \frac{\mu nV}{s} + R_\rho \rho n^2 V^{5/3}. \quad (10)$$

Applying the dimensionless variables given by the equations (6) and (7) to this approach yields a description of the specific friction torque

$$M_{mh}^+(\Delta p^+, Re, \psi) = C + R_\mu \frac{Re}{\Delta p^+ \psi} + R_\rho \frac{Re^2}{\Delta p^+}. \quad (11)$$

C , R_μ and R_ρ are the dimensionless loss coefficients of the different loss terms. These loss coefficient also include the ratios of loss specific geometric parameters to the characteristic length of the pump $V^{1/3}$ (see [8,9,10,11]).

Altogether, there are five model parameters, namely L , m , C , R_μ and R_ρ , that need to be identified, i. e. calibrated, using measurement data. The model parameter identification is based on robust linear regression. In summary, the semi-analytical equations (9) and (11) together with equation (8) give the mathematical formulas which are applied on spindle screw pumps in this paper.

Before that, we take a closer look on the prerequisite of a useful application of the concept of similarity and dimensional analysis. This is the presence of geometric similarity being a part of full similarity. For this study and the future study, this is crucial for two reasons:

firstly, the average gap height \bar{s} of a series of identical pumps will always vary from the nominal value due to manufacturing (including assembly) uncertainty. Hence, full geometric similarity in spindle screw pumps is never present. Furthermore, from the practical point of view, it is advantageous to set a detailed geometrical modelling of the several different gaps of a screw pump aside in favor of determining the average gap height by means of the measured pump characteristic. For this purpose, Pelz et al. [1] introduce the relative gap $\psi := \bar{s}/V^{1/3}$ and the class of gap ψ/ψ_{ref} to provide an approach for relative considerations. In this paper we follow this approach and validate it based on experimental investigations of identical screw pumps with modified gaps. On the one hand, the approach allows a characterization of the manufacturing uncertainty and, on the other hand, a comparison of pumps based on one single characteristic quantity.

Secondly, spindle screw pumps need to be investigated in regard to their geometric similarity when aiming to scale the pumps characteristic based on the displacement volume. Typically, within a model series of spindle screw pumps, changes of the displacement volume are realized by varying the main geometry parameters of a spindle. Those are the head circle diameters d_A , the pitch H and the length l . In this context, Thurner [20] presents a pure analytical description of the displacement volume of a three-spindle screw pump as a function of the pitch and the square of the head circle diameter of a spindle:

$$V \propto Hd_A^2. \quad (12)$$

Furthermore, within a model series of spindle screw pumps, the ratios of these above main geometry parameters remain equal and, thus, screw pumps of the same model series can be considered geometrically similar in terms of the main geometry parameters. Regarding the leakage and the viscous-driven losses, a closer look on the gap geometries is also necessary. In this regard, Rossow [21] identifies the length and wide of the gaps with major influence on the above losses which are the tooth sided gaps and the contour gaps [4]. The length and the width of the gaps can also be represented as a function of the pitch and the head circle diameter. From their actual mathematical description follows that the ratios of these geometric parameters remain equal if geometric similarity of the above main geometric parameters is fulfilled. From this point of view, only the ratio of the gap heights is expected to change when varying the displacement volume. As mentioned above, this is due to

manufacturing uncertainty. Consequently, within a model series the relative gap ψ is expected to change as well and, hence, the relative gap is identified as the one measure that represents the incomplete geometric similarity of spindle screw pumps. Following this reasoning, the five model parameters, L , m , C , R_μ and R_Q are expected to remain equal for screw pumps of one model series that fulfills the above requirements. The validation of this approach will be core of the future study on model series of spindle screw pumps.

4. Experimental setup

The following section presents the experimental setup of Corneli et al [4] of the investigations on identical spindle screw pumps with modified gaps. The validation results are presented in the following section.

The hydraulic circuit diagram of the experimental setup is shown in figure 1. Pressure and temperature are measured at the inlet and outlet of the pump. A piezoresistive sensor and a Pt-100 resistance thermometer are utilized, respectively. A torque meter with built-in speed sensor operating on the strain gage principle measures the shaft torque and rotational speed of the pump. Furthermore, a screw type flow meter is used to measure the volume flow rate at the outlet of the pump. The pressure at the outlet of the pump is adjusted with a throttle valve. The cooling of the pumping fluid is realized using a pump-transfer cooler filtration unit. Table 1 summarizes the measurement range and accuracy of the used measurement equipment.

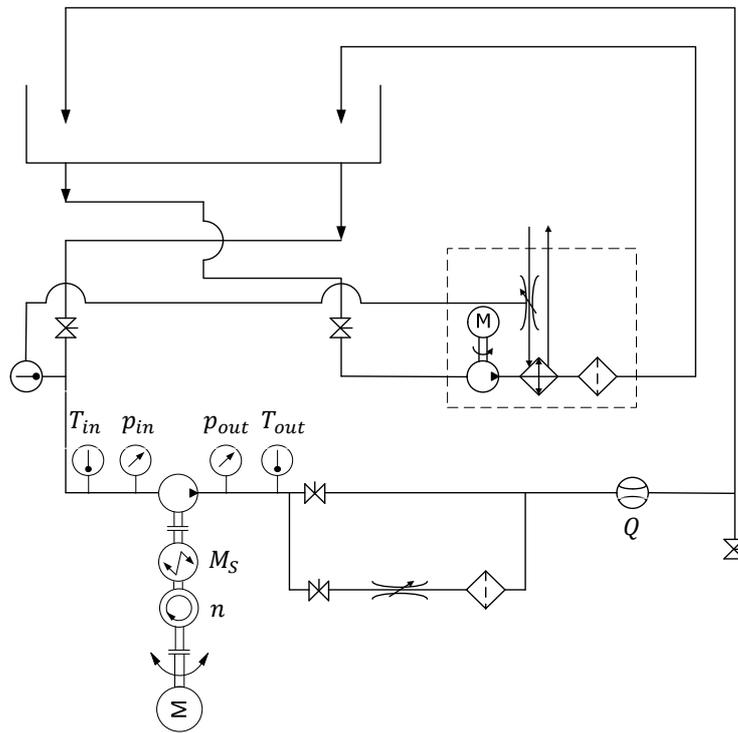


Figure 1: Hydraulic circuit diagram of the test bench.

The pumping fluids utilized in this study were standard hydraulic oils of the viscosity classes 2, 7, 22 and 68. The viscosity-temperature curves and the density-temperature curves of all hydraulic oils were measured with a highly accurate glass capillary viscometer and a density meter.

In total, the investigations of Corneli et al. include pump characteristic measurements of eight three-spindle screw pumps: one original pump without modified gaps and seven pumps with modified gaps. A detailed description of the gaps and their location in the screw pump can be found

in [4] (see fig. 4 and 5). However, for reasons of confidentiality a detailed description of the modifications is not given.

All measurements were carried out at rotating speeds between 650 and 1450 rpm, a fluid temperature of 40 °C and discharge pressure between 2 and 28 bar. Table 2 summarizes the modifications of all three-spindle screw pumps.

Table 1: Measurement ranges and uncertainty of measured operation variables (MW= measured value, FS=full scale).

MEASURED VARIABLES	MEASUREMENT RANGE	MEASUREMENT ACCURACY
p_{in}	0 ... 5 bar	0.15 % FS
p_{out}	0 ... 30 bar	0.15 % FS
Q	120 l/min	0.5 % MW
M_S	40/200 Nm	0.2/0.1 % FS
n	12000 rpm	0.1 % MW
T_{in}, T_{out}	0 ... 100 °C	± 0.6 °C

Table 2: Eight three-spindle screw pumps with modified gaps investigated by Corneli et al. [4].

STATE OF MODIFICATION	MODIFICATION	AFFAECTED GAPS
original	-	-
1	reduced tip diameter of the choke plunger	Throttle gap - dsc
2	reduced tip diameter of the choke plunger	Throttle gap - isc
3	regrinded tooth side of the drive screw	thooth sided gap - dsc
4	reduced inside diameter of the drive screw	thooth sided gap - dsc, radial gap - dsc
5	reduced tip diameter of the idle screw	radial gap - dsc, contour gap - isc
6	reduced inside diameter of the idle screw	thooth sided gap - dsc, radial gap - isc
7	reduced tip diameter of the drive screw	radial gap - isc, contour gap - dsc

5. Results

As the introduction states, this paper raises three research questions: firstly, how accurate can a spindle screw pump characteristic be represented based on similarity? Secondly, how accurate is an estimation of the volumetric and mechanical-hydraulic efficiency based on similarity? Thirdly, is a representation of manufacturing tolerances based on the model parameter relative gap ψ useful? The following section discusses the first two research questions in detail based on one spindle screw pump measured by Corneli et al. This is a three-spindle screw pump without modified gaps (cf. tab 2.: ‘Original’). However, the approach and the results are transferable to the screw pumps with modified gaps as well. The third research question is discussed on the dimensionless pump characteristics of all identical screw pumps with modified gaps. The operating range of all measurements carried out in this study was limited by the maximum pressure of 28 bar and the maximum rotating speed of 1450 rpm. As section 4.1 states, four different hydraulic oils were utilized.

First of all, the model of the leakage flow rate is validated. Equation (9) gives the mathematical relationship between specific leakage and specific pressure by means of a power law. Figure 2 shows the specific leakage versus specific pressure for all measured operating points, i.e. varying the pressure, rotating speed and viscosity, in one double logarithmic diagram. The power law in equation 9 describes the connection between the specific leakage and the specific pressure with good accuracy. The exponent m has a value of 0.72 and the leakage constant L has a value of $10^{-4.7}$. Furthermore, it is evident that the Reynolds number does not affect the specific leakage.

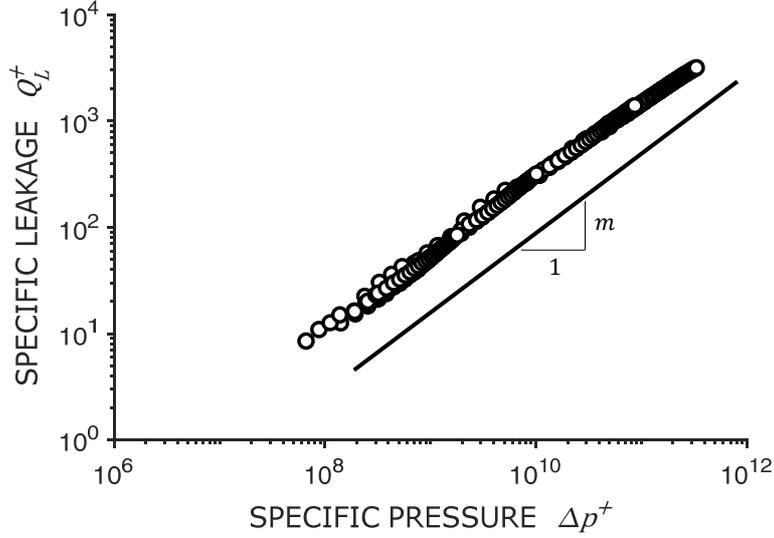


Figure 2: Specific leakage versus specific pressure in a double logarithmic diagram.

In the next step, the accuracy of the leakage model is determined. For this purpose, the relative deviation of the leakage model $\delta(Q_L)$ and the relative deviation of the volumetric efficiency model $\delta(\eta_{vol})$ (see eqn. 8 and 9) are considered. Those relative deviations are defined as follows

$$\delta(Q_L) := \frac{|Q_{L,model} - Q_{L,measurement}|}{m}, \quad (13)$$

$$\delta(\eta_{vol}) := \frac{|\eta_{vol,model} - \eta_{vol,measurement}|}{\eta_{vol,measurement}}.$$

The model parameters L and m are identified using all available measurement points. Hence, the relative deviation is a measure for the accuracy of the leakage model to represent the screw pump characteristic. Figure 3 shows both relative deviations between the measurement points and the model.

The relative deviation of the leakage model is below 10 %, predominantly. Solely for low pressures, the deviation increases. Consequently, the relative deviation of the volumetric efficiency model is very low and mostly below 2 %.

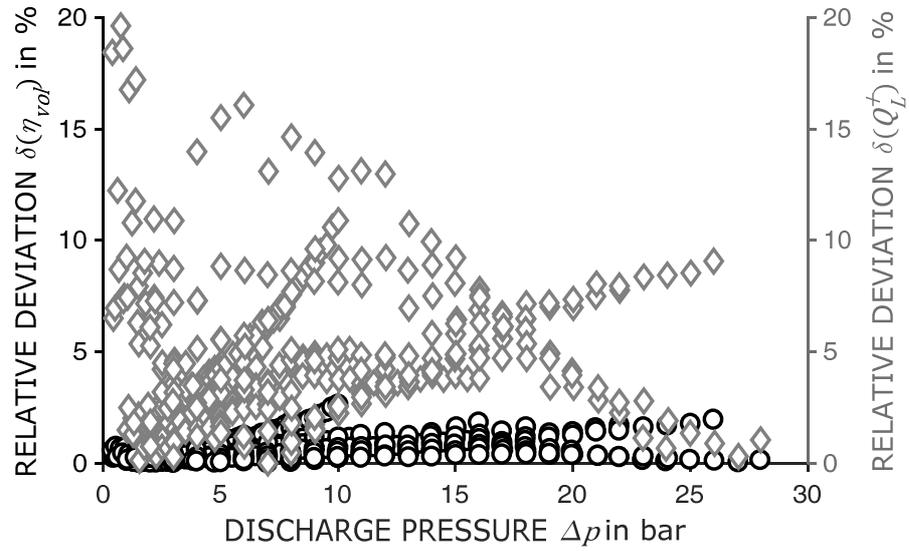


Figure 3: Relative deviation of the leakage model (grey diamond marker) and the volumetric efficiency model (black circle marker) versus the discharge pressure.

The following section presents the validation of the dimensionless friction torque model. The mathematical description is given in equation 11. Similar to the leakage, the model parameters of friction torque model, i.e. the loss coefficients C , R_μ and R_ρ , are identified using all available measurement points: C has a value of 6.08×10^{-4} , R_μ has a value of 2.87×10^4 and R_ρ has a value of 6.35

Figure 4 shows a 3-dimensional logarithmic diagram with all measured operation point and the friction torque model surface as a function of the Reynolds number and the specific pressure.

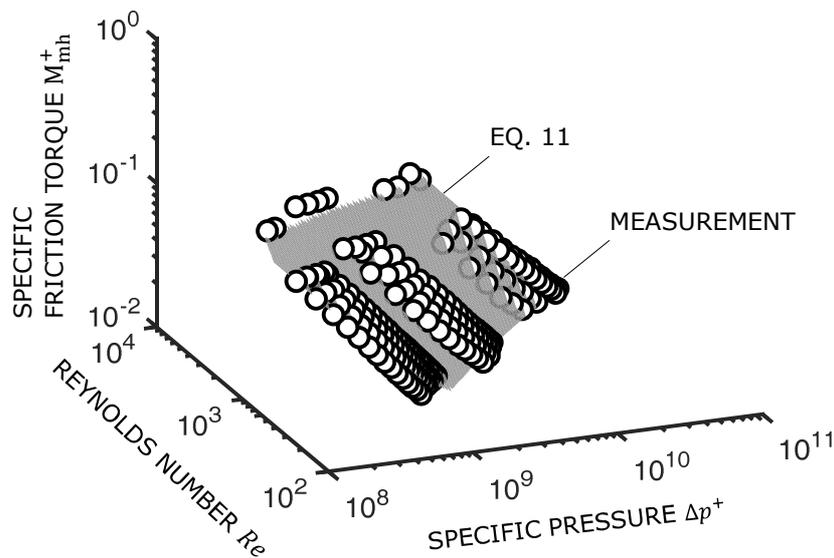


Figure 4: 3-dimensional logarithmic diagram of specific friction torque vs. Reynolds number and specific pressure.

The accuracy of this model description can be determined in the same way as the accuracy of the leakage model. Hence, the relative deviation of the friction torque model $\delta(M_{mh})$ and the relative deviation of the mechanical-hydraulic efficiency model $\delta(\eta_{vol})$ (see eqn. 8 and 11) are defined as

$$\delta(M_{mh}) := \frac{|M_{mh,model} - M_{mh,measurement}|}{M_{mh,measurement}},$$

$$\delta(\eta_{mh}) := \frac{|\eta_{mh,model} - \eta_{mh,measurement}|}{\eta_{mh,measurement}}.$$
(14)

All model parameters of the friction torque model, namely C , R_μ and R_ρ are identified using all available operating point excluding the hydraulic oil of viscosity class 2. For operating points with a very low viscosity, the mechanical-hydraulic efficiency is mostly above 96 %, and therefore the friction torque is low and the measurement uncertainty increases. For this reason, these measurement points are neglected in the model parameter identification.

The relative deviation of the friction model is below 15%, predominantly. The relative deviation of the mechanical-hydraulic efficiency is also very low and mostly below 2%. Due to the high mechanical-hydraulic efficiency even high deviation of the loss model have only little influence. In comparison with the leakage model, the friction torque model is less accurate. Nonetheless, the loss models and the corresponding partial efficiency models are able to represent the three-spindle screw pump characteristic in the given operating range.



Figure 5: Relative deviation of the friction torque model (grey diamond marker) and the mechanical-hydraulic efficiency model (black circle marker) versus the discharge pressure.

The second research question is relevant for the practical use of similarity. Measuring pump characteristics for more than one or two commonly used pumping fluids is costly for the manufacturers. Therefore, when selecting a suitable spindle screw pump for a specified application, they are often confronted with a lack of comparable measurement data for the desired combination of operating conditions and pumping fluid viscosity. For this reason, the practical benefit of the volumetric efficiency model and the mechanical-hydraulic efficiency model is examined on the basis of the following case study: The pump characteristic measured with a hydraulic oil of viscosity class 7 are used to identify the five model parameters L , m , C , R_μ and R_ρ . Subsequently, the calibrated models are used to estimate the pump characteristic at a hydraulic oil of viscosity class 22.

Figure 6 shows the estimation of the volumetric efficiency together with the measurements at 22 cSt at four different Reynolds numbers. As can be seen, the estimations match the measurements with high accuracy.

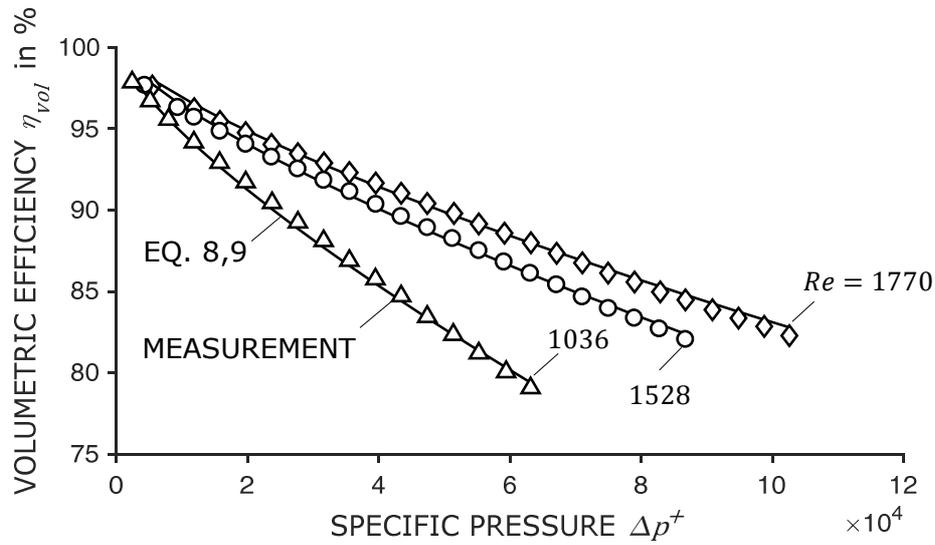


Figure 6: Efficiency model and measurements at 22 cSt.

Figure 7 shows the estimations of the mechanical-hydraulic efficiency together with the measurements at 22 cSt at four different Reynolds numbers. Similar to Figure 6, the estimation of the mechanical-hydraulic efficiency matches the measurements with high accuracy, too.

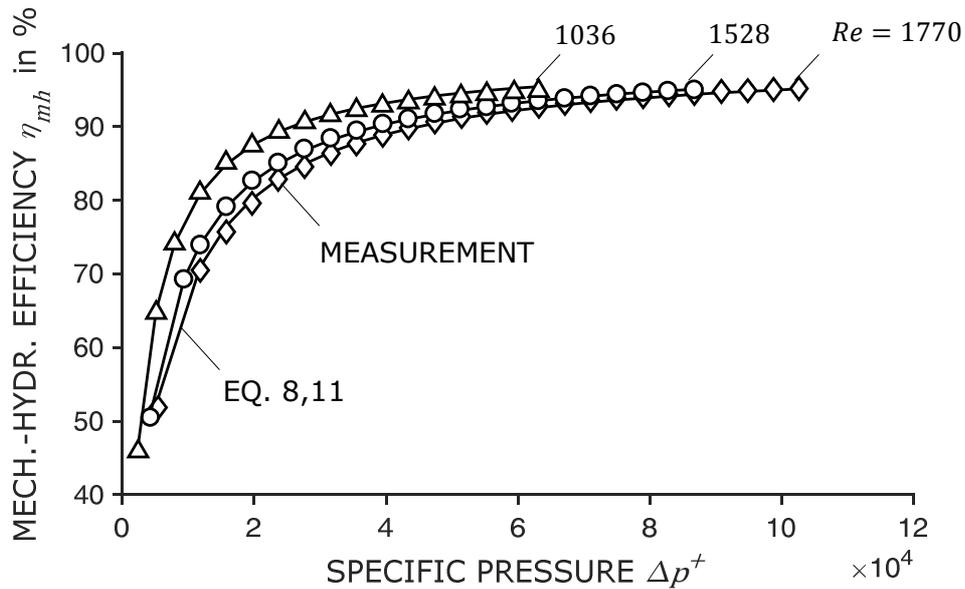


Figure 7: Friction torque model and measurements at 44 cSt.

In summary, the above results prove that the application of similarity and of the presented model is useful and beneficial to calculate the pump characteristics at different viscosities. Following the approach of this paper, the prevalent lack of comparable measurement data for desired combinations of operating conditions and pumping fluid viscosity can be compensated.

The following section discusses the third research question regarding the representation of manufacturing tolerances based on the relative gap ψ . For this reason, the measurements of all identical three-spindle screw pumps with and without modified gaps (see. tab. 2) are evaluated under the following considerations: The pump characteristic of the original three-spindle screw pump serves as a reference with an assigned relative gap $\psi := 1$. The other three-spindle screw pumps with

modified gaps represent deviations from this reference due to manufacturing tolerances. For the spindle screw pumps with modified gaps the relative gap is determined on the basis of a relative comparison with the reference pump. This approach was already applied by the authors (cf. [1]) comparing different positive displacement pump types.

However, at this point it is to be emphasized that the study of this paper is a hypothetical one, as the real manufacturing tolerances are actually lower than the gap modification of the pumps [4]. Nonetheless, the approach is transferable to pumps of a real production process.

Against this background, figure 8 shows the double logarithmic diagram of specific leakage versus specific pressure for all measured three-spindle screw pumps. For reasons of clarity, all spindle screw pumps are represented by their model curve (Eq. 9). All model curves of the spindle screw pumps with modified gaps, modification 1 – 7 (see tab. 2) lie above the original spindle screw pump. The model parameter m is nearly constant for all spindle screw pumps varying between 0.71 and 0.74. Assuming the model parameter m is equal for all spindle screw pumps with $m = 0.72$ (original spindle screw pump), the relative gap ψ for all spindle screw pumps with modified gap is determined on the basis of a relative comparison with the original spindle screw pump. Hence, the relative gap is a measure for the relevance of the corresponding modification. Table 3 lists the relative gap for all three-spindle screw pumps.

For manufacturers, the above approach can be used as follows: In the monitoring process of manufacturing quality newly manufactured pumps can be rated against a referential pump characteristic. Usually a manufacturer proves that each pump delivers the required volume flow rate at the customer's operating point, i.e. discharge pressure and rotating speed, by means of a single measurement. Is a pump's performance significantly worse than expected, the deviation results from the sum of all manufacturing uncertainties of those geometric sizes that influence the gap heights of a pump. A back tracking of a possible exceedance of a manufacturing tolerance of a single geometric size is time consuming and costly and, furthermore, not always effective.

On the other side, measuring the whole characteristic curve of newly manufactured pumps on a random basis additionally, the manufacturing process can be monitored reasonably by only one single parameter: the relative gap ψ of each random sample. The relative gap represents the accumulated manufacturing (including assembly) uncertainty that influence the gap heights in a pump. Consequently, an evaluation of the manufacturing process is possible based on the distribution of the relative gap of all random samples.

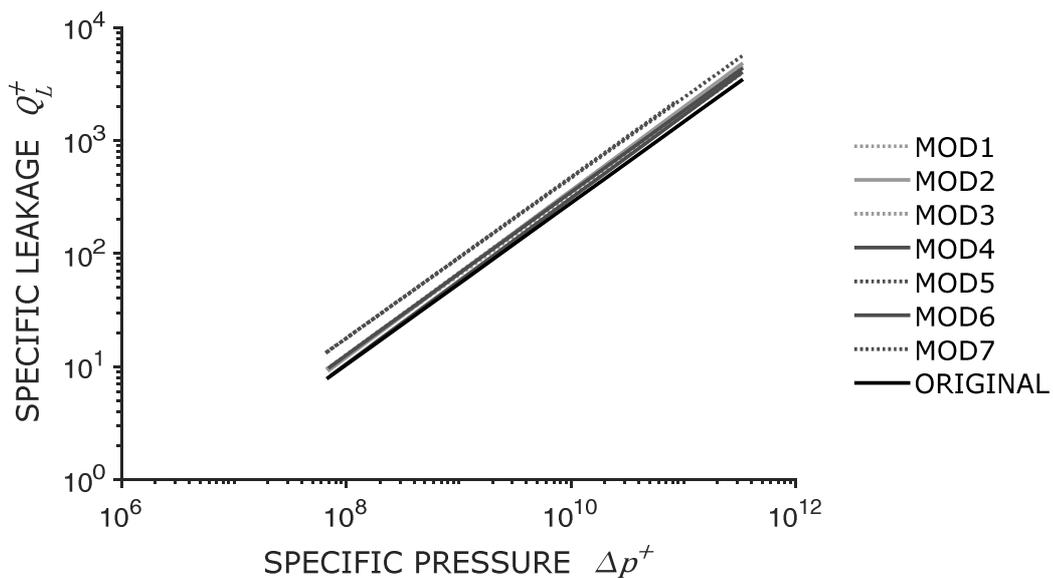


Figure 8: Specific Leakage versus specific pressure in a double logarithmic diagram.

Table 3: Relative gap ψ for all spindle screw pumps.

MODIFICATION	RELATIVE GAP ψ
original	1 (by definition)
1	1.12
2	1.13
3	1.08
4	1.05
5	1.28
6	1.11
7	1.27

6. Conclusion and Outlook

The validation results prove that the presented model approach based on similarity and dimensional analysis are useful to represent a pump's characteristic over a wide operating range. The estimation of the volume flow rate and power consumption at a changed viscosity show high accuracy and are of high value for the manufacturer and operators. Furthermore, this paper presents a new approach to monitor and to evaluate the manufacturing uncertainty on the basis of the model parameter relative gap ψ .

An experimental study on model series of three spindle screw pumps will be carried out in the second half of 2018 at TU Darmstadt. The focus will be on similarity based scaling of spindle screw pump characteristics when varying the displacement volume. These investigations will be carried out within the research project No. 10887/16, funded by budget resources of the Bundesministerium für Wirtschaft und Technologie (BMWi) approved by the Arbeitsgemeinschaft industrieller Forschungsvereinigungen "Otto von Guericke" e.V. (AiF).

Acknowledgments

We would like to thank the Leistritz Pumpen GmbH for funding and their substantial technical contributions to carry out the investigations presented in this paper.

Nomenclature

d_A	head circle diameter of spindle
dsc	drive screw
\bar{h}	average height of the lubrication gap
H	pitch of spindle
isc	idle screw
l	length of spindle
L	dimensionless leakage coefficient
m	exponent for power law of specific leakage
M_{hyd}	hydraulic torque
M_{mh}	friction torque
M_{mh}^+	specific friction torque
M_S	shaft torque
n	rotational speed
Δp	discharge pressure
Δp^+	specific pressure
p_{in}	pressure at inlet of pump
p_{out}	pressure at outlet of pump
P_S	shaft power
Q	volume flow rate
Q_1	volume flow rate at inlet of pump

Q_L	Leakage
Q_L^+	specific leakage
Q_{th}	theoretical volume flow rate
C	pressure-related loss coefficient
R_μ	viscous friction-related loss coefficient
R_ρ	pressure-related loss coefficient
Re	Reynolds number
\bar{s}	Average gap height
V	displacement volume
$\delta()$	relative deviation of a quantity
η	total efficiency
η_{mess}	measured efficiency
η_{mh}	mechanical-hydraulic efficiency
η_{vol}	volumetric efficiency
κ	compressibility
μ	dynamic viscosity
ν	kinematic viscosity
ρ	Density
ψ	relative gap
ψ/ψ_{ref}	class of gap

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