Manuel Metzler and Peter F. Pelz

Draft Tube Design Method for Operation at optimal Coefficient of Performance in Low Head Hydropower

Commonly practised design methods for low head hydropower draft tubes are aiming to minimize dissipative losses, hence to maximize hydraulic efficiency. Betz established the coefficient of performance for wind turbines. It is a more general view on efficiency, since it not only takes dissipative losses into account, but also nondissipative losses, due to wake energy flux. The second author derived the coefficient of performance for an open channel-flow in analogy to Betz. The present paper presents a new approach for operation at optimal coefficient of performance.

1 Introduction

With technological progress in hydropower, initiated by the invention of the dynamo by Werner von Siemens in 1867, a trend from machines of large diameter at low specific speed to machines of smaller diameter at high specific speed can be observed. With the decreasing size of machines investment cost per installed power decreased as well. Otto Cordier [3] showed the correlation of specific speed and specific diameter of hydraulic machines, based on empiric data. The correlation between specific diameter and investment cost is considered by Pelz and Metzler [4]. Due to low diameter numbers, the meridian speed of flow along the hydraulic machine increased compared to ancient machines of low rotational speed and large diameter. Hence, the fraction of dynamic pressure in the total pressure is high at the rotor outlet. Experiments indicate that draft tubes with large diffusers increase the shaft power significantly, especially for machines of high rotational speed. This is why they are common practice in design of low head hydropower plants. On the other hand those diffusers cause high investment effort, so again there is a target conflict in between efficiency and investment cost.

The most common explanation for the beneficial effects of diffusers is an assumed recuperation of energy within the draft tube, due to conversion of dynamic pressure into static pressure. Deniz [5], as one of the recognized authors writes:

"To achieve high hydraulic efficiency it is necessary to recuperate the major part of the outlet energy. Velocity energy has to be converted in pressure energy at highest possible efficiency."

The nondissipative outlet loss is defined by Deniz [5] and Raabe [6] as the ratio of kinetic energy at the rotor outlet to the gross head defined as the difference of total head between head water and tail water (see position 1 and 2 of **Figure 1**)

$$k := \frac{u_2^2}{2g(H_1 - H_2)} \qquad (1)$$

Following the classic hydropower design methods, this loss is the measure to be minimized. By reducing the velocity at the outlet the net head $H_T = H_+ - H_-$ (Figure 1) can be increased.

While designing draft tube diffusers using the classic approach (Eq. (1)) and other state of the art design strategies, two quantities are assumed to be given by nature:

(i) Volume flow rate $Q = \text{const.}$,
(ii) gross head $H_g := H_1 - H_2 = \text{const.}$,
(iii) within system boundaries in between 1 and 2 (Figure 1).

These assumptions, without a doubt, are feasible for plants with large ground drop $\Delta z/(h_1 + u_1^2/2g) >> 1$, i.e.. Regarding Eq. (1), they lead to the conclusion that by lowering the diffusers outlet velocity to zero, the flux of kinetic energy, hence the lost kinet-

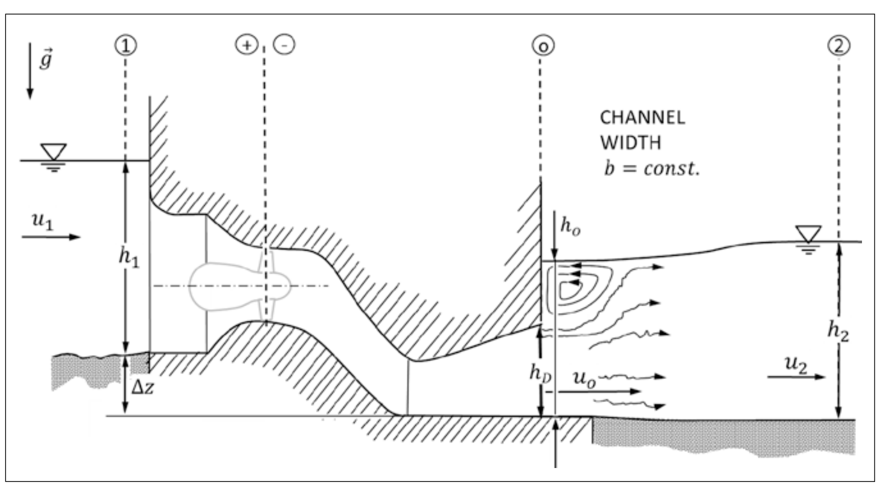

Figure 1: Control points in hydropower plant (Source: Authors)

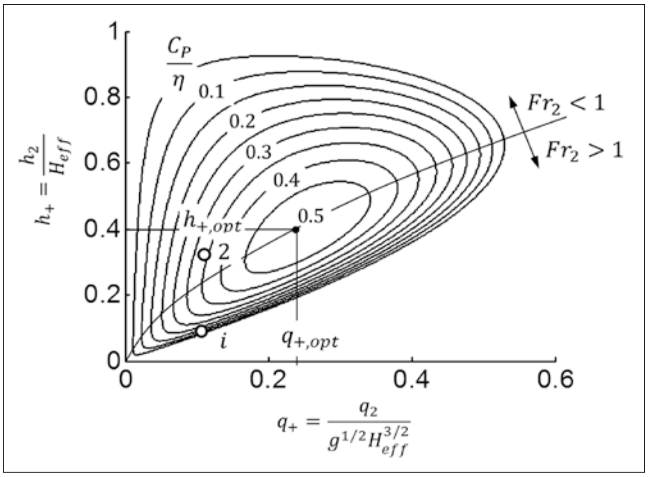

Figure 2: Coefficient of performance as function of operating point (Source [2])

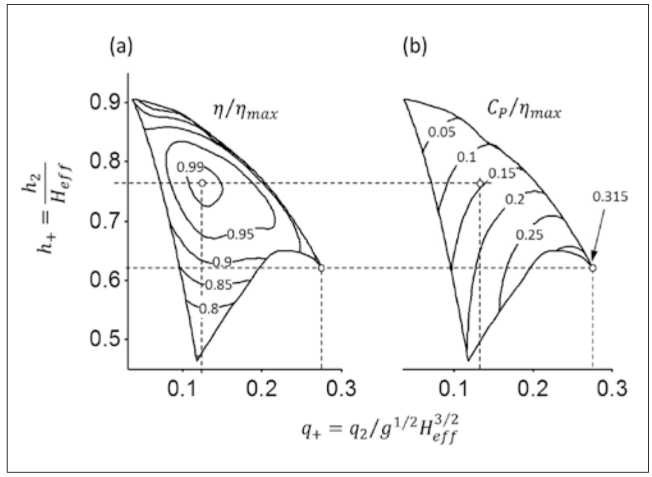

Figure 3: Hydraulic efficiency η vs. coefficient of performance C_p (Source: [2], [8])

ic energy in the tail water can also be reduced to zero.

Contrary to the above assumptions for low head hydropower plants in open channels it can be observed that the gross head is not only determined by nature, but also influenced by the operation point of the hydropower plant. As the operator knows while changing the load of the machine, the tail water level height can alter in significant ranges. Furthermore, in open channel flows, there is a fixed relation between flow velocity, water level height and specific energy $H_2 = Q/bu_2 + u_2^2/2g + z_2$. Hence, it is not possible for the velocity to change without changing the head as well. Hence, assumption (ii) of the classic approach is violated. Furthermore in literature there is not always a distinction of position o at the draft tube outlet and position 2 in the tail water (Figure 1). Distinguishing these positions allows integrating flow phenomena that could occur in between them, like hydraulic jumps or Carnot shock losses, into the analytical modelling. Some detailed examples are given in section 3.

In his numerical research on draft tubes Ruprecht [7] kept the tail water level h_2 fixed, but already noticed that the tail water condition has an influence on the draft tubes outlet flow. The present paper aims to describe this influence analytically.

Therefore the new approach is based on the assumption, that for a low head hydropower plant in stationary operation the following quantities are given by nature:

(i) Volume flow rate ,

(ii) effective head $H_{eff} := h_0 + u_0^2/2g + \Delta z = h_1 + u_1^2/2g + \Delta z = \text{const.}$ (cf. Pelz [2], 0 stands for undisturbed condition),

(iii) with system boundaries always at 1 and 2 (Figure 1).

The proposed new approach provides a new design principle for diffusers by means of minimization of specific tail water energy $H_2 = h_2 + u_2^2/2g + z_2$ and dissipative losses at the same time. The new approach allows taking the tail water condition, hence, any possible change of gross head, hydraulic jumps or Carnot shock losses, into account.

2 Optimal Operation for Low Head Hydropower

A common criterion for efficiency of low pressure hydropower plants is the hydraulic efficiency η of the plant defined as:

$$\eta := \frac{P_T}{\rho g(H_1 - H_2)Q} \qquad (2)$$

It is the ratio of shaft power P_T to hydraulic power defined as $P_H := \rho g(H_1 - H_2)Q$ at a particular operation point. It is a dimensionless measure for the dissipation within the plant only. Maximizing the hydraulic efficiency of the plant means to minimize dissipative losses, such as friction and inertia related losses. It does not cover the possibility of gross head changes as yet mentioned in the above section.

Contrary to the classic approach, the new approach assumes (i) the volume flow rate and (ii) the effective head as given by nature and hence, constant for stationary operation. In contrast to the classic approach for these assumptions the gross head i.e. the hydraulic power P_H is no longer constant. Thus, a new benchmark for the available power is needed. Still the shaft power is defined as given by Eq. (2):

$$P_T = \eta\rho g Q(H_1 - H_2) \qquad (3)$$

But now with H_{eff} and $Q = q_2 b$ the very same Eq. (3) is written in its equivalent form:

$$P_T(h_2, q_2) = \eta\rho g q_2 b\left(H_{eff} - h_2 - \frac{q_2^2}{2gh_2^2}\right) \qquad (4)$$

A straight forward analysis yields the following result: The maximum of the desired shaft power P_T is reached for $h_2 = 2/5\, H_{eff}$ and $q_2 = (2/5)^{3/2} g^{1/2} H_{eff}^{3/2}$, being the solution of the linear system of equations $\partial P_T/\partial h_2 = 0$ and $\partial P_T/\partial q_2 = 0$. At that optimal point the shaft power reaches the optimum $\eta\, P_{avail}/2$ with the available power $P_{avail} := 2\,(2/5)^{5/2}\rho g^{3/2} H_{eff}^{5/2}\, b$, defined by Pelz [2]. If we measure shaft power in multiplies of the available power yields a dimensionless measure called commonly coefficient of performance $C_p := P_T/P_{avail}$. With the dimensionless parameters $q_+ := q_2/g^{1/2}H_{eff}^{3/2}$ and $h_+ := h_2/H_{eff}$ the energy Eq. (4) is written now in the form [2]:

$$C_p := \frac{P_T}{P_{avail}} = \frac{\eta}{2}\left(\frac{5}{2}\right)^{5/2} \cdot q_+\left(1 - h_+ - \frac{1}{2}\frac{q_+^2}{h_+^2}\right) \leq \frac{\eta}{2} \qquad (5)$$

Figure 2 shows iso-lines of C_p in the q_+-h_+-plane clearly showing the desired optimum. Now there are other possibilities to plot Eq. (5). With $Fr_2 := q_+/h_+^{3/2}$ the volume flow rate can be replaced by the Froude number. In a similar way h_+ is substitutable by the dimensionless gross head $H_{g+} := H_g/H_{eff}$. The very same optimum is reached for $Fr_2 = 1$, $H_{g+} = 2/5$.

Hence one can easily remember as a rule of thumb:

In the optimum the by nature given effective head H_{eff} splits into three parts:

- 2/5 H_{eff} gross head $H_g = H_1 - H_2$,
- 2/5 H_{eff} tail water depth h_2,
- 1/5 H_{eff} tail water dynamic head $u_2^2/2g$.

Finally Eq. (5) once more clarifies the difference between hydraulic efficiency of the plant and coefficient of performance. The coefficient of performance C_p represents both, dissipative losses and nondissipative losses due to energy flux in the tail water. The hydraulic efficiency is a factor to the coefficient of performance and hence only of secondary importance.

The efficiency data sketched in **Figure 3** illustrates the different consequences that results from the classic and the new approach. According to the classic approach, the hydraulic efficiency sketched in Figure 3a induces the operator to adjust the operation point to maximal hydraulic efficiency $q_{+,\eta,max} = 0.12$, $h_{+,\eta,max} = 0.76$.

In contrast to the hydraulic efficiency the coefficient of performance, sketched in Figure 3b induces the operator to adjust the operation $q_+ = 0.28$, $h_+ = 0.62$. At this operation point the coefficient of performance is 31% while in the point of maximal hydraulic efficiency it is 14% only. Hence, the increase of power output in between the classic approach and the new approach is 17%!

3 Analytical Case Study

In this section various cases of possible operation points i.e. tail water conditions are discussed. The effect of the diffuser on the gross head and net head is exemplified. Especially the partition of the energy content of the flow into net head H_T, dissipative and nondissipative losses is addressed. All of the cases of section 3 are discussed for stationary operation of the plant, hence for constant flow rate Q. The net head for the hydraulic machine writes (cf. [2]):

$$\frac{H_T}{H_{eff}} = \frac{H_g - h_L}{H_{eff}} = 1 - h_+ - \frac{1}{2}\frac{q_+^2}{h_+^2} - h_{L+} \qquad (6)$$

All of dissipative losses in the whole device including screen, inlet, hydraulic machine the draft tube, hydraulic jump losses and Carnot shock losses are summarized in the head loss $h_{L+} = h_L/H_{eff}$. As a matter of fact the representation of net head used in Eq. (6) can be transformed into the form of Eq. (3) employing the definition of the hydraulic efficiency $\eta := 1/(1+h_T/H_T)$. Thus, considerations made in section 3 are completely consistent to those of section 2.

The graphs on the bottom half of **Figure 4** show an example for the possible states of an open channel-flow, i.e. the possible water level heights for each total head. The function of the diffuser is discussed for three exemplary cases. Case I is a draft tube without diffuser (see Figure 4, left column). Case II is a draft tube with diffuser (see Figure 4, right column). Case III is a draft tube with submerged diffuser (see Figure 1).

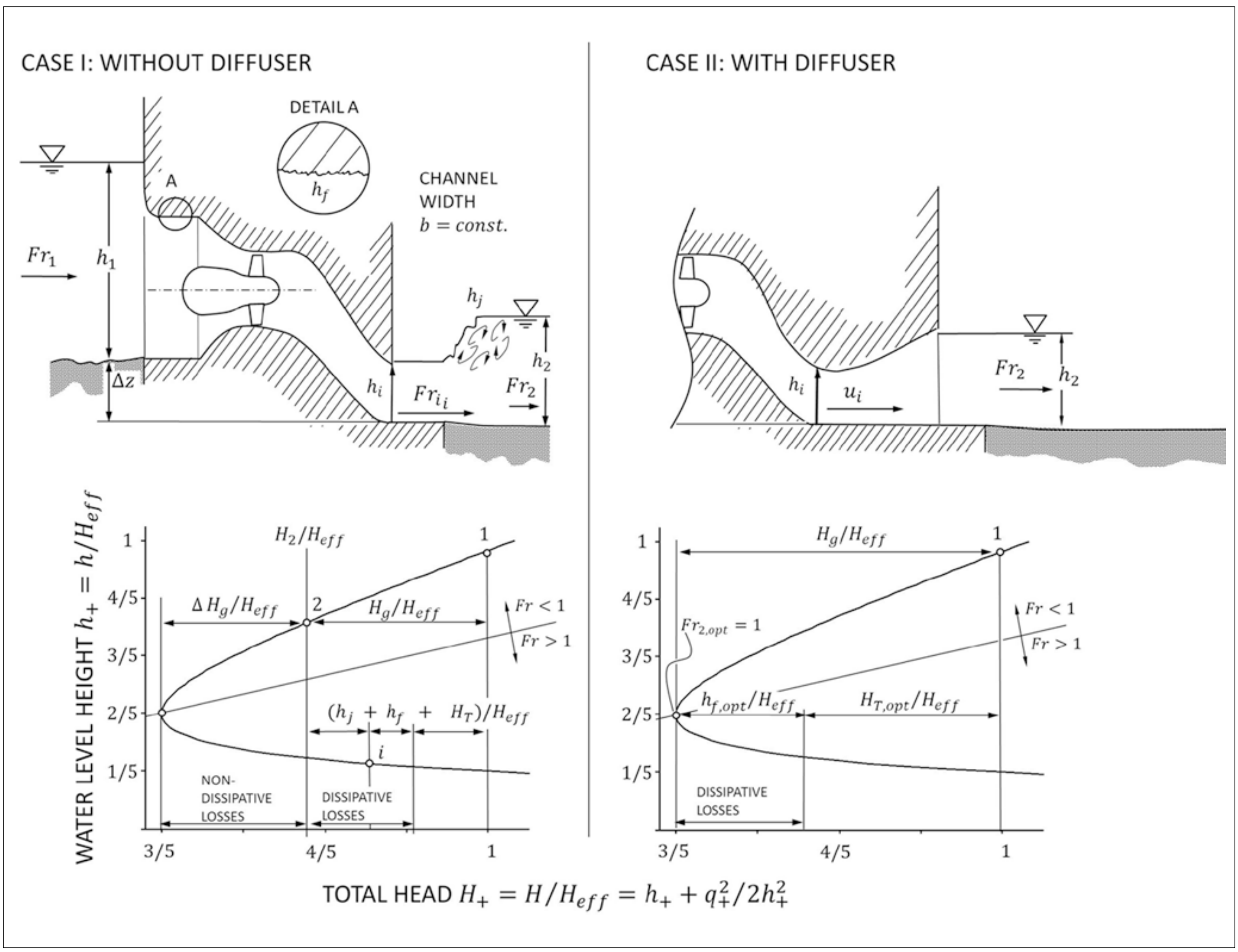

Figure 4: Draft tube without (Case I) and with diffuser (Case II) (Source: Authors)

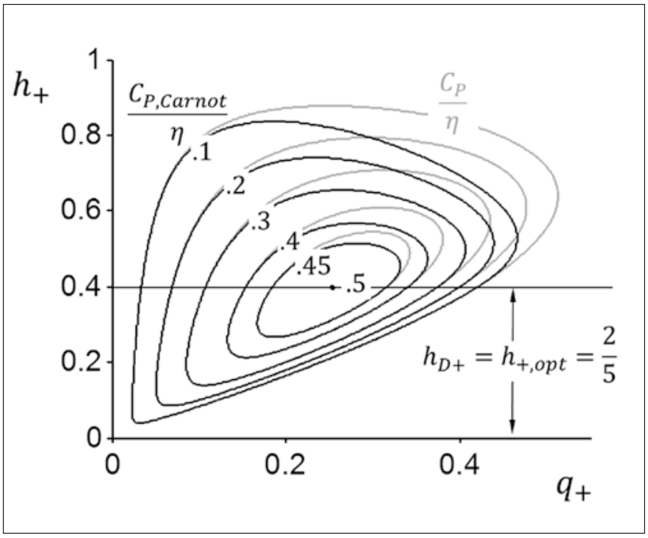

Figure 5: Coefficient of performance with Carnot shock losses included (Source: Authors)

Case I: Draft tube without diffuser (Figure 4, left column)

In this case the index i represents the position at the outlet of the draft tube without diffuser. The velocity head at the turbine outlet $u_i^2/2g$ is comparatively high, while the pressure head h_i is low.

Point 2 in the left column of Figure 4 is the tail water state of case I. The transition from state i to 2 is known as hydraulic jump. For given pair of effective head H_{eff} and volume flow rate Q and $Fr = u/\sqrt{gH} \neq 1$ there are two possible water depths. One of them with $Fr_i > 1$ is supercritical, the other with $Fr_2 < 1$ is subcritical. The highest possible gross head occurs when the tail water flow is critical ($Fr_{2,opt} = 1$, Figure 4). This statement has been established by the second author [2].

In case I, the flow is supercritical at the draft tube outlet ($Fr = u_i/\sqrt{gh_i} > 1$). The total head in the tail water H_2/H_{eff} is comparatively high. The shaft power is low due to both, high dissipative and nondissipative losses.

The dissipative losses related to the hydraulic jump from state to state 2, h_j are included in $h_{L+} = (h_f + h_j)/H_{eff}$ with friction losses denoted by the subscript f. The nondissipative loss, hence the gross head loss, which cannot be described by the hydraulic efficiency are $\Delta H_g/H_{eff} = H_2/H_{eff} - 3/5$.

Case II: Draft Tube with Diffuser (Figure 4, right column)

In case II the flow is converted from state i to state 2 inside the diffuser. In this case it is assumed that the diffuser adjusts the tail water flow to its optimal condition given by Pelz [2] $h_+ = h_{+,opt} = 2/5$ and $Fr_2 = Fr_{2,opt} = 1$. As a consequence of this conversion, the specific energy of the tail water decreases to its minimum $H_2/H_{eff} = 3/5$. Also there are no hydraulic jump losses. The nondissipative losses decrease to zero and, hence the gross head attains its maximum value while the net head increases to:

$$\frac{H_{T,opt}}{H_{eff}} = \frac{2}{5} - \frac{h_{f,opt}}{H_{eff}} \tag{7}$$

The pressure drop at the rotor outlet, which occurs when a draft tube is attached to the turbine, can be explained by this open surface driven increase of net head. The most remarkable fact about the optimization method is, that even with lower hydraulic efficiency the energy output can be higher, when the coefficient of performance is optimized. Figure 4 shows a higher hydraulic efficiency in case I, since the dissipative losses of case I $h_f + h_j$ are smaller than the dissipative losses $h_{f,opt}$ of case II. Anyway the energy yield is higher for case II since nondissipative losses are minimized. A simple design strategy for a diffuser operating at $C_p/\eta = 0.5$ is proposed in section 4.

Case III: Submerged Diffusers and Carnot Shock Losses (Figure 1)

In the third case an example of how to include dissipative losses in the coefficient of performance (Figure 2) is given. An important dissipative loss mechanism for draft tube outlets is the Carnot shock loss. It occurs when the draft tube outlet is submerged by the tail water column ($h_D < h_o \approx h_2$, Figure 1). Submerging can occur due to backwater from downstream dams, or as a consequence of the operating point of machines, i.e. volume flow rate of watercourses. In many cases it is even necessary to submerge the diffuser to avoid flow separation at the diffuser walls.

The Carnot shock loss can be established by strictly axiomatic equations. It is a consequence of balance of momentum and Bernoulli's equation which itself can be established under use of balance of momentum [9]. The result of this derivation is the Carnot shock loss for an open channel-flow of constant width b, volume flow rate Q and for $h_o \approx h_2$:

$$h_C = \frac{Q^2}{2g\, b^2 h_D^2}\left(1 - \frac{h_D}{h_2}\right)^2 \tag{8}$$

With the dimensionless operating parameters q_+ and h_+, defined in section 2, and the dimensionless diffuser outlet height $h_{D+} := h_D/H_{eff}$ the dimensionless Carnot shock loss reads:

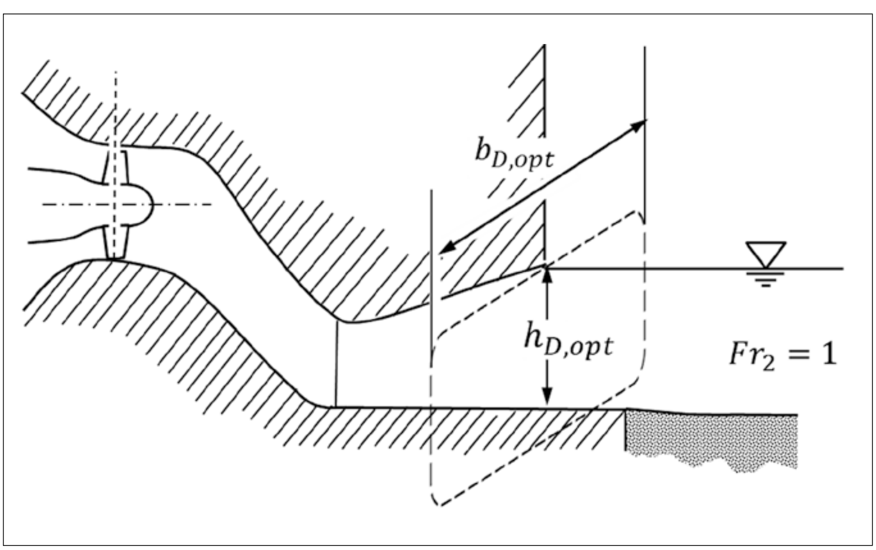

Figure 6: Rectangular diffuser for optimal operation (Source: Authors)

$$h_{C+} = \frac{h_C}{H_{eff}} = \frac{1}{2}\frac{q_+^2}{h_{D+}}\left(1 - \frac{h_{D+}}{h_+}\right)^2 \quad (9)$$

The hydraulic efficiency due to Carnot shock losses is:

$$\eta_C = \frac{1}{1 + \frac{h_C}{H_T}} \quad (10)$$

with $H_T = H_1 - H_2 - h_C - h_L$. Substituting Eq. (10) into Eq. (5) the coefficient of performance with Carnot shock loss included is:

$$\frac{C_{p,Carnot}}{\eta} = \eta_C\,(q_+, h_+)\,\frac{1}{2}\left(\frac{5}{2}\right)^{5/2} \cdot q_+\left(1 - h_+ - \frac{1}{2}\frac{q_+^2}{h_+^2}\right) \quad (11)$$

Figure 5 shows grey contour lines for the coefficient of performance without dissipative losses C_p/η (Eq. (5)). The black contour lines in Figure 5, with the same values contour levels, show an exemplary coefficient of performance with Carnot shock loss included (Eq. (11)). For values $h_+ < h_{D+}$ there is no Carnot shock loss, hence both contour plots are identical ($CP_{,Carnot}/\eta = C_p/\eta$). For values $h_+ > h_{D+}$ nonzero Carnot shock losses occur, hence $C_{P,Carnot}/\eta < C_p/\eta$ for all values of dimensionless tail water height h_+ and dimensionless tail water flow rate per width unit q_+.

4 Diffuser Design for Optimal Operation

As it is mentioned in the first section the optimal operation point is established by Pelz [2]. A diffuser should be designed to work in this operating point. To be more concrete: instead of aiming to lower the outlet velocity to zero, it is better to aim for an outlet velocity that leads to $Fr_2 = Fr_{2,opt} = 1$, since the specific energy of the tail water flow gets minimal at this point. This is a paradigm shift from the classic to the new approach justified by an increase of power output. For a rectangular cross section at the outlet (**Figure 6**), the optimal height of the diffuser is the optimal tail water height $h_{+,opt} = 2/5$ (Figure 2), hence $h_{D,opt} = 2/5\ H_{eff}$. The optimal volume flow rate per width unit $q_{2,opt} = g^{1/2}\ (2/5\ H_{eff})^{3/2}$ can be adjusted by variation of the width of the diffuser. For optimal operation the width calculates:

$$b_{D,opt} = b_{2,opt} = \frac{Q}{q_{2,opt}} \quad (12)$$

5 Conclusion

The coefficient of performance, compared to the hydraulic efficiency, allows a more general view on the efficiency of low head hydropower plants. It can be established by exclusive use of axioms, such as the first law of thermodynamics. Beside the dissipative losses that are considered in the hydraulic efficiency, the coefficient of performance also includes nondissipative losses of the system. The coefficient of performance is not meant to be a disproof of the hydraulic efficiency, but an enhanced approach that satisfies the special hydraulic conditions of low head hydropower, as the influence of operation points on the gross head and the open channel-flow boundary condition at the outlet. Nevertheless the hydraulic efficiency is a factor into the coefficient of performance and still a part of the new approach.

The design method for diffusers proposed in section 4 is quite basic. It is meant to adjust the tail water flow to near critical Froude numbers. By doing so, specific tail water energy can be lowered, to take advantage of the increased energy output, discussed in the examples of Figures 3 and 4. Nevertheless further design effort might be necessary to avoid flow phenomena causing dissipative losses, like separation and others. When the optimization no longer is aiming for near zero velocities at the outlet but near critical velocities the area ratio and hence, the length of the diffusers could be reduced significantly. Thus, beside the beneficial effect of higher energy output, lower investment costs are to be expected.

Manuel Metzler und Peter F. Pelz

Neue Methodik zur Saugrohrauslegung für einen optimalen Erntefaktor in der Wasserkraft

Die gängigen Methoden zur Auslegung von Saugrohren für kleine Fallhöhen zielen darauf ab, dissipative Verluste zu minimieren, d. h. den hydraulischen Wirkungsgrad zu maximieren. Die neue Methode setzt stattdessen die Maximierung des Erntefaktors als Ziel. Betz führte den Erntefaktor erstmals für Windturbinen ein und ermöglichte somit eine allgemeingültigere Diskussion der Effizienz, da er nicht nur die dissipativen, sondern auch die Verluste durch den Energiefluss im Nachlauf der Maschinen in die Bewertung der Effizienz mit einbezog. Der zweite Autor leitete analog zu Betz den Erntefaktor für offene Gerinneströmungen her. Die vorliegende Arbeit stellt einen Ansatz zur Auslegung von Kleinwasserkraftwerken und deren Saugrohre vor, mit dem der Betrieb bei optimalem Erntefaktor und somit die maximal mögliche Energieausbeute erreicht werden kann.

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