INFLUENCE OF COMPRESSIBILITY ON INCIDENCE LOSSES OF TURBOMACHINERY AT SUBSONIC OPERATION

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SUMMARY

The in situ scaling-up methods for the efficiency and pressure rise for axial and centrifugal fans give satisfying results in the design point for low Mach number flow only. At higher Mach numbers these methods result in a significant difference between prediction and measurement. To end up with a better understanding of the effects caused by the compressibility and incidence between incoming flow and profile a predictive physical model for the incidence loss for compressible flows is introduced. This model is validated by cascade measurements at different incidence angles and Mach numbers up to 0.7.

INTRODUCTION

Pelz and Stonjek developed a physical based scaling method for Reynolds number effects which has proven to be valuable for both axial and centrifugal fans [1]. Even though the Mach number is in most cases small we observed an influence of the Mach number on the measured efficiency curves of centrifugal fans. For very low speed fans there is still no need for a compressibility correction, but the industry aims at better fan performance resulting in a higher loading and pressure rise. This often results in higher rotational speed and hence higher flow velocities which make it necessary to include compressible effects into the scaling method. As it was said: this effect become visible in different fan characteristics measured by us (cf. [2], [3]).

In our turbocharger experiments [4] the circumferential Mach number $Ma \coloneqq u/a$ (the circumferential velocity is $u = \Omega\, D/2$, with Ω the rotational speed, D the diameter and the speed of sound a for inlet temperature) varies between 0.66 and 1.51 (cf. Figure 1). For such a high Mach number the Reynolds number effects, with $Re \coloneqq uD/\nu$

(kinematic viscosity for the the inlet temperature is denoted by ν), predicted by the model of Pelz and Stonjek [1] are negligible. Hence if we can predict the Mach number effects for compressors we can adapt this model to fans and considere both: Mach number and Reynolds number effects. Both effects are expected to be present for the fan efficiency curves shown in Figure 2.

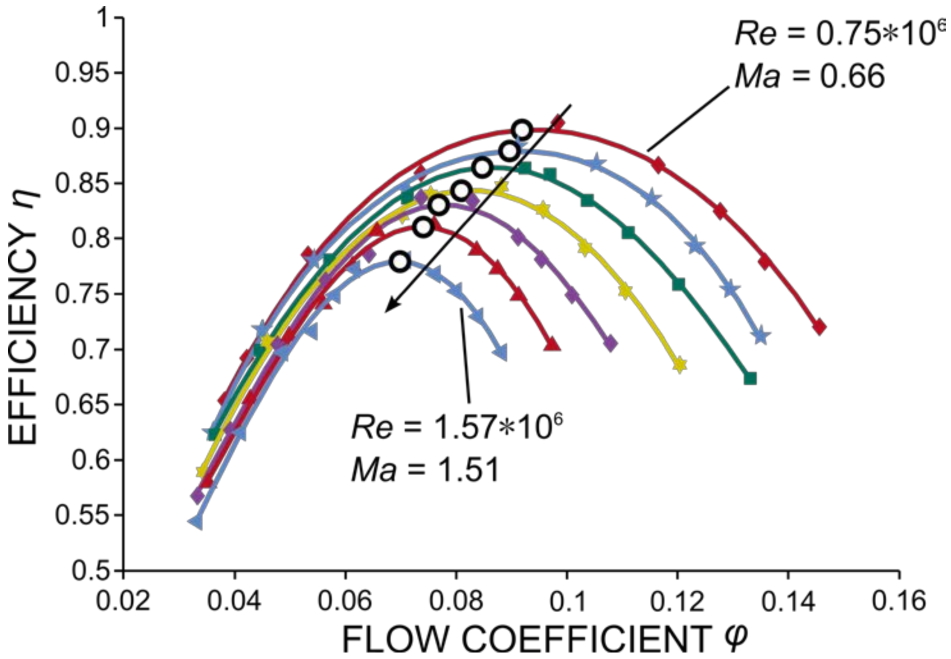


Figure 1: Measured efficiency versus flow coefficient for a compressor with Reynolds and Mach number varied. Peak efficiencies are marked by a circle. They are all aligned along a straight line [7].

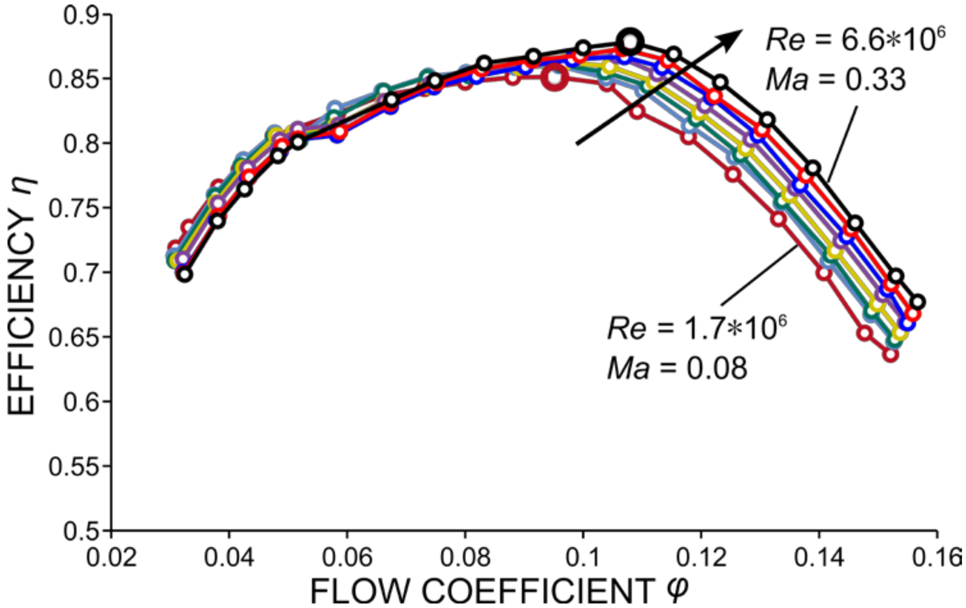


Figure 2: Variation of the best efficiency point of a centrifugal fan [6].

The compressor performance (Figure 1) shows a decrease of efficieny with increase of Mach number. For high Mach number flow the compressiblilty effects are in most cases dominant in comparison with friction, i.e. Reynolds number losses. On the other side it is well known that the friction losses decrease with increasing Reynolds number which is the basis of the Ackeret scaling methods and our methods published so far. If we denote the total enthalpy losses by h_l and define a loss coefficient as $\zeta \coloneqq 2h_l/u^2$ we can express the inefficiency of the machine $\varepsilon \coloneqq 1-\eta = \zeta/\lambda$ with $\lambda \coloneqq 2\Delta h_t/u^2$ (Δh_t rise in total enthalpy). The logarithmic derivative of the inefficiency yields

$$\frac{\mathrm{d}\varepsilon}{\varepsilon} = \frac{\mathrm{d}\zeta}{\zeta} - \frac{\mathrm{d}\lambda}{\lambda} \text{ or } \mathrm{d}\varepsilon = \varepsilon\frac{\mathrm{d}\zeta}{\zeta} - \varepsilon^2\frac{\mathrm{d}\lambda}{\zeta} \approx \varepsilon\frac{\mathrm{d}\zeta}{\zeta}.$$

The order of magnitude analysis gives $\varepsilon \sim 0.1$ and hence $\varepsilon^2 \sim 0.01$ for typical turbo machines. The term ε^2 can usually be neglected. Since $\zeta = \zeta(Re, Ma)$ the scaling method is hence

$$\Delta\varepsilon \approx \frac{1}{\lambda}\Delta\zeta \approx \frac{1}{\lambda}\frac{\mathrm{d}\zeta}{\mathrm{d}Re}\Delta Re + \frac{1}{\lambda}\frac{\mathrm{d}\zeta}{\mathrm{d}Ma}\Delta Ma.$$

For the Taylor expansion the preassumption was made, that both loss mechanismen $\zeta = \zeta_f(Re) + \zeta_w(Ma, shape)$ are independent of each other. This is known as the Froude assumption which is indeed valid for friction losses and wave losses. Infact compressible losses are in most cases wave losses and very much shape dependent wheras frictional losses are shape independent and in most cases can be treated by similarity to such flows like the one along a flat plate.

There is one more important information within the measurements shown in Figures 1 and 2. The point of best efficiency is shifted towards higher flow coefficient $\varphi \coloneqq 4\dot{V}/u\pi D^2$ (volume flow rate $\dot{V}$) as it is shown in Figure 1 and 2. Our measurements indicate (cf. internal report [3]) a shift of the peak efficiency towards higher flow coefficient and is indicated by an increase in Mach number.

Once more we give a motivation to understand the influence of Mach number effects on the efficiency maps of turbo machines. We do this by an analytic model and the outline of our paper is as follows: In the next section we derive a loss model to predict the Mach number dependent losses $\zeta_w(Ma)$. The result is discussed and compared to the case of incompressible flow. The fourth section contains a validation which includes the results of experimental investigations in a pressurized test rig. These experimental investigations are made with a constant Reynolds number and variable Mach number to investigate the compressible effect without a variation of the friction losses. Furthermore the results and the global behavior of the incidence loss for compressible flows are discussed and a proposal is made how to match the described model with a real axial or centrifugal fan.

LOSSES DUE TO A COMPRESSIBLE FLOW THOURH A CASCADE

Figure 3 shows schematically the flow through a cascade of infinitesimal thin and flat plates. The incidence angle is denoted by α. The thermodynamic state of the approaching flow is given by its pressure p_1 and temperature T_1. The Mach number of the approaching flow is denoted as $Ma = Ma_1 \coloneqq c_1/a_1$ (c_1 as the absolute flow velocity). To predict p_2, T_2, Ma_2 continuity equation, energy equation and momentum equation will be solved simultaneously for the control volume sketched in Figure 3. Thoma considered this case for incompressible flow already in 1922 [5].

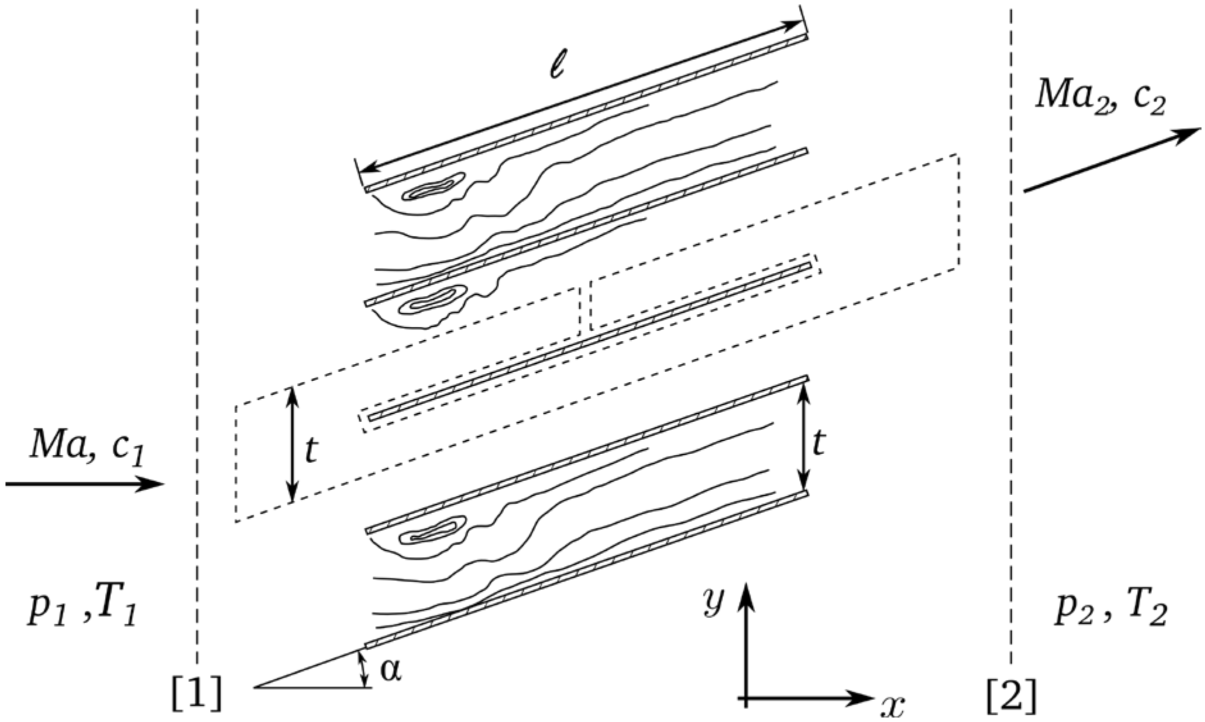


Figure 3: Schematic flow through a cascade of flat plates.

The spacing t and length l is such that congruent flow is reached. Since we consider wall friction within the loss coefficient $\zeta_f(Re)$ it is justified to neglect for the sake of simplicity this effect in our analysis. If one would like to include this effect here this would be no problem in principle.

The conservation of mass for the sketched control volume yields

$$c_1 \varrho_1 = c_2 \varrho_2 \cos\alpha. \tag{1}$$

The momentum equation yields the forces to the plate in *x*-direction

$$F_x = (p_1 - p_2)t + \varrho_1 c_1^2 t - \varrho_2 c_2^2 t \cos^2\alpha, \tag{2}$$

and in *y*-direction

$$F_y = -\varrho_2 c_2^2 t \cos\alpha \sin\alpha. \tag{3}$$

For negligible wall friction (see discussion above) this yields

$$F_t = F_x \cos\alpha + F_y \sin\alpha = 0. \tag{4}$$

If one would like to consider wall friction it would enter Equation (4) on the right side. We here choose the way to superimpose the wall friction loss (Froude's assumption).

Combining Equation (2), (3) and (4) results in the pressure difference

$$p_1 - p_2 = \varrho_1 c_1^2 - \varrho_2 c_2^2. \tag{5}$$

The conservation of energy for an adiabatic system is

$$h_1 + \frac{c_1^2}{2} = h_2 + \frac{c_2^2}{2}. \tag{6}$$

The three conservation laws (1), (5) and (6) for the unknown parameters c_2, ϱ_2, T_2, p_2, h_2 are completed by two constitutive equations, i.e. that the thermal and the caloric equation of state for an ideal gas $p = \varrho RT$ and $h = c_p T + const.$ with the definition of the Mach number $Ma \coloneqq c/a = c/\sqrt{\gamma RT}$ and the isentropic exponent $\gamma = c_p/c_v$ are used to derive the conservation laws for compressible flows.

The conservation of mass (1) yields

$$\frac{p_1}{p_2}\frac{\varrho_1}{\varrho_2}\left(\frac{Ma_1}{Ma_2}\right)^2 = \cos^2\alpha, \tag{7}$$

the momentum equation (5) gives

$$\frac{1}{Ma_1^2} + \gamma\left(\frac{Ma_1}{Ma_2}\right)^{-2} = \left(\gamma + \frac{1}{Ma_1^2}\right)\frac{p_1}{p_2}, \tag{8}$$

and the energy equation (6) reads

$$\left(\frac{Ma_1}{Ma_2}\right)^{-2}\left(\frac{p_1}{p_2}\right)^{-1}\frac{\varrho_1}{\varrho_2} = \frac{1 + \frac{\gamma - 1}{2}Ma_1^2}{\left(\frac{Ma_1}{Ma_2}\right)^2 + \frac{\gamma - 1}{2}Ma_1^2}. \tag{9}$$

The solution of this non-linear system of equations for the Mach number ratio yields

$$\begin{aligned}\frac{Ma_1}{Ma_2} = \big[(\gamma Ma_1^2 - Ma_1^2 + 2)^{-1}\big\{\gamma^2\cos^2\alpha\, Ma_1^4 + 2\gamma\cos^2\alpha\, Ma_1^2 \pm \\ \pm [\cos^2\alpha\,(\gamma Ma_1^2 + 1)^2(\gamma^2\cos^2\alpha\, Ma_1^4 + 2\gamma\cos^2\alpha\, Ma_1^2 + \\ + \cos^2\alpha - \gamma^2 Ma_1^4 + Ma_1^4 - 2\gamma Ma_1^2 - 2Ma_1^2)]^{1/2} + \cos^2\alpha - \\ - \gamma^2 Ma_1^4 + \gamma Ma_1^4 - 2\gamma Ma_1^2\big\}\big]^{1/2}.\end{aligned} \tag{10}$$

In Equation (10) the positive root is valid for subsonic incoming flow. The negative root should not be considered since supersonic flow shocks would strongly influence the flow. In the following discussion only the subsonic case is important. With the result (10) the static pressure ratio (8) and the density ratio is given by Equation (7) or (9).

Denton introduced the dimensionless measure $T_2\Delta s/(h_{t1} - h_1)$ for losses in turbomachines (Δs is the increase in entropy) [6]. From a thermodynamic point of view this definition is not justified, since we have $\Delta h_t = q + w$ from the energy equation and $\Delta s = q/T_* + \Delta s_{irr}$ from the second law of thermodynamics (with q as the specific heat flow). If we introduce the exergy ex as the thermodynamic "distance" to the ambient state T_0, s_0: $ex := h - h_0 - T_0(s - s_0)$ and if we substitute the second law of thermodynamics into the energy equation we end up with $\Delta ex = \eta\, w + \eta_C\, q$ with the aerodynamic efficiency

$$\eta := 1 - \frac{T_0\Delta s_{\mathrm{irr}}}{w} \tag{11}$$

and the Carnot efficiency

$$\eta_C := 1 - \frac{T_0}{T_*}. \tag{12}$$

Here the exergy-concept clearly guides the way to a physical motivated definition whereas Denton`s concept seems to be more practical motivated. He uses $T_* \approx T_2$ in his definition, i.e. the temperature at which the heat is transferred to the gas. Thermodynamics indicate the ambient temperature T_0 to be more appropriated. For an adiabatic system $\Delta s = \Delta s_{irr}$ and (11) yields

$$\varepsilon = \frac{T_0 \Delta s}{w} = \frac{\zeta}{\lambda}, \tag{13}$$

with

$$\zeta \coloneqq \frac{2T_0 \Delta s}{c_1^2} \approx \frac{2h_l}{c_1^2} \approx \frac{2\Delta p_l}{\varrho_1 c_1^2}, \quad \text{for } Ma_1^2 < 1. \tag{14}$$

With the definition (13) and (14) for the loss coefficient we are on truly physical grounds. In addition the definition fits perfectly into all definitions for the loss coefficient known in engineering science.

The total pressure in Equation (14) is

$$p_t = p\left(\frac{\gamma - 1}{2} Ma^2 + 1\right)^{\gamma/(\gamma-1)}. \tag{15}$$

The density and the velocity are always the value at the incoming area of the control volume and the total pressure drop $\Delta p_{t,l}$ can be calculated with the equations before.

Figure 4 shows the incidence loss as a function of the incidence and the Mach number. With increasing Mach number and incidence the losses increase as Figure 1 already indicates. Figure 5 highlights the ratio of the compressible and incompressible flow which is defined as

$$f \coloneqq \frac{\zeta(Ma)}{\zeta(0)}. \tag{16}$$

At an incidence angle of 5° and a Mach number $Ma = 0.3$ (red circle) the difference between both loss coefficients is about 6.5%.

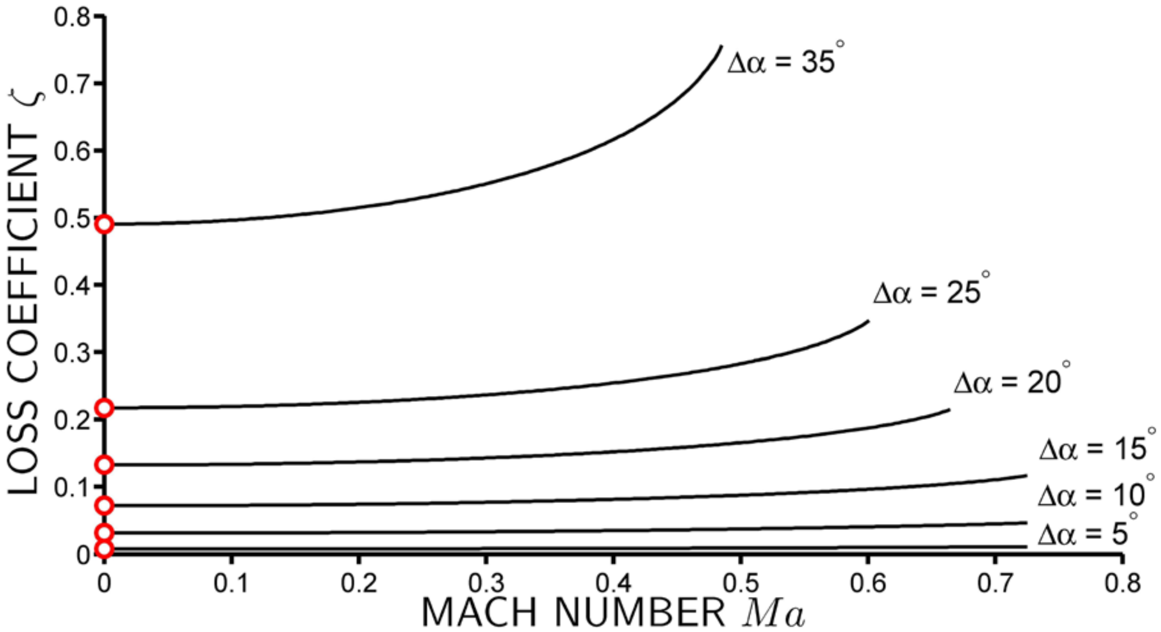


Figure 4: Incidence loss for compressible flows

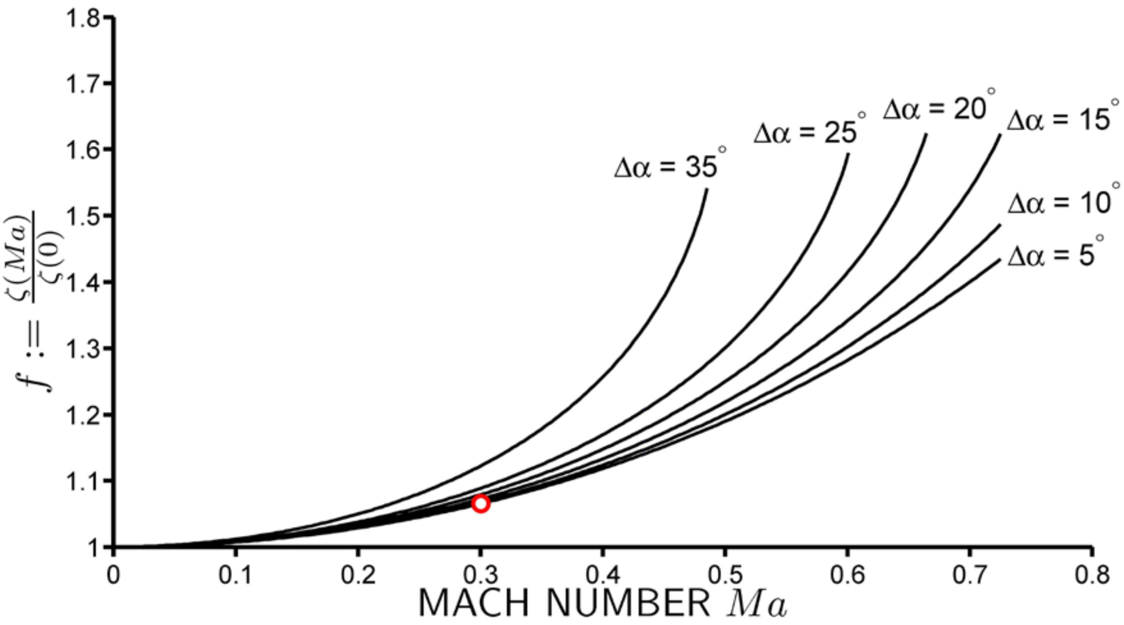


Figure 5: Correction function for the incidence loss

VALIDATION AND DISCUSSION

The previous described model made the assumptions of infinitesimal thin and flat plates (i), a plate congruent outflow (ii), a negligible wall friction (iii), a two dimensional flow with endless plate depth (iv) and an incoming flow Mach number which is less than the critical Mach number (v). That means shocks does not occur. The first three assumptions are not valid for a comparison with experimental data. Hence the blade respectively plate thickness and the kinetic energy loss due to the boundary layer is considered in form of a Borda-Carnot loss which is the Carnot loss for a compressible flow. The wall friction is determined by knowing the Reynolds number and the Mach number.

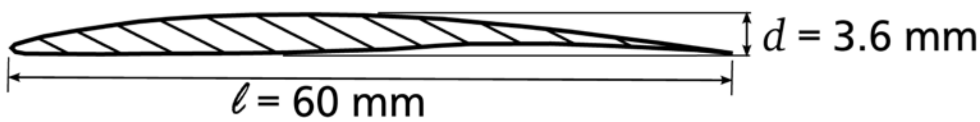


Figure 6: NACA 65-606 Profile in Bahr's investigations [7]

For the validation the experimental data from Bahr's publication „*Investigation on the influence of the profile thickness on the compressible two-dimensional flow through a compressor cascade*" are used [7]. Bahr focused on compressible effects in a compressor cascades. He used the pressurized test rig in Braunschweig, Germany which holds the possibility to run the experiments at different Mach numbers while keeping the Reynolds number constant. In the following investigation the Reynolds number is about $Re = 2 * 10^5$. The blade cascade consists of several thin and low curved

NACA profiles. The chosen one is shown in Figure 6 and the solidity $l/t = 1$. The reason for the use of this data are the thin and low curved profile and the high number of experimental results.

With the known Reynolds number and the assumption that the flow is turbulent[1], the turbulent wall friction factor for a flat plate can be estimated with Schlichting's equation for the hydraulically smooth case [8] and a further correction has to be done because of a change in Mach number:

$$c_{f,turb} = 0.455\,(\log Re)^{-2.58} = 0.0062, \tag{17}$$

$$c_{f,compr} = \frac{c_{f,turb}}{\sqrt{1 + 0.12\,Ma^2}}. \tag{18}$$

Equation (15) is from Krasnov [9] and is valid for Reynolds numbers up to 10^6. Inside the blade cascade the assumption of endless blade depth and a flat plate is not fulfilled. To include these facts the wall friction coefficient for both blade surfaces is raised by 50% and written by $c_f = 3c_{f,compr}$. A consideration of the blade thickness is done by the Carnot loss for compressible flows which is

$$\Delta p_{Carnot} = \frac{\varrho}{2}(c_1 - c_2)^2\,. \tag{19}$$

Rist [10] derives the Carnot loss for compressible flows in form of the velocity ratio

$$\frac{c_2}{c_1} = \frac{1}{(\gamma+1)Ma^2}\left[\gamma Ma^2 + \frac{A_2}{A_1} - \sqrt{(Ma^2-1)^2 + \left(\frac{A_2}{A_1} - 1\right)\left(1 + 2\gamma Ma^2 + \frac{A_2}{A_1}\right)}\right]. \tag{20}$$

This equation depends only on incoming flow parameters (Ma^2 and c_1), the flow material itfelf and the flow area ratio A_2/A_1.

The final equation of the model is

$$\zeta = \zeta_{carnot} + \zeta_{incidence} + c_f\,. \tag{21}$$

The results of the loss model are shown in Figure 7. All calculations with the loss model for the Mach numbers 0.3, 0.5, 0.6 and 0.7 show a satisfied agreement with the experimental data in the range of -8° or -9° up to +4°. The important point is that the low curved profile cascade is showing the same behavior at changing Mach number and incidence angles than the model. With a higher incidence the loss coefficient rises which is basically the modelled incidence loss. The wall friction loss is constant for every experiment with constant Mach number as we have seen above (Equation (14) & (15)). The Borda-Carnot loss is also constant for a specific Mach number but the area ratio changes with different incidence angles.

[1] Bahr wrote that the boundary layer over the profile had a laminar and a turbulent part but he did not mentioned the place of the transition. The beginning of a separation bubble and the reattachment depends on the degree of turbulence, the Reynolds Number, the roughness, the Mach number and the profile itself [11] & [13].

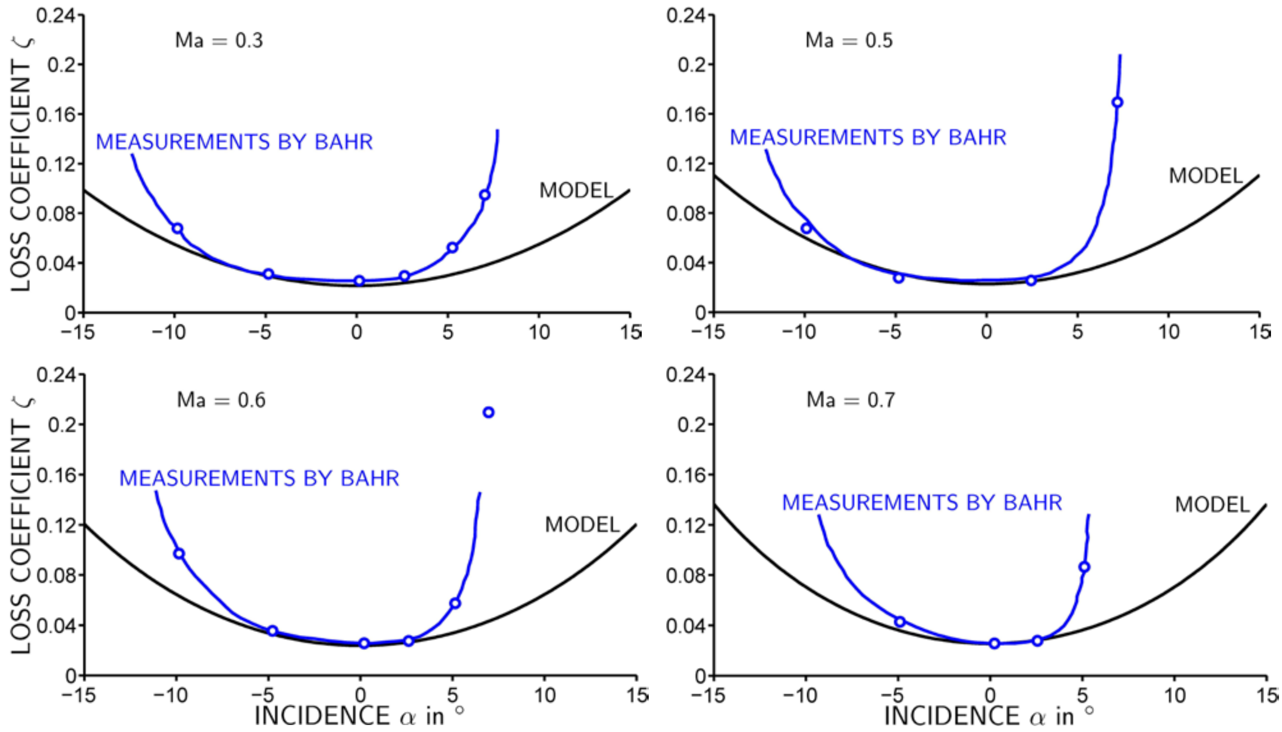


Figure 7: Comparison of the model and validation datas at $Re = 2 * 10^5$ *and a solidity of* $l/t = 1$.

This behavior is again displayed in Figure 8 with the summarized loss coefficient. The lowest line shows the Carnot loss. The line above indicates the combined Carnot and friction loss coefficient and finally the black line is again Equation (21). This figure shows the significance of the compressible treatment. An increase of the Mach number from 0.3 to 0.7 shows a difference of about 18% at an incidence angle of 5°.

The model is made for a non-curved plate thus positive and negative incidence angle show exactly the same values. Bahr's measurements are showing a dissymmetry because of the curvature of the profiles which are low but nevertheless have an impact on the redirection. For high incidence angles the model fails since the model does not consider trailing edge separations even though these kind of separations occur with rising incidence angles. These separations cause less flow redirection and the assumption of an outflow angle which is equal to the blade angle, is no longer fulfilled. Bahr detected this separations by measuring the static pressure over the profile surface. For higher inflow Mach numbers than 0.7 or 0.8 the model is difficult to validate because shocks can cause larger losses inside the cascade. The additional effects listed above can be added to the modular build up compressible flow model.

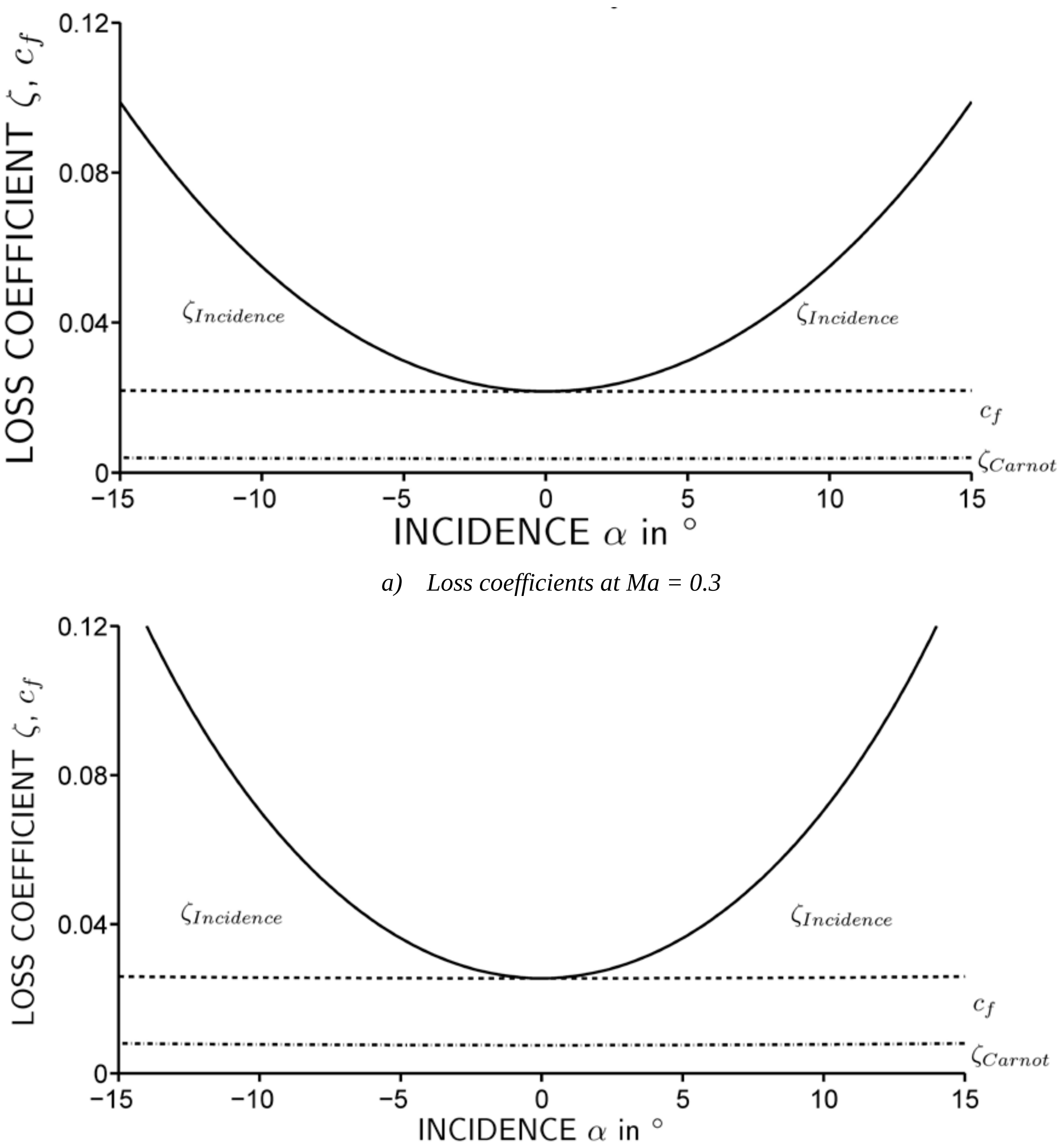
For an easier usage of the incidence loss model a function can be used to take the compressible effects into account. For example a fitting function of $f = f(Ma, \alpha)$ which is shown in Figure 5 is a good assumption below $Ma < 0.6$ and $\alpha < 25°$. The function is

$$\frac{\zeta(Ma)}{\zeta(0)} \approx 1 + Ma^2 + A(\alpha)Ma^2 \tag{22}$$

with

$$A(\alpha) = 9/2 - 5\alpha/2 + 23\alpha^2. \tag{23}$$

Below the Mach number and incidence angle limit the error is less than 1.6%. Furthermore this fitting function shows the important parameters inlet Mach number Ma and the incidence α with their exponents which describes their global behavior. With this equation a correction of the incompressible calculations can be done easily afterwards to take the compressible effects of the incidence between flow and blade into account.



a) Loss coefficients at Ma = 0.3

b) Loss coefficients at Ma = 0.7

Figure 8: Overview of the modelled loss coefficients at Ma = 0.3 and 0.7

CONCLUSION

In this paper a physically based model is analytically derived to calculate the incidence loss for a compressible flow through a cascade of flat plates and a correction function for this model is introduced. This model describes the behavior of the flow inside a fan if it is operated at part- or overload conditions. Under these conditions the incidence angle is not optimal and causes losses due to redirection and separation bubbles.

The introduced model made the following assumptions:

1. infinitesimal thin and flat plates,
2. plate congruent outflow,
3. negligible wall friction,
4. the inlet Mach number is smaller than the critical Mach number $Ma < Ma_{crit}$,
5. two-dimensional flow with endless plate depth.

For a more physical description an entropy based loss coefficient is used and compared to the incompressible results. The resulting correction function can be used, for instance, for correcting an incompressible fan model afterwards. The incidence loss of a compressible flow shows the same limits as the incompressible case for a Mach numbers equal to zero which approves this model.

With the help of the compressible Carnot loss and a compressible friction coefficient the non-zero height of the real profiles and the existing wall friction are modelled afterwards. A comparison to experimental data show a good agreement with the global behavior but it has to be optimized further to include the friction loss directly in the conservation laws. The height and the curvature of the blades could be included as well, but at this time both additions make it difficult to find analytical solutions for this model. Additionally, separations or the degree of redirection of the outflow have to be considered as well to reach better results at lower incidence angles than -9° or higher than +4°.

The limits for the compressible incidence loss are the shocks which occur in transonic flows. It has not yet been proven if these effects are described correctly, but for an application for axial or centrifugal fans a limit up to Mach numbers 0.7 is high enough.

The advantage of the introduced loss coefficient is that it can be used for axial and centrifugal fans and for flows with an incidence angle between flow and flow guidance parts. For an application the fan geometry, the flow velocity and direction as well as the used fluid and its ambient conditions have to be known.

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