Analytical method towards an optimal energetic and economical wind-energy converter

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Abstract

An innovative concept to convert wind energy in wind-rich ocean regions is presented and analyzed. This concept involves the operation of wind-propelled vessels equipped with hydrokinetic turbines so that the kinetic energy of the water flow relative to the hydrokinetic turbine is converted into electricity. This electric power then is used to split sea water electrolytically into hydrogen and oxygen. A currently missing upper limit of energy conversion of the proposed system is presented, which is based on axiomatic conversion laws. To ensure the requirement of economic profitability the energetic description is widened by an economic description. Normally, the technical analysis precedes the economic assessment of a system. In contrast, a holistic approach is presented which yields the techno-economic optimal design as a trade-off between energetic efficiency and economic profitability. For system optimization Pareto optimization is applied to obtain convergence of the energetic and economic system quantities. The Pareto-frontier, defined as the multitude of all optimal energetic and economical systems, is presented. The application of this analysis shows that typical sailboats with 50 m² sail area operating in 10 m/s winds deliver a mechanical power output of about 13 kW and sailing ships with 3200 m² produce about 1 MW.

1. Introduction

The transformation of the fossil fuel-based energy supply system into a sustainable system is one of the major societal challenges of the twenty first century. Renewable energy systems need to be developed by meeting the objectives of energetic efficiency, robustness, and affordability. In this paper an analytical optimization of an energy conversion system is demonstrated by considering its energetic and economical aspect. The concept converts wind energy into electric energy using sail-powered vessels. A hydrokinetic turbine is used to convert the relative kinetic water energy into electricity. In a further step the electricity is used to produce hydrogen using sea water. Using electrolysis the electrical energy is converted into transportable and storable chemical energy. The basic concept seems to have been first proposed by Salomon [1] as long ago as 1982, followed by Meller [2], Holder [3], and Gizarra [4], but the first more quantitative analyses have been presented only during the past six years by Platzer [5–7] and Kim [8]. The main advantage of this type of energy supply system is the substitution of air by water for power production. While the vessel is driven by wind power, the hydraulic turbine is driven by the water flow. For the same turbine shaft power the diameter ratio $D_l/D_g$ scales with the square root of the density ratio $\rho_l/\rho_g$ where the subscripts g and l indicate “gas” and “liquid”

$$\frac{D_l}{D_g} = \sqrt{\frac{\rho_g}{\rho_l}} = 3.5\%.$$  

Equation (1) shows that the diameter of a hydrokinetic turbine exposed to the same flow speeds is only 3.5% that of a wind turbine to deliver the same amount of power with the water density $\rho_l = 1000$ kg/m³ and the air density $\rho_g = 1.2$ kg/m³. The reduced hydrokinetic turbine size therefore reduces the investment costs, which scale with the component size. The electric energy produced by the turbine electricity needs to be stored using an adequate energy storage technology, like the conversion into chemical energy. It is reasonable to use the electricity to convert sea water into hydrogen and oxygen using commercially available electrolyser. The hydrogen is then stored in tanks in the hull of the vessel. In the
present paper a standard sailing ship configuration is assumed. The sails can either be standard, semi-rigid or rigid sails. Their aerodynamic performance is specified by the sail area and the sail lift coefficient, thus implying that the actual lift coefficient is ultimately determined by accounting for the sail drag. Ouchi et al. [10] and Smulders [11] have shown the feasibility of rigid wing sails with lift coefficients between 1.8 and 2.5. Kim and Park [8] proposed the use of parawings to exploit the larger wind power at altitude. Later, Kim and Park [9] showed the economic profitability of the concept through case studies. Also, the hydrodynamic drag of sailing vessels can be minimized by the use of hydrofoils.

2. Methodology and modeling

In the following sections only conventional single or multi-hull vessels are considered where the hydrodynamic drag is specified by the wetted area \( A \) and the hydrodynamic drag coefficient \( C_D \). Also, the drag due to additional control surfaces and hydrofoils needed for ship stabilization is neglected. As can be seen in Fig. 1, the sail area is denoted as \( A_s \) and the turbine area as \( A_T \). The relative wind velocity \( w \) produces the aerodynamic lift force \( L \), which can be divided into the thrust \( T \) and the heeling force \( S \). Due to the thrust the vessel speed \( V \) will adapt, which is also the inflow velocity of the hydrokinetic turbine under the assumption of no water velocity. The outflow velocity of the turbine is specified through the axial induction factor \( \zeta \), which is defined as the ratio of the inflow and outflow velocity and is therefore a measure of the flow deceleration through the turbine. The generated electricity is used to split sea water into hydrogen, which is compressed and stored in tanks. For a mathematical description the energy converter is separated into two functional units, each of them modeled by an axiomatic conversion law. The energy system, shown in Fig. 1, can be divided into the vessel propulsion system and the hydrokinetic turbine which is described through the disk actuator theory.

Without loss of generality a rigid sail is considered which produces the thrust \( T \) needed to propel the vessel. The thrust is balanced by both the drag force of the vessel \( W = A_V C_D V^2 g_0 / 2 \) and the resistance force of the turbine \( W_T \), leading to the force balance \( T = W + W_T \). The drag force of the turbine is given by the momentum balance for the turbine \( W_T = \Delta p A_T \) with the pressure difference determined by Bernoulli’s equation \( \Delta p = \frac{V^2}{2} (1 - \zeta^2) g_0 / 2 \). For the first approach, the influence of the free surface is neglected. For a more detailed model the interaction of the free surface with the turbine should not be neglected as shown by the first author [12,13] and Metzler [13]. Also the tangential induced velocity, caused by the circulation on sail and turbine, are neglected. The theory considering the rotation of the stream tube is shown in detail by Schmitz [14] for wind turbines. Also, Fig. 2 shows the vessel and wind speed as well as the forces on a sail section.

Neglecting friction and induced drag for a first approach, the thrust force is given by the sine of the lift force

\[ T = \sin \beta \frac{q_s}{2} A c_l(\alpha) w^2, \]

\( \beta \) denotes the relative wind direction as shown in Fig. 2. \( w \) denotes the relative wind speed and \( c_l \) the lift coefficient, which is a function of the angle of attack \( \alpha \). The cosine rule gives a relation between the velocities at the sail section

\[ w^2 = V^2 + c^2 - 2 V c \cos \alpha_0. \]

where \( c \) denotes the absolute wind velocity and \( \alpha_0 \) the absolute wind direction. The absolute wind velocity is the vector sum of the relative wind and the vessel speed \( \vec{v} = \vec{V} + \vec{w} \). By introducing the dimensionless absolute velocity \( \nu := V/c \), the dimensionless air density \( \rho := \rho_0 / \rho_1 \) (measured in multiplies of the water density) the dimensionless thrust is given by

\[ \frac{T}{\nu^2 c A / 2} = \sin \beta c_l \left( \nu^2 + 1 - 2 \nu \cos \alpha_0 \right). \]

By applying the dimensionless quantities, we end up with the dimensionless force balance

\[ \sin \beta c_l \nu^2 \left( \nu^2 + 1 - 2 \nu \cos \alpha_0 \right) = \nu^2 c_D a_V + \nu^2 \left( 1 - \nu^2 \right) a_T, \]

(5)

giving a relation between the dimensionless vessel speed and the axial induction factor of the turbine. Here the dimensionless areas are \( a_V := A_V / A \) and \( a_T := A_T / A \). Equation (5) is equivalent to

\[ \nu^2 - 2 \nu \cos \alpha_0 + q = 0, \]

(6)

with the substitution

![Fig. 1. Free-body representation of the energy ship.](image1)

![Fig. 2. Wind speed and forces on a sail section.](image2)
\[ q := \frac{\sin \beta \ c_L \ g}{\sin \beta \ c_L \ g - c_D \ a_V - (1 - \cos^2 \gamma) a_T} < 1. \]  
(7)

Hence the dimensionless vessel speed is

\[ v_{1/2, F} = q \cos \alpha_0 \sqrt{a^2 \cos^2 \alpha_0 - q}. \]  
(8)

giving the relationship between the dimensionless vessel speed and all influencing physical parameters. Only the positive solution for the dimensionless vessel speed is physically reasonable. The negative solution represents the dimensionless vessel speed in inverted direction. The advantage of using dimensionless quantities is obvious: The amount of physical parameters is reduced, so that we end up with a simpler equation to describe the system. Because Equation (8) is a result of the force balance, it is denoted as \( v_F \).

Through this description the vessel speed is a function of seven parameters

\[ v_T = v_T(\alpha_0, \beta, c_L, c_D, a_V, a_T, \gamma), \]  
(9)

which characterize the wind direction, sail and vessel forces and turbine output. Only six of them are independent parameters. The relative wind direction will adapt in dependence of the vessel speed. Another equation, which can be found in the velocity triangle of Fig. 2, has to capture this relation. By applying the basic triangle equations for the velocities, we can find for the vessel speed \( V \)

\[ V = c \cos \alpha_0 + w \cos \beta. \]  
(10)

with the velocity ratio \( w/c = \sin \alpha_0 / \sin \beta \) the dimensionless vessel speed is

\[ v_V = c \alpha_0 + \cot \beta \sin \alpha_0. \]  
(11)

This dimensionless vessel speed is denoted as \( v_V \), because it results of the velocity triangle at the sail section. The required equality of these two calculated dimensionless vessel speeds \( v_T = v_V \) leads to an expression for \( \zeta \) as a function of the relative wind direction

\[ \zeta = \sqrt{\frac{\sin \beta c_L g}{a_T} \left( \frac{2 \nu c \cos \alpha_0 - 1}{\nu V^2} \right) + \frac{c_D}{a_T} + 1}. \]  
(12)

Equation (12) is a relation between the axial induction factor and the resulting relative wind direction. For maximal turbine resistance (\( \zeta = 0 \)) the maximal angle \( \beta \) will adapt to the minimal vessel speed. For a hypothetical turbine (\( \zeta = 1 \)) the relative wind direction \( \beta \) will minimize, representing the maximal vessel speed for the given wind speed. The permissible solution space of \( \zeta \) is limited to \( 0 < \zeta < 1 \). Therewith the vessel speed is a function of the six independent parameters

\[ v = v(\alpha_0, c_L, c_D, a_V, a_T, \gamma). \]  
(13)

3. Optimal system design

The energy system, shown in Fig. 1, can be divided into the vessel propulsion system, the hydrokinetic turbine and the energy storage system. The first author showed that because of the interaction between these subsystems the optimization needs to comprise the total system [15]. As indicated in Fig. 3, the available wind power, proportional to the cube of the absolute wind speed, is converted into electric power and ultimately stored in the energy storage system. In terms of an energy source - sink model the available wind power is the energy source and the energy sink is the storage system. The system describes the energy transmission, so that an optimization improves the transmission from the energy source to the sink. The energetic block-diagram is extended by economical quantities. The yearly investment costs of each component as well as the yearly revenue \( R \) are shown in Fig. 3. The yearly vessel investment is denoted as \( I_V \), of the turbine as \( I_T \) and of the storage system as \( I_S \). Operation and maintenance costs are neglected in this first approach. As can be seen in Fig. 3 the power flow and the cost flow converge in the revenue. The more efficient the system operates, the more hydrogen is produced, which can be finally sold.

It is important to be aware of the difference between the system boundaries shown in Fig. 3. A fluid system consists at all times of an inlet flow, an outlet flow and a machinery connecting the two flows. Consequently, the energy storage system is not included in the fluid system. The ratio of the entering and exiting flow velocities is a quantity which defines the converted kinetic energy of the flow. The energetic optimization therefore is performed considering only the fluid system, whereas the economic optimization requires the consideration of the whole system.

3.1. Energetic optimization

For energetic optimization the output power needs to be compared with the available input power. Prandtl has shown that the lift generated by a finite span wing is due to giving an air mass flowing through a circular area of a diameter equal to the wing span a downwash velocity equal to the induced velocity in the Trefftz plane infinitely far downstream. Hence the available input power is given by the kinetic energy flowing through a circle with a radius equal to the wing’s half-span. However, another commonly used area on sailing vessels is the sail area. This suggests to introduce a coefficient of performance defined as

\[ C_P := \frac{P_S}{P_{avail}} \]  
(14)

where \( P_S = \eta_T \Delta P_{AT} \) is the turbine output shaft power and \( P_{avail} = \delta E A/2 \) is the kinetic power flowing through the sail area. \( \eta_T \) denotes as usual the turbine efficiency which accounts all dissipative losses within the turbine. It is important to remember that the coefficient of performance is not identical with the energy conversion efficiency because the true available power is the one deduced by Prandtl. With the average current velocity through the
turbine $\mathcal{V} = \text{cn}(1 + \zeta)/2$, according to Betz [16] the coefficient of performance becomes

$$C_p = \frac{\eta_{\text{T}} A \mu(T) \text{cn}(1 + \zeta)/2}{\rho \mathcal{V}^3 A/2}. \hspace{1cm} (15)$$

With respect to the dimensionless quantities we end up with

$$C_p \frac{1}{\eta_{\text{T}}} = \frac{\rho \mathcal{V}^3 \mu(T) (1 - \zeta^2)}{1 + \zeta^2}. \hspace{1cm} (16)$$

The term $(1 - \zeta^2)(1 + \zeta^2)/2$ is already known from Betz’ wind turbine analysis. However, in contrast to Betz’ analysis the axial induction factor given in Equation (12) contains the induction factor of performance. The efficiency factor of the generator is defined as the ratio of the chemical power of hydrogen and the electrical power consumption of the electrolyzer $\eta_{\text{Gen}} := \frac{H_2}{H_{\text{H}_2}}/P_{\text{Gen}}$. The mass flow of hydrogen is denoted as $m_{H_2}$ and the caloric value of hydrogen as $H_{\text{H}_2}$. The average hydrogen price is $p_{H_2}$, measured in €/kg or any other currency. The total capital investment consists of the investment costs of the vessel, turbine and the investment costs of the energy storage technology

$$C = I_V + I_T + I_{\text{St}}. \hspace{1cm} (22)$$

If we assume the investment costs as specific area related costs, the total investment can be calculated as

$$C = A_V p_V + A_T p_T + A_{P_{\text{St}}}. \hspace{1cm} (23)$$

with $p_V := I_V/A_V$, $p_T := I_T/A_T$ and $p_{P_{\text{St}}} := I_{P_{\text{St}}}/A$. The costs of the energy storage technology are related to the sail area, because the capacity of the storage technology is proportional to the converted power and therewith to the sail area. The economic profit function is then composed as

$$\dot{G} = \frac{\theta g}{2H_{\text{H}_2}}C_p(a_V, \alpha_T)\eta_{\text{Gen}}\eta_{\text{Elec}} P_{H_2} + CRF(A_V p_V + A_T p_T + A_{P_{\text{St}}}). \hspace{1cm} (24)$$

By relating this function to the sail area one obtains

$$\frac{\dot{G}}{A} = \frac{\theta g}{2H_{\text{H}_2}} C_p(a_V, \alpha_T)\eta_{\text{Gen}}\eta_{\text{Elec}} P_{H_2} + CRF(a_V p_V + a_T p_T + P_{P_{\text{St}}}). \hspace{1cm} (25)$$

To determine the system which maximizes the economical profit the profit function needs to be maximized by considering every parameter. Because the yearly revenue as well as the yearly costs depend on the sail and vessel wetted areas the optimization can be limited to these two parameters. The necessary conditions are

$$\frac{\partial}{\partial a_V} \left( \frac{\dot{G}}{A} \right) = 0, \hspace{1cm} (26)$$

$$\frac{\partial}{\partial \alpha_T} \left( \frac{\dot{G}}{A} \right) = 0, \hspace{1cm} (27)$$

which leads to

$$\frac{\partial C_p(a_V, \alpha_T)}{\partial a_V} \frac{\theta g}{2H_{\text{H}_2}} C_p(a_V, \alpha_T)\eta_{\text{Gen}}\eta_{\text{Elec}} P_{H_2} = CRF p_V. \hspace{1cm} (28)$$

The generator efficiency factor $\eta_{\text{Gen}}$ needs to be considered, because it is not included in the previous definition of the coefficient of performance. The efficiency factor of the electrolyzer is defined as the ratio of the chemical power of hydrogen and the electrical power consumption of the electrolyzer $\eta_{\text{Elec}} := \frac{H_2}{H_{\text{H}_2}}/P_{\text{Gen}}$. The mass flow of hydrogen is denoted as $m_{H_2}$ and the caloric value of hydrogen as $H_{\text{H}_2}$. The average hydrogen price is $p_{H_2}$, measured in €/kg or any other currency. The total capital investment consists of the investment costs of the vessel, turbine and the investment costs of the energy storage technology

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with $p_V := I_V/A_V$, $p_T := I_T/A_T$ and $p_{P_{\text{St}}} := I_{P_{\text{St}}}/A$. The costs of the energy storage technology are related to the sail area, because the capacity of the storage technology is proportional to the converted power and therewith to the sail area. The economic profit function is then composed as

$$\dot{G} = \frac{\theta g}{2H_{\text{H}_2}}C_p(a_V, \alpha_T)\eta_{\text{Gen}}\eta_{\text{Elec}} P_{H_2} + CRF(A_V p_V + A_T p_T + A_{P_{\text{St}}}). \hspace{1cm} (24)$$

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\[
\frac{\partial C_\text{P}}{\partial \alpha_T} = C_\text{P, max} \frac{C_\text{F}}{C_\text{F, max}} \frac{\eta_{\text{Gen}}}{{\eta_{\text{Gen}}}^\text{Elec}} P_{\text{H}} = CRF \; P_{\text{T}}.
\] (29)

With these two equations the optimal sail and wetted areas \(\alpha_{\text{U, opt}}\) and \(\alpha_{\text{T, opt}}\) can be estimated to provide the maximal profit. Table 1 summarizes the optimization requirements. The energetic optimization delivers the optimal axial induction factor and therewith the information which will influence the control system. The economical optimization makes it possible to specify the vessel and turbine design.

The combined energetic and economical optimization yields the economic profit function. In this way this function becomes the objective function of the Pareto-optimization problem. The sum of all systems which satisfy the requirements listed in Table 1 is called the Pareto-frontier.

### 4. Results and discussion

In this section the influence of several parameters on the coefficient of performance and the economical profit are shown and discussed. The Pareto-frontier, as the multitude of all optimal systems, is shown.

#### 4.1. Coefficient of performance

The parameter influences on the energetic system quantity, the coefficient of performance, are analyzed to determine the optimal energy conversion conditions. For example, assume a constant average absolute wind velocity of \(c = 10 \text{ m/s}\), a lift coefficient \(c_{\text{l}} = 1.5\) and a drag coefficient \(c_{\text{D}} = 0.01\), unless otherwise stated. To determine the optimal ship course the optimal coefficient of performance can be calculated for varying absolute wind directions. Fig. 4 shows the result. Since energy can only be converted for an axial induction factor \(0 < \zeta < 1\) the limiting lines for \(\zeta = 0\) and \(\zeta = 1\) are also plotted in Fig. 4.

The line \(\zeta = 0\) represents the maximal turbine drag force whereas the line \(\zeta = 1\) corresponds to the case of a vessel without turbine and therefore zero turbine drag. Energy can be converted only between these two lines so that \(C_\text{P} > 0\). The maximal coefficient of performance is obtained for an absolute wind direction of approximately \(\alpha_0 \approx 107^\circ\). Vector addition of the vessel velocity to the wind vector yields a relative wind direction of approximately \(\beta \approx 55^\circ\). Cruising without turbine at the same absolute wind angle increases the vessel speed and therefore decreases the apparent wind angle to approximately \(\beta \approx 40^\circ\) (as can be seen by the intersection with the line \(\zeta = 1\)), whereas cruising with maximal power extraction (intersection with the line \(\zeta = 0\)) decreases the vessel speed and therefore increases the apparent wind angle to approximately \(\beta \approx 65^\circ\). This behavior is further elucidated in Fig. 5.

In Fig. 5 (left) the dimensionless aerodynamic lift and its thrust and heeling force components are plotted as a function of the absolute wind direction. The thrust force is seen to be a maximum at approximately \(\alpha_0 \approx 107^\circ\), thus confirming the determination of this optimal angle in Fig. 4. It is also of interest to note that at an absolute wind direction of approximately \(\alpha_0 \approx 70^\circ\), the heeling force is zero. This corresponds to a apparent wind angle \(\beta \approx 90^\circ\) where the aerodynamic lift points in the ship direction. In this case no counterweight or additional hydrofoil is needed for roll stability. In Fig. 5 (right) the optimal coefficient of performance and the corresponding vessel speed are plotted as a function of absolute wind angle, showing that the vessel speed reaches values close to half the wind speed at the maximal coefficient of performance.

Fig. 6 shows the coefficient of performance for varying turbine areas. The optimal axial induction factor \(\zeta\), providing the maximal value of the coefficient of performance, is plotted for each turbine area.

It is shown that the coefficient of performance at first increases rapidly with an increasing turbine area. For further increasing turbine areas the coefficient of performance further increases only slightly. Nevertheless, the coefficient of performance maximizes for large turbine areas. Fig. 7 summarizes the influence of the drag coefficient of the vessel and the lift coefficient of the sail for the coefficient of performance. Since the vessel can be considered as a mechanical dissipator, the coefficient of performance can be increased by minimizing the drag coefficient and maximizing the lift coefficient. The shape and type of the vessel determines the drag coefficient and the maximal speed of the vessel, as mentioned by Püschl [18].

The drag force of the vessel depends on the vessel speed, the drag coefficient and the wetted vessel area. Fig. 8 (left) shows the impact of varying the dimensionless vessel area on the coefficient of performance. It is seen to decrease for increasing vessel areas, due to higher energy dissipation on the vessel hull.

Fig. 8 (right) shows the axial induction factor for varying vessel areas. The axial induction factor for the optimal coefficient of performance is also plotted. The Figure shows that the axial induction factor changes for varying vessel areas. Also, the optimal axial induction factor differs from Betz’ prediction for wind and hydrokinetic turbines in unbounded low [16] that the highest coefficient of performance is reached if the velocity of the outlet flow is 1/3 of the inlet flow. The difference between Betz’ fluid system and the fluid system of the present paper is the dependence of the latter system on many parameters which enable the determination of an optimal vessel speed and heading. Therefore two control systems are required to maximize the coefficient of performance.

<table>
<thead>
<tr>
<th>Optimization criteria and the system consequences.</th>
<th>Optimization criteria</th>
<th>Energetic</th>
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<tr>
<td>Optimized parameter Requirements</td>
<td>(\zeta_{\text{opt}})</td>
<td>(\alpha_{\text{U, opt}}, \alpha_{\text{T, opt}})</td>
<td>(\frac{\partial\zeta}{\partial\alpha_T} = 0)</td>
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<tr>
<td>Consequence</td>
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Fig. 4. Coefficient of performance for varying vessel courses.

- A closed loop control sail pitch control system for maximal sail thrust.
- A closed loop control turbine power output control system to regulate the axial induction factor of the turbine. This parameter depends on the dynamic torque of the turbine, which, in turn, can be influenced by the generator.
4.2. Economical profit

For the economic analysis the profit function is used to determine the optimal sail, turbine and vessel areas. The area related prices are assumed to be constant, so that the total prices grow linearly with the reference area. The number of loans and the interest on the capital determine the Capital Recovery Factor, which is a measure of the yearly costs. Obviously, the yearly costs depend on the loan duration. A longer loan decreases the yearly costs, whereas a shorter one increases them. The advantage of a shorter loan is the smaller amount of interest payment. Fig. 9 shows the optimal ship design for two different rates of interest. It is seen that a greater capital interest leads to greater yearly costs and thereby to a shift to smaller turbine areas.

The optimal turbine area for every vessel area is also plotted in Fig. 9. This optimal turbine area delivers the maximal profit and therefore represents the optimal trade-off between revenue (energetic efficiency) and costs. These optimal turbine areas forms the Pareto-frontier. An important parameter which determines the revenue of the economic profit function is the price of hydrogen. Fig. 10 shows the impact of varying the hydrogen price on the optimal dimensionless turbine areas. While the loan amount and the rate of interest influence the yearly costs, the hydrogen price influences the economical revenue. It is seen that an increase in hydrogen price postpones the optimal design to larger turbine areas. Also, increasing hydrogen prices expand the possible range of profitable energy ship designs.

4.3. Exemplary design of an optimal energy ship

In this section a specific example of an energetic and economic optimal energy ship is shown. One has to note that the method is suitable to optimize all possible energy ships. A typical sailboat with a sail area of $A = 50 \, \text{m}^2$ and a wetted area of $A_V = 20 \, \text{m}^2$ is assumed. Hence the dimensionless vessel area is $a_V = 0.4$. The lift and drag coefficients are 1.5 and 0.01, respectively. Further the price data of Fig. 9 are assumed with a rate of interest of $i = 4\%$ and a number of loan payments $n = 20$. According to Fig. 9 the optimal dimensionless turbine area can now be calculated to be $a_T = 0.0124$ and therefore the optimal turbine area becomes $A_T = 0.62 \, \text{m}^2$. A yearly revenue of $G = 14165 \, \text{e}/\text{a}$ is gained if the hydrogen is sold for a price of $p_{H_2} = 10 \, \text{e}/\text{kg}$. In Fig. 11 the maximal coefficient of performance $C_{p_{\text{max}}}$ is marked.

As shown in Fig. 12 the optimal axial induction factor for the chosen design parameter is $\zeta_{\text{opt}} = 0.81$. Therewith the apparent wind angle is $\beta = 50^\circ$. 

**Fig. 5.** Dimensionless Forces on the sail section (left), coefficient of performance and corresponding vessel speed for varying vessel courses (right). All forces are scaled by $q c^2 A/2$.

**Fig. 6.** Coefficient of performance for varying dimensionless turbine areas.

**Fig. 7.** Coefficient of performance for varying drag (left) and lift (right) coefficients.
The optimal dimensionless vessel speed can be calculated to be $v_{opt} = 0.52$ meaning that the vessel speed in this specific case should be 52% of the absolute wind velocity for optimal energy conversion.

5. Conclusion and future scope

An innovative concept to convert ocean wind energy into storable chemical energy has been presented, modeled and discussed. The following main results are obtained.
A physical model as well as an upper limit for energy conversion, based on axiomatic conversion laws, is shown. Also parameter influences on the upper limit are investigated.

• Requirements for the control system for optimal turbine operation are shown.

• It is shown how the simultaneous energetic and economic system description leads to the optimal system design as a trade-off between energetic efficiency and economic profitability.

• For a typical sailboat it is shown how the presented method can be used to design an optimal energy conversion system under consideration of energy and economy. This type of analysis is likely to be useful for the design of future MegaWatt-scale energy ships.

As future scope the further development of an application-independent method for energetic and economic optimal system design is intended. Also the uncertainties of the method and of the used data for optimal solutions found will be quantified.

Acknowledgments

The authors would like to thank the Deutsche Forschungsgemeinschaft (DFG) for the financial support in the framework of the Excellence Initiative, Darmstadt Graduate School of Excellence Energy Science and Engineering (GSC 1070).

References


Nomenclature1

A: sail area in L2
A: dimensionless turbine area
A: wetted vessel area in L2
A: dimensionless vessel area
C: total investment costs in C
C: yearly costs in CT
C: coefficient of performance
CRF: capital recovery factor
C: absolute wind velocity in LT
C: turbine drag coefficient
C: sail lift coefficient
D: wind turbine diameter in L
D: hydro turbine diameter in L
H: caloric value of hydrogen in L2T
I: storage investment costs in C
i: dimensionless density
L: lift force in MLT
L: vessel costs in CL
L: dimensionless lift force
L: lift coefficient
L: vessel lift coefficient
L: vessel area specific investment costs in C
L: rate of interest
L: lift force in MLT
L: hydrogen mass flow in MT
L: number of annuities
P: available power in MLT
P: mechanical turbine shaft power in ML2T
P: pressure in ML
P: hydrogen price in CM
P: sail area specific storage costs in CL
P: turbine area specific turbine costs in CL
P: vessel area specific vessel costs in CL
R: revenue in C
R: yearly revenue in CT
S: heeling force in MLT
T: thrust force in MLT
V: vessel speed in LT
V: dimensionless vessel speed
W: vessel resistance force in MLT
W: wind relative wind velocity in LT
α: angle of attack in degree
α: absolute wind direction in degree
β: relative wind direction in degree
β: sail pitch angle in degree
η: electrolyser efficiency factor
η: generator efficiency factor
η: turbine efficiency factor
η: air density in ML
ρ: water density in ML
γ: dimensionless density
ζ: axial induction factor

1 The abbreviations are shown in the dimension of length (L), mass (M), time (T) and currency (C).