

## Developing a Control Strategy for Booster Stations under Uncertain Load

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**Abstract.** Booster stations can fulfill a varying pressure demand with high energy-efficiency, because individual pumps can be deactivated at smaller loads. Although this is a seemingly simple approach, it is not easy to decide precisely when to activate or deactivate pumps. Contemporary activation controls derive the switching points from the current volume flow through the system. However, it is not measured directly for various reasons. Instead, the controller estimates the flow based on other system properties. This causes further uncertainty for the switching decision. In this paper, we present a method to find a robust, yet energy-efficient activation strategy.

### Introduction

The world's energy consumption keeps rising. Today we are facing a wide range of energy sources and consumers are diversive: Economical sectors as well as private households. About 12 % of the electrical energy in Europe drives pumps [1]. A specific type of pump system is a booster station: A parallel setup of two or more pumps. The individual machines are located close to each other and the whole system is delivered as a package. Booster stations cover a wide performance map and thus are flexible machines with widespread possibilities of usage. Booster stations convey fluid from reservoirs to its destination or support existing networks: If the pressure of a building's water supply is too low, a booster station increases the pressure.

Due to the many applications and thus large energy input, booster stations need to be designed and controlled appropriately. This paper focuses on the control of existing booster stations. Binary decisions are always a discontinuity and hence a burden in the calculation of energy efficient control strategies. We address the following question: How many pumps should cover the uncertain load?

### Technical Description

**Topological Layout of Booster Stations.** Fig. 1 shows the connection scheme of the booster station. Six pumps are connected in parallel. The incoming water flows from the suction pipe, through the single pump units, into the pressure pipe. The pressure at the outlet is always higher than at the inlet. To avoid reverse flow, a check valve is installed behind each pump. The technical topology of the booster station is unquestioned in this paper.

In a pump's field of operation four parameters are important: The volume flow  $Q$ , the pressure increase  $\Delta H$ , the rotational speed  $n$  and the power consumption  $P$ . Any two of these parameters describe the point of duty. Following industrial standards we measure the pressure increase in meter by scaling the pressure increase  $\Delta p$  in bar with the density  $\rho$  and the specific gravitational constant  $g$ :

$$\Delta H = \frac{\Delta p}{\rho g}. \quad (1)$$

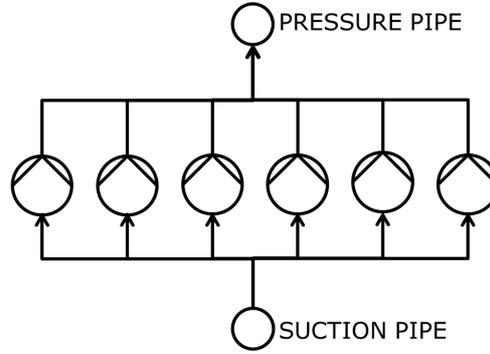


Fig. 1: Topology of a booster station composed of six pumps.

Table 1 shows the characteristics of the considered pump as sample points of the reference curve at maximal rotational speed  $n = n_{max}$ .

Table 1: Pump Characteristics

Volume flow in $\text{m}^3 \text{h}^{-1}$	0	2.05	4.33	6.5	8.7	11	14
Pressure head in m	95.535	94.206	90.937	87.360	82.660	74.486	59.569
Power demand in kW	1.41	1.85	2.45	2.95	3.40	3.75	3.9

For other rotational speeds, the well known scaling laws hold:

$$Q(n) = \left( \frac{n}{n_{max}} \right) Q(n_{max}), \quad (2)$$

$$H(n) = \left( \frac{n}{n_{max}} \right)^2 H(n_{max}), \quad (3)$$

$$P(n) = \left( \frac{n}{n_{max}} \right)^3 P(n_{max}). \quad (4)$$

In case of a booster station the number of active pumps is an additional parameter of operation.

**Operational mode.** The control variable for the operation of the booster station is the pressure in the pressure pipe. With the premise of a constant supply pressure, this means that the pressure increase of the booster station is controlled to be constant. The controller adjusts the rotational speed of the pumps and switches pumps on and off. The six pumps are of the same type, so the rotational speed in all running pumps is the same. While the rotational speed is a continuous variable which could be controlled, e.g. by a PID controller, the operational status is a discrete variable that causes a discontinuous transition. To react on this discontinuity the designer might use one of the two following control layouts:

1. The first option is a simple layout with few input parameters: Whenever the rotational speed of the active pumps is set to maximum, the controller turns on an additional pump and adjusts the rotational speed. The set-point of the continuous part of the control becomes an input parameter for the discontinuous part. This simple rule guarantees the functionality of the system, but does not consider the energy consumption. The point of duty remains unknown as the total volume flow is unknown.
2. The second option considers the energy consumption, but needs more information on the actual working point of the booster station: The volume flow becomes an input parameter of the control. Volume flow and pressure increase define the working point of the booster station. Two parameters remain to set the working point in any pump. These are (i) the actual number of working pumps and (ii) the rotational speed of the pumps. We optimize the setting of these two parameters, so that the energy consumption of the system becomes minimal.

While the measurement of the pressure in pumps is state of the art, one cannot measure the volume flow in the booster station easily. The conventional measurement devices are way too expensive or would cause an additional flow resistance. However, the controller can make a proper guess for the current volume flow from the already known parameters: With the knowledge about the machine characteristics, the currently measured pressure and the control signal for the rotational speed, the corresponding volume flow is calculated and the exact point of duty of the station is found. With the additional assumption that the volume flow in each pump is the same, we know the point of duty for all pumps.

The estimation of the volume flow assumes stationary flow and exactly measured pump characteristics. Thus it has an error, which leads to a binary decision under uncertainty. We want to apply the second control strategy in technical system to reduce the energy consumption. Our optimization allows to find a control strategy for this case and considers the uncertainty in the calculation of the volume flow.

**Optimization task.** Following the the TOR-Methodology [2], we have to define (1) the function of the systems, (2) the aim, and (3) the playing field for the optimization:

1. The function of a booster station is to increase pressure to establish a volume flow.
2. The aim of this optimization is to fulfill the function as energy efficient as possible.
3. The playing field is the control strategy. In several duty points the controller can use two degrees of freedom to find a set-point to cover the load. On the one hand, the rotational speed of the pumps, and on the other hand, the number of working pumps.

Based on this, we create an optimization model, which includes both degrees of freedom and covers the uncertain load of the booster station. The mathematical model consists of two stages: First, find a discrete decision for the number of running pumps and secondly, find a continuous decision for the rotational speed.

## Mathematical Model

**Basic Model.** The model for the optimization of the booster station follows the description of [3] with modifications and uses the linearization techniques of [4] to generate a Mixed-Integer Linear Program (MILP). The constraints of the optimization program contain the physical and technical description of the problem. The booster station is modeled as a graph  $G(V, E)$ . The edges either represent the technical components or simple connections within the graph. Each edge has a variable for the volume flow  $Q$  and the head difference  $\Delta H$  between the vertices. For simple connections the pressure difference equals 0.

The only technical components in this model are centrifugal pumps  $R$ : The characteristics within the dependent variables for volume flow  $Q$ , pressure head  $H$ , rotational speed  $n$  and power consumption  $P$  describe the flow in every active pump (cf. table 1). We model the dependence of the control variables  $Q$ ,  $H$ ,  $n$ ,  $P$  by linearization of the max-curve and the scaling laws as a constraint in the optimization program.

To deactivate a pump, the volume flow in the pump is necessarily set to zero and the pressure of the two connection ports must be uncoupled. This leads to a Big-M formulation with a binary decision for the activation of a pump. The set  $V$  represents the connection ports of all technical components plus sources and sinks of the system. In every vertex the volume flow conservation

$$\sum_{(i,v) \in E} Q_{i,v} - \sum_{(v,j) \in E} Q_{v,j} = 0 \quad \forall v \in V \quad (5)$$

holds as a constraint. Two exceptions from this rule are the vertices for the source  $s$  and the sink  $t$ : The pressure  $p$  in the vertex is given as a boundary parameter of the system's load and volume flow conservation does not hold. Instead of this, the volume flow demand becomes part of the flow conservation in source and sink:

$$\sum_{(s,j) \in E} Q_{s,j} = \sum_{(i,t) \in E} Q_{i,t} = Q_{\text{Load}}. \quad (6)$$

The objective of the optimization is to use as less energy as possible to cover the load. Thus, we minimize the total energy consumption of all pumps:

$$\min \sum_{(i,j) \in R} P_{i,j}. \quad (7)$$

**Integration of the uncertain load.** We assume that the estimation of the volume flow  $Q$  based on the pressure head  $p$  and the rotary speed  $n$  has no systematic error. The statistical error can be modeled by the probability density function

$$f(Q) = \frac{1}{\sigma_Q \sqrt{2\pi}} \exp\left(-\frac{(Q - \mu_Q)^2}{2\sigma_Q^2}\right) \quad (8)$$

of the normal distribution with the mean volume flow  $\mu_Q$  and the standard deviation  $\sigma_Q$ . A large standard deviation indicates a fluctuating flow or a high uncertainty in the measured pump characteristic.

A continuous distribution is difficult to incorporate in our optimization under uncertainty setting: On the one hand, we have discrete and continuous decision variables depending on the random variable. On the other hand, we want to be able to use the model of this paper in a multi-stage setting. Therefore, the continuous distribution has to be approximated by a discrete distribution.

A good discrete approximation of a continuous probability distribution preserves the lower moments. The idea is to reproduce decisive factors of the distribution's shape like mean, variance, skew and kurtosis. According to [5], we can preserve  $2N - 1$  moments using  $N$  value-probability pairs. The calculation of the discrete approximation is straight-forward. The discrete distribution replaces the system's deterministic load in the model. This results in a stochastic model. In the following computations, we consider  $N = 5$  load scenarios:

$$p(Q = \mu_Q) = 53.33\%, \quad (9)$$

$$p(Q = 104.07\% \mu_Q) = p(Q = 95.93\% \mu_Q) = 22.21\%, \quad (10)$$

$$p(Q = 108.57\% \mu_Q) = p(Q = 91.43\% \mu_Q) = 1.13\%. \quad (11)$$

The total power consumption in the objective function is replaced by the average total power consumption over all scenarios. The deterministic equivalent program is generated and optimized with the commercial solver gurobi.

## Results and Discussion

**Results for model without uncertainty** The application of linearization techniques enables us to use the commercial optimization solver *Gurobi*. It allows one to include numerous discrete decision variables into the model and still finds a solution in a reasonable time span. In order to find control guidelines for the whole field of operation, we have to run the optimization model many times. Every combination of pressure and volume flow demand needs one optimization run. We discretize the pressure and the volume flow with a step size of  $\delta H = 1$  m and  $\delta Q = 1$  m<sup>3</sup>/min and calculate the optimal number of running pumps. Thus we obtain the fields of equal numbers of running pumps within the field of operation as shown in Fig. 2.

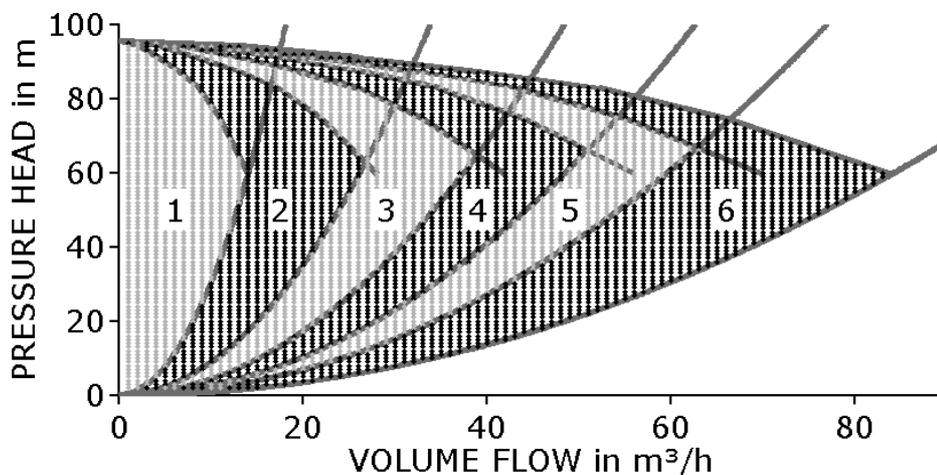


Fig. 2: Optimal control strategy for certain load.

We can identify two reasons for the switch of a pump: (1) For high constant pressure and increasing volume flow the controller increases the rotational speed of the active pumps. Once the maximum is reached, an additional pump is switched on. The decreasing gray lines show the maximum volume flow for fixed pressure head for one, two, three... pumps. (2) For low constant pressure and increasing volume flow, we find an efficiency argument: An additional pump is switched on, because it reduces the total power consumption, even though more pumps are working. We identify the switching line as a parabolic function with

$$H = aQ^2 . \quad (12)$$

The slope of the parabola  $a$  is calculated with a fit as shown in Table 2. These parabolas should be integrated into the control strategy of a booster station.

**Decisions under uncertainty** We apply the same technique of operating field discretization to the model under uncertainty. The same two reasons for the switch of a pump apply to the case with uncertain load: (1) For high pressure the optimization problem reduces once again to a feasibility problem. Due to the assumption of uncertain load, the optimization algorithm activates the next pump at a slightly smaller volume flow than before. (2) For low pressure the switching lines are parabolas again, but with different slopes compared to the results of the model without uncertainty. An uncertain load also leads to efficiency-caused pump activations at lower volume flows.

Table 2: Slopes  $a$  of the parabolas in  $h^2m^{-5}$ 

	certain load	uncertain load
1 - 2	0.3017	0.3565
2 - 3	0.08724	0.08934
3 - 4	0.04264	0.04281
4 - 5	0.02547	0.02557
5 - 6	0.01687	0.01699

## Summary and Outlook

In this paper we showed how to lay out a robust control strategy for booster stations: For high pressure rises the optimization problem reduces to a feasibility problem. The described first option for the operational mode is the best solution for the optimization problem. For low pressure rises the measurement of the volume flow in the station is a necessary additional input parameter for the control. The control strategy for a closed loop network should be based on the calculated parabolas to drive the booster station with optimal energy consumption even for low pressure rises.

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