Applied Mechanics and Materials Vol. 807 (2015) pp 99-108
© (2015) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/AMM.807.99

Submitted: 2015-04-15
Accepted: 2015-08-04
Online: 2015-11-23

Uncertainty Scaling – Motivation, Method and Example Application to a Load Carrying Structure

Angela Vergé[1,a]*, Julian Lotz[2,b], Hermann Kloberdanz[2,c], Peter F. Pelz[1,d]

[1]Technische Universität Darmstadt, Chair of Fluid Systems

Magdalenenstr. 4, 64289 Darmstadt, Germany

[2]Technische Universität Darmstadt, Product Development and Machine Elements

Magdalenenstr. 4, 64289 Darmstadt, Germany

[a]angela.verge@fst.tu-darmstadt.de, [b]lotz@pmd.tu-darmstadt.de, [c]kloberdanz@pmd.tu-darmstadt.de, [d]peter.pelz@fst.tu-darmstadt.de

Keywords: Scaling, Size Range, Uncertainty, Uncertainty Scaling, Dimensional Analysis, Laws of Growth, Product Development Process

Abstract. Scaling methods allow the estimation of the impact of changes in individual parameters on system performance. In the technical context, physical similarity is the focus. This paper demonstrates the extension of scaling methods to include uncertainty scaling. The advantages of using scaling uncertainty for the development of scaled products and the contribution of extended scaling methods to the analysis and assessment of uncertainty are illustrated. Uncertainty scaling based on dimensional analysis and complete similarity is derived. The potential of this method is demonstrated using a load carrying structure - a buckling beam.

Introduction

An engineer developing a product that comes in different sizes (size range) or transferring information from a model prototype to a life-sized product has to predict the properties of the scaled draft correctly. A more efficient method than designing the product with different numerical parameter values for each iteration exists: scaling using dimensional analysis or laws of growth, both of which refer to similarity laws. Being based on a basic design that is fully worked out, the designer can create new product sizes or predict the consequence of altering design parameter efficiently, leading to faster design processes for scaled products.

These methods are described well in literature. At the one hand, the best known methodology for size range development is that of Pahl and Beitz [1], which introduced the use of laws of growth derived from physical relations with respect to similarity laws. They also provide a brief overview of literature relevant to size range development. Simpson et al. [2] present an approach that is more strongly related to computational optimization methods, though they do not employ similarity relations to reduce complexity and design effort. One major concern in size range development is size-related uncertainty. Having variances in product properties that do not scale equally to the product property value or having disturbances that stay constant no matter the size of the product (e.g. the intensity of sunlight, temperature of the environment, etc.), critical design areas often have to be adapted and reworked. To prevent the designer from getting in too much trouble while mitigating the impact of size-related uncertainty, Lotz et al. developed laws of growth for product properties influenced by uncertainty [3,4]. The existing approaches to scaling uncertainty in product properties using static and dynamic laws of growth do not scale the uncertainty itself; they only address product property, including uncertainty, which is a less effective way of scaling uncertainty than being able to scale the variance caused by uncertainty directly.

On the other hand scaling is based on similarity laws derived from dimensional analysis. This powerful method can be traced back to the forefather of the modern world, Galilei Galileo. Joseph Fourier, in 1822, was the first to write of it in "Analytical Theory of Heat", where he reflected on a model: "…This relation depends in no respect on the units of length, which from its very nature is contingent, that is to say, if we took a different unit to measure linear dimensions, the equation … would still be the same.” Today, this is called the Bridgman postulate [5].

The technical focus is physical similarity, including geometrical similarity. The most prominent example of a complete physical similarity is the explosion by Taylor, von Neumann and Sedvo [6, 7, 8, 9]. Incomplete similarity is treated by Pelz et. al. [10, 11], particularly in the context of turbomachinery. The strength of scaling, not only in a geometric sense, became obvious recently in the paper of Pelz and Vergé [12]: Type and discrete numbers of proper formula units on vehicle speed was precisely predicted through physical and allometric scaling.

However, as found with laws of growth, none of the mentioned methodologies for scaling with dimensional analysis investigates the effects of uncertainty. Within this paper, scaling is expanded to the contemplation of uncertainty, with the focus being on parameter uncertainty. The strength of the uncertainty scaling is demonstrated by analyzing a load carrying structure.

Motivation for Uncertainty Scaling

A product passes through several phases in its lifecycle, including the production of material and then the product itself, as well as its usage and later disposal or recycling. These phases are characterized by processes in which uncertainty occurs [13]. As well as the product lifecycle phases, there is the product development (PD) process of the product itself. The uncertainty occurring in product lifecycle processes can be addressed within the development of the product by anticipation and analysis of the processes through which the product goes or that it enables. This can be done using product-process models, such as the process model of Collaborative Research Centre SFB 805 [14].

As well as the development of singular products, there is a class of products that face another type of uncertainty: scaled products. Scaling, as mentioned above, uses methods based on similarity to predict the product properties that the product will have when its size or pressure, material, etc., are changed. This prediction has the big advantage that the complexity of physical relations is reduced by monitoring only the relative change of parameters (Eq. 3). This makes the dimensional analysis a valuable and efficient scaling method. The reduction of scaling to similarity laws as it is performed in literature has a downside: In being able to determine the values of scaled parameters, the possibility of their deviation when the product is scaled is often overlooked. For example, production tolerances, as in the example later in this paper, grow less than proportionally compared to the product’s size because the relative precision of production processes increases with the size of the manufactured product. This shows that the parameters describing the product’s properties have to be scaled, and that there is also a need to scale the deviation from the nominal value (scaling uncertainty).

The scaling of uncertainty is not only a necessity, the methods existing in literature [3, 4] (that only target laws of growth) as well as the method developed in this paper using dimensional analysis can be a valuable contribution to the PD process for scaled products. They can be integrated into the existing methodologies for the analysis, assessment and control of uncertainty, which currently lack methods that address scaling uncertainty.

Uncertainty scaling methods need information to contribute to the analysis and assessment of uncertainty, which is their main contribution to the PD process. The information needed can be prepared systematically, using the existing models and methods. The scaling of products, while simultaneously using uncertainty scaling methods, can be displayed in a procedural model (Fig. 1). The advantage of integrating uncertainty analysis into the scaling of products is that iterations, including expensive ones that become necessary in the latter stages of the development process, can be reduced by anticipating critical processes and parameters in early design stages.

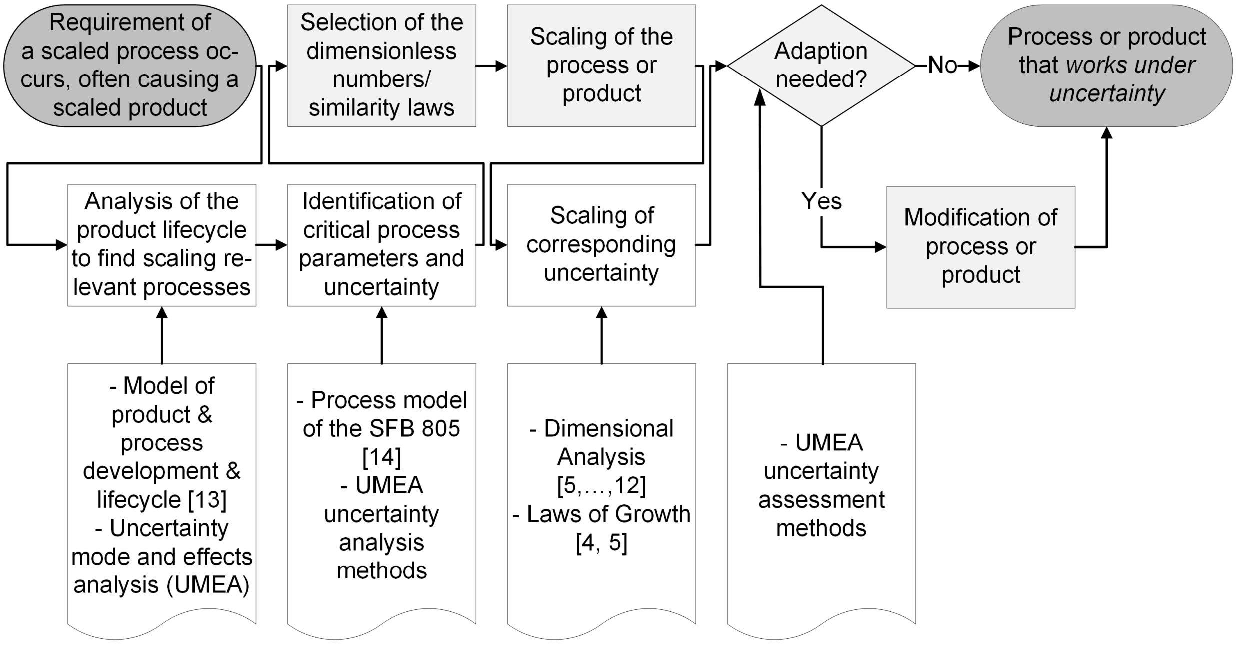

Figure 1: A procedural model of the scaling process, including the analysis and assessment of uncertainty. The light grey processes are also part of the scaling process without considering uncertainty. Supporting methods are displayed in the third row of the diagram.

A few additional details about uncertainty scaling are relevant: The scaling process can be performed at different stages of the process or product concretization. An estimation of basic product or process properties can be carried out in the early stages of the development process. This also allows an initial estimate of the occurring uncertainty, which is often based on expert estimates of the uncertainty of certain parameters. If the process or product that is to be scaled is known in detail (which is often the case when the need to scale arises), a brief analysis and assessment of uncertainty can be achieved as concretization occurs during the development process at a higher level, where more information is available. Scaling uncertainty can be used as a method for uncertainty analysis within the UMEA framework created by Collaborative Research Centre SFB 805, and therefore expands the methodical framework for controlling uncertainty to include the control of scaling uncertainty.

Method and Example Application

Fig. 2 shows the example application of a load carrying structure - a buckling beam.

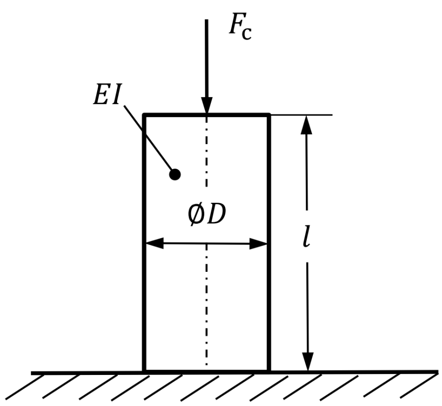

Figure 2: Load carrying structure.

In the first place we make a picture representing the reality. The model of the load carrying structure in our example is a cylindrical beam of circular cross-section (diameter D), of length l and constant flexural rigidity $EI \sim ED^4$. The question remains if this picture is complete or incomplete. The incompleteness of a picture = model is shown in Fig. 3. The model does not cover everything of the relevant reality to the buckling problem example shown, for example predeformation.

This discussion, summarized in Fig. 3, demonstrates that the model is a source of uncertainty. The model parameters are the second source of uncertainty. The third and, in most cases, dominant source of uncertainty is those that are critical or transferred through the boundary conditions: In this example, one boundary is the fixing of the beam on the ground. The second boundary is the applied load. The first and third sources of uncertainty are not considered here; this paper concentrates on parameter uncertainty only.

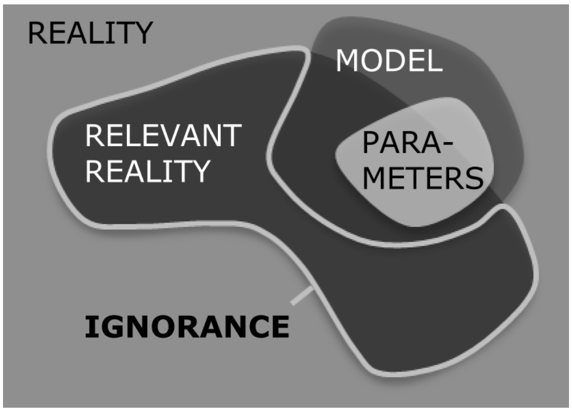

Figure 3: The uncertainty map according to Pelz and Hedrich [15].

Dimensional Analysis. Using the example of a buckling beam, the dimensional analysis content is extended to account for uncertainty. To do this, a brief review of the basic concept of scaling itself is required. To describe the physical problem with $j = 1 \dots n$ dimensioned values p_j, generally the value of interest, here the critical load F_c, is calculated from other measurable values: the flexural rigidity EI and the beam length l. As well as the measured values, $i = 1 \dots m$ fundamental units P_i are also required. The buckling of a beam is a static problem, thus, the fundamental units are given with force and length: $[F, L]$. Accordingly, it is possible to describe the relation of the physical values with

$$p_1 = fn(p_2 \dots p_n), \quad (1)$$

where the critical load F_c is a function of the flexural rigidity EI and the length of the beam l

$$F_c = fn(EI, l). \quad (2)$$

Corresponding to the Bridgman postulate [5], only relative values have absolute significance and the fundamental units have no relevance. This reduction follows an equivalent relation to Eq. 1

$$\Pi_1 = fn(\Pi_2 \dots \Pi_d), \quad (3)$$

with $d = n - r$ dimensionless products Π_i. r is the minimum number of fundamental units needed to describe the physical values and the rank of the dimension matrix $(a_{ij})_{m,n}$. In general, the number of the fundamental units corresponds to the rank of the dimension matrix $m = \mathrm{rg}\left((a_{ij})_{m,n}\right) = r$.

For $n = 3$ and $r = 2$ Eq. 3 simplifies to $\Pi = \mathrm{const.}$ and the dimensionless products are products of the physical values, i.e. parameters and quantities

$$\Pi = \prod_{j=1}^{n} p_j^{k_j}, \tag{4}$$

as well as

$$[\Pi] = 1 = \prod_{j=1}^{n} [p_j]^{k_j}. \tag{5}$$

The square brackets are used here as an operator to give the dimensions of any operand. For the dimension quantities apply

$$[p_j] = \prod_{i=1}^{m} P_i^{a_{ij}}. \tag{6}$$

From Eq. 5 and Eq. 6 follows

$$[\Pi] = 1 = \prod_{j=1}^{n} \prod_{i=1}^{m} P^{a_{ij} k_j}. \tag{7}$$

In the special and most simple case $n = m + 1$, which is sufficient for our example, Eq. 7 is satisfied, as long as

$$\sum_{j=1}^{n} a_{ij} k_j = 0, \qquad i = 1 \dots m, \tag{8}$$

has no trivial solutions for the n unknown k_j. The dimension matrix $(a_{ij})_{m,n}$ is represented with

	$p_1, \quad \dots, \quad p_n$
P_1	
.	
.	$(a_{ij})_{m,n}$
.	
P_m	

for the buckling beam

	F_c	EI	l
F	1	1	0
L	0	2	1

Eq. 8 has r linearly independent equations with $d = n - r$ linearly independent solutions, thus the values of k_j can be determined. In the general case $n > m$ which has more unknown values than independent equations, Eq. 8 change to an inhomogeneous linear equation from which $d = n - r$ dimensionless products Π_i are derived [16].

The result of Eq. 8 in our example is given $k_{F_C} = 1$, $k_l = 2$, $k_{EI} = -1$, i.e.

$$\Pi = \frac{F_c l^2}{EI}. \tag{9}$$

Using model $EI \sim ED^4$ yields

$$\Pi = \frac{F_c l^2}{ED^4}. \tag{10}$$

Scaling. In the technical context the basic design and behavior of size ranges and scaling of physical quantities is of interest. It is important to mention that not only geometric scaling can be treated but also Young's modulus can be scaled. However, looking at every sequential design is not efficient: using scaling methods enables the prediction of the behavior of a scaled design. The physical properties of the basic design correlate with the full-scale by

$$p_j := p_{0j} M_j, \qquad j = 1 \dots n, \tag{11}$$

which defines the scale factors M_j. The basic and full-scale designs behave in a completely physically similar manner under the condition that all dimensionless products are similar. This means

$$\Pi_i = \Pi_{0i}, \qquad i = 1 \dots d, \tag{12}$$

and is equal to

$$1 = \prod_{j=1}^{n} M_j^{k_{(i),j}}, \qquad i = 1 \dots d. \tag{13}$$

Usually, scaling in the context of heat and mass transfer, turbomachinery, etc. is based on Eq. 13. If there is complete similarity between basic design and scaled design, is given as long as the systems of equations (Eq. 13) are fulfilled. The requirements in Eq. 12 and 13 are the basis of model theory [16].

In our example the scale factor of the critical buckling load is given by

$$M_{F_c} = \frac{M_E M_D^4}{M_l^2}. \tag{14}$$

Uncertainty Scaling. The concept of scaling based on Eq. 13 is ready to be extended to include uncertainty scaling. The assumption of deterministic values does not correspond to the reality that any value is uncertain (Fig. 4).

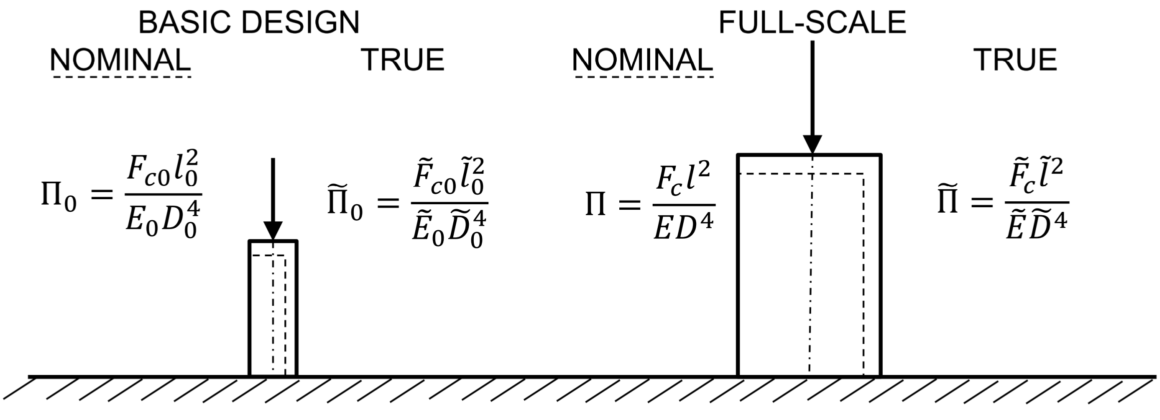

Figure 4: Nominal and true values of the basic design and a geometric scale of a buckling beam.

To account for uncertainty it is necessary to differentiate between the nominal value p_j and the tolerance range Δp_j. The true value $\tilde{p}_j$ is between

$$\tilde{p}_j = p_j \pm \Delta p_j, \qquad j = 1 \dots n. \tag{15a}$$

With the definition of uncertainty $U_j := \Delta p_j / p_j$ the true value is

$$\tilde{p}_j = p_j (1 \pm U_j), \qquad j = 1 \dots n. \tag{15b}$$

In the same manner, applies a nominal value for the dimensionless products, Eq. 4, and a true value

$$\tilde{\Pi}_i = \prod_{j=1}^{n} \tilde{p}_j^{k(i),j}, \qquad i = 1 \dots d, \tag{16}$$

as well as

$$\tilde{\Pi}_i = \Pi_i \prod_{j=1}^{n} \left(1 \pm U_j\right)^{k(i),j}, \qquad i = 1 \dots d. \tag{17}$$

The same description holds for the basic design

$$\tilde{\Pi}_{0i} = \Pi_{0i} \prod_{j=1}^{n} \left(1 \pm U_{0j}\right)^{k(i),j}, \qquad i = 1 \dots d. \tag{18}$$

When considering uncertainty the product of the scaling factors reads

$$1 = \prod_{j=1}^{n} \tilde{M}_j^{k(i),j} = \prod_{j=1}^{n} M_j^{k(i),j} \prod_{j=1}^{n} \left(1 \pm U_j\right)^{k(i),j}, \quad i = 1 \dots d. \tag{19}$$

Since $\prod_{j=1}^{n} M_j^{k(i),j} = 1$ the desired result for the uncertainty scaling follows:

$$1 = \prod_{j=1}^{n} \left(1 \pm U_j\right)^{k(i),j}, \qquad i = 1 \dots d. \tag{20}$$

Eq. 20 allows the calculation of the uncertainty of the interesting, most critical, values of a physical system in terms of complete similarity. In this paper we focuses on the production uncertainty that affects the geometric properties of the beam. The nominal geometric value, where p represents either length l or diameter D, is linked to the related tolerance factor i through the empirical relationship

$$i = L_1 \left(\frac{p}{L_{10}}\right)^{1/3}. \tag{21}$$

The constants L_1 and L_{10} in Eq. 21 are defined by DIN ISO 286 to $L_1 := 10^{-3}$ mm and $L_{10} := 10$ mm [17]. The specified tolerance range Δp is a multiple of the tolerance factor i

$$\Delta p = f i. \tag{22}$$

The constant f is set according to DIN ISO 286 [17]. For a diameter with the fundamental tolerance IT7 $f = 16$, for IT9 $f = 40$. Larger components can be manufactured more precisely than smaller components [18].

From the condition for complete geometrical similarity and similarity of the material $M_E = 1$ the scaling factor of the critical buckling load is

$$\tilde{M}_{F_c} = \frac{\tilde{M}_D^4}{\tilde{M}_l^2}, \tag{23}$$

and the uncertainty of the buckling load is given by

$$\pm U_{F_c} = \frac{\left(1 \pm U_D\right)^4}{\left(1 \pm U_l\right)^2} - 1. \tag{24}$$

In relation to the basic design with a scaling factor, $\tilde{M}_D = \tilde{M}_l = 1$, the uncertainty of the buckling load decreases with higher scaling factors, up scaling, and increases considerably more strongly with down-scaling, as seen in Eq. 24 and Fig. 5. Fig. 5 also shows that the uncertainty of the critical load varies in a higher range than the production uncertainties. Non-symmetrical behavior of the positive and negative curves of the worst case buckling load uncertainty results from Eq. 24. In this case, the effect is not recognizable in Fig. 5.

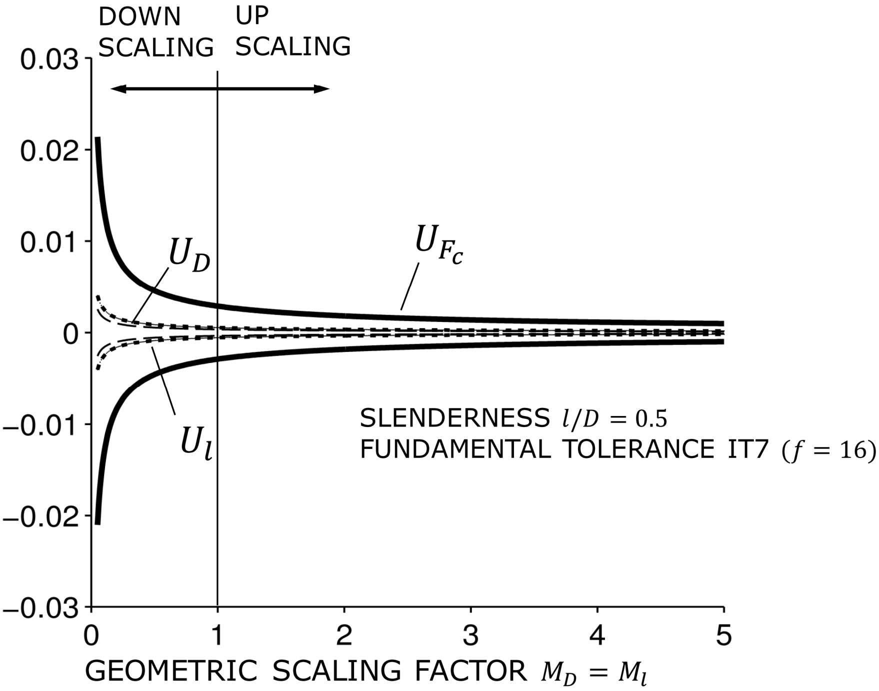

Figure 5: Uncertainty of the geometric values and the critical buckling load of the beam.

Fig. 5 reflects the need for uncertainty scaling. The uncertainty of the critical load due to the manufacturing tolerances increases sharply in down-scaling. If this is not considered in the scaled design, it can lead to failure due to high load. The method of uncertainty scaling introduced here allows the direct capture of the critical impact of scaling so that necessary actions can be performed. The method further provides the advantage of considering uncertainty directly during the scaling process for the entire region, so that it does not need to be considered anew for each execution.

Summary and Outlook

The main motivation for uncertainty scaling, ensuring the proper function of scaled products, leads to the advantage of integrating uncertainty scaling into product or process scaling: deviations that prevent proper working of process or product are not monitored with conventional scaling but can be analyzed with the expanded scaling methods, i.e. uncertainty scaling, derived in this paper. The expansion of the scaling methods is done by creating additional similarity relations for uncertainty, leading to a product model that is more detailed and therefore represents reality in a more precise way. This allows existing uncertainty control methodologies like UMEA to address not only solitary but also scaled products. Information about the class of size-dependent uncertainty influences uncertainty scaling (Fig. 5) and can be fed into the product development process to determine the true behavior of products and processes.

Within this paper, uncertainty scaling was derived, based on a load carrying structure – a buckling beam. Our focus was on complete geometric similarity and demonstrating the influence of production uncertainty on a size range. It is important to keep in mind that not only geometric scaling can be treated.

The advantage of uncertainty scaling is being able to show the uncertainty of critical correlations directly. It was also demonstrated that it is insufficient to only care about individual sources of uncertainty – in the example, production uncertainty – it is also necessary to consider the scaled uncertainty of the critical value, which is often calculated through simulation or measurement and can be a multiple higher, as seen in the example of the buckling beam. The uncertainty of the buckling load varies in a higher range than the production uncertainties: Our example shows the nonlinear amplification of geometric manufactured uncertainty by the power of two (Eq. 24 $U_D \approx U_l$). Generally, the product development process for size ranges and scaled products can be shortened by anticipating the effects of uncertainty in a size-related manner, reducing the risk of creating scaled products that exceed the limits of scaling, which would lead to reduced functionality or even failure. Iterations in the development process can be reduced.

Future work treats uncertainty scaling in products with more than one dimension and uncertainty scaling for incomplete similarity in dimensional analysis, if Eq. 13 is not fully satisfied.

Acknowledgment

We would like to thank Deutsche Forschungsgemeinschaft (DFG) for funding this project within the Collaborative Research Centre (CRC) 805.

References

[1] G. Pahl, W. Beitz, J. Feldhusen, K.H. Grote, Pahl Engineering Design. A Systematic Approach, third ed., Springer-Verlag, London, 2007.

[2] T.J. Simpson, J.R.A. Maier, F. Mistree, Product platform design: method and application, Research in Engineering Design ,Vol. 13 (2001), pp. 2-22.

[3] J. Lotz, H. Kloberdanz, T. Freund, K. Rath, Estimating Uncertainty of Scaled Products Using Similarity Relations and Laws of Growth, Design Conference 2014, 19.-22.05.2014, Dubrovnik. Proceedings of the 13th International Design Conference DESIGN 2014 Dubrovnik.

[4] J. Lotz, H. Kloberdanz, in T.J. Howard, T. Eifler (eds.), Scaling under Dynamic Uncertainty using Laws of Growth, Proceedings of the International Symposium on Robust Design – IsoRD14, Copenhagen (2014), pp. 17-27.

[5] P.W. Bridgman, Dimensional Analysis, Yale University Press, New Haven, 1922.

[6] G.I. Taylor, The formation of a blast wave by a very-intense explosion, Proceedings of the Royal Society, London, Vol. A201 (1950), pp. 175–186.

[7] J. von Neumann, The point source solution, NDRC Division B Rept AM-9, 1941, Reprinted in Taub AH (ed) John von Neumann collected works, Pergamon, Oxford, (1963), pp 219-237.

[8] L.I. Sedov ,The movement of air in a strong explosion, Journal of Applied Mathematics and Mechanics, Vol. 10 (1946), pp. 241 – 250.

[9] L.I. Sedov, Similarity and dimensional methods in mechanics, Academic Press: New York, (1959), Chap. 4.

[10] P.F. Pelz, S. Karstadt, Tip Clearance Losses – A Physical Based Scaling Method, In: International Journal of Fluid Machinery and Systems, Vol. 3 (2010), pp. 279-84.

[11] P.F. Pelz, S. Stonjek, A Second Order Exact Scaling Method for Turbomachinery Performance Prediction, In: International Journal of Fluid Machinery and Systems (2013).

[12] P.F. Pelz, A. Vergé, Validated biomechanical model for efficiency and speed of rowing, Journal of Biomechanics, Vol.47 (2014), pp. 3415-3422.

[13] H. Hanselka, R. Platz, Ansätze und Maßnahmen zur Beherrschung von Unsicherheit in lasttragenden Systemen des Maschinenbaus, Konstruktion, November/December (2010), pp. 55-62.

[14] A. Bretz, S. Calmano, T. Gally, B. Götz, R. Platz, J. Würtenberger, Darstellung passiver, semi-aktiver und aktiver Maßnahmen im SFB 805-Prozessmodell, Preprint 2015, SFB 805, TU Darmstadt.

[15] P.F. Pelz, P. Hedrich, Unsicherheitsklassifizierung anhand einer Unsicherheitskarte, technical Report of Instituts for Fluid Systems, Darmstadt, 2015.

[16] J.H. Spurk, Dimensionsanalyse in der Strömungslehre, Springer Verlag, Berlin Heidelberg, 1992.

[17] Deutsches Institut für Normung, DIN ISO 286

[18] W. Steinhilper, B. Sauer, Konstruktionselemente des Maschinenbaus 1, Springer-Lehrbuch, Springer Verlag, Berlin Heidelberg, 2012.

Uncertainty in Mechanical Engineering II

10.4028/www.scientific.net/AMM.807

Uncertainty Scaling – Motivation, Method and Example Application to Aload Carrying Structure

10.4028/www.scientific.net/AMM.807.99