TWO COMMON FALLACIES IN LOW HEAD HYDROPOWER – A CRITICAL DISCUSSION

Manuel Metzler, Peter F. Pelz
Chair of Fluid Systems
Technische Universität Darmstadt, Germany

SUMMARY: *The optimization of draft tubes in low head hydro hydropower stations is up to now commonly carried out by CFD analysis, or based on empiric data. The reasons for that is a lack of analytical optimization methods. The present paper provides a new analytic approach to optimization of draft tubes. Furthermore it names and corrects two common fallacies in hydropower literature. The optimized draft tube design satisfies the optimal operation condition for hydropower in an open channel flow [1].*

Keywords: low head hydropower, open surface, draft tube, optimization

Optimal Operation for Low Head Hydropower

A common criterion for efficiency of low pressure hydro power plants is the hydraulic efficiency η of the facility

$$\eta \coloneqq \frac{P_T}{\rho g H_T Q}\,. \tag{1}$$

It is the ratio of shaft power to the hydraulic energy at a particular operating point. It is a measure for the dissipation within the hydraulic machine. The turbine head $H_T := H_1 - H_2$, (see Equation (7) and Figure 1) and the volume flow rate Q are considered as given by the hydrologic conditions of the tidal current or river. Anyway, while operating low pressure power stations it can be observed, that the hydraulic machine's operating point itself has an influence on the available Head and volume flow rate. The operating point determines which fraction of the available power P_{avail} can be converted into shaft power P_T. The coefficient of performance C_P is defined as that ratio

$$C_P := \frac{P_T}{P_{avail}}\,. \tag{2}$$

Since the efficiency of the system is normalized by the available power P_{avail}, the first step is to define the available power. Figure 1 shows a schematic hydro power plant with its state variables. Velocities at different points are described by the symbol u, water levels symbol is h and ground levels symbol is z. The effective head is defined as $H_{eff} := h_1 + \frac{u_1^2}{2g} + z_1 - z_2$. The second author defines the available power, as the power a hypothetic ideal machine (without tail water) can convert into shaft power [1]

$$P_{avail} := 2\left(\frac{2}{5}\right)^{5/2} \rho g^{3/2} H_{eff}^{5/2} b\,. \tag{3}$$

The coefficient of performance C_P can be expressed as a function of volume flow rate per width unit $q_2 = Q/b_2$ and tail water level height h_2. Those two parameters define the operating point of the installed hydraulic machine. With dimensionless operating parameters q_+ and h_+ Pelz [1] establishes the graph shown in figure 2 (see equation 4).

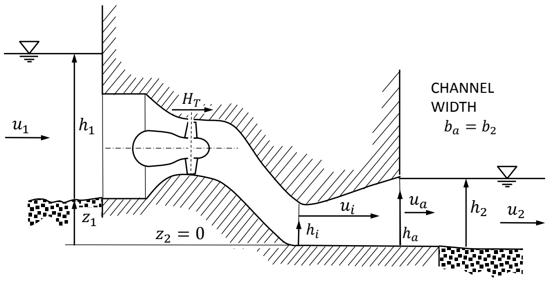

Figure 1: Schematic cross section of low head hydropower plant (Case II)

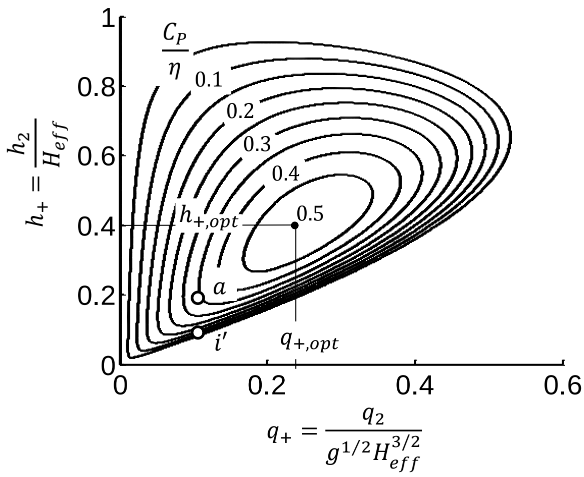

Figure 2: Coefficient of performance as function of operating point

As a consequence of the first law of thermodynamics, the coefficient of performance is

$$C_P = \frac{\eta}{2}\left(\frac{5}{2}\right)^{5/2} q_+ \left(1 - h_+ - \frac{1}{2}\frac{q_+^2}{h_+^2}\right)\,. \tag{4}$$

The optimum of equation (4) is $C_P/\eta = 0.5$. In other words: half of the available power in an open channel-flow can be converted into shaft power.

$$q_{+,opt} = \left(\frac{2}{5}\right)^{3/2} \text{and} \quad h_+ = \frac{2}{5}\,. \tag{5}$$

The parameters in equations (5) describe the optimal operating point for a hydropower plant in an open-channel flow. As an example for the significance of the coefficient of performance, figure 3 shows two cases. In case A, half of the available power is convertible ($C_P/\eta = 0.5$). The nonconvertible power is represented by the black bar P_{NC}. With a hydraulic efficiency of $\eta = 0.5$, shaft power P_T and dissipated power P_L, both are 25% of the available power P_{avail}. In case B the fraction of convertible power is $C_P/\eta = 0.25$. Although hydraulic efficiency in case B $\eta = 0.8$ is far better than in case A, shaft power P_T is less than in case A. The coefficient of performance determines the actual efficiency while the limitation on hydraulic efficiency for optimization of hydro power plants can be misleading.

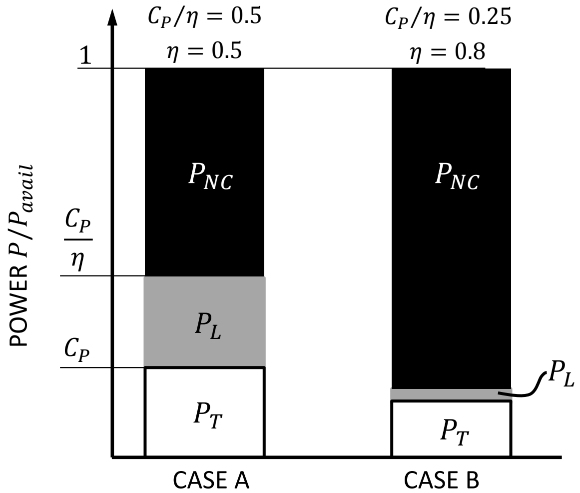

Figure 3: Example for coefficient of performance

Terminology of Total Pressure

In hydro power literature there is a multitude of terms in the context of pressure. In some cases they are contradictory. The terms for the present paper follow the definitions of [2] and will be explained within this section.

$$p_t := p_s + p_d + p_z \qquad (6)$$

Equation 6 is the definition for total pressure p_t, with static pressure p_s, often named pressure or hydrostatic pressure. It is caused by the water column. The second term on the right side of Equation 6 $p_d = \rho u^2/2$ is the hydrodynamic pressure with velocity u and specific weight of the fluid ρ. Becker has no particular term for p_z[2], but defines the sum $p^* = p + \rho g z$ as piezometric pressure.

The ratio of the total pressure divided by specific weight and the gravitational constant is the total head

$$H = \frac{p_g}{\rho g} = h + \frac{u^2}{2g} + z \qquad (7)$$

The total head H has the dimension of energy per volume unit [1], [2]. The summand h is the symbol for pressure head, which is equivalent to the height of the water column i.e. the water level height in an open channel-flow. It is the pressure energy per volume unit. The velocity head $u^2/2g$ represents the kinetic energy per volume unit. The ground level z is the potential energy per volume unit.

State of the Art Draft Tube Theory

With technological progress in hydro power a trend from machines of big diameter at low specific speed to machines of smaller diameter at high specific speed can be observed. With the decreasing size of machines investment cost per installed power decreased as well. Otto Cordier proofs, based on empirical data [3], the correlation of specific speed and specific diameter of hydraulic machines. The correlation between specific diameter and investment cost is considered by Pelz [4]. Due to small diameters, the meridian speed of flow along the hydraulic machine increased compared to ancient machines of low rotational speed and big diameter. Hence, the percentage of kinetic energy in the total head is high (see Equation (7)) at the rotor outlet. Experiments proofed that draft tubes with big diffusors enhanced the shaft power significantly, for machines of high rotational speed. The common explanation for this effect is a recuperation of energy within the draft tube, due to conversion of pressure head into velocity head. Since the cross section area of the diffusor increases in direction of the flow, averaged speed decreases. Kinetic energy that leaves the power plant is considered as lost. This loss is expressed by Deniz [5] and Raabe [6] as the ratio of kinetic energy at the rotor outlet and the turbine head defined as the difference of total head between head water and tail water [5].

$$k = \frac{u_2^2}{2g(H_1 - H_2)} \ . \qquad (8)$$

Due to Deniz [5] this loss is to be minimized. Furthermore Deniz claims, that the pressure gain leads to a pressure drop at the rotor outlet. Without a doubt this pressure drop has been measured and can be considered as proofed. Only the explanation is not adequate how the following section will show.

The Impossibility of Energy Recuperation by Pressure Conversion

The term energy recuperation, that is used in hydropower literature to describe the function of the diffusor ([5], [7]), is misleading since the energy content of a flow cannot be changed by the conversion of hydrodynamic pressure into hydrostatic pressure. Neither the amount of energy that flows out the hydro power plant with the tail water can be lowered, as this section will show.

In hydropower literature plants are modeled analytically under use of the Bernoulli Equation. For the scheme given in figure 1 the Bernoulli Equation writes

$$H_T = h_1 + \frac{u_1^2}{2g} + z_1 - h_2 - \frac{u_2^2}{2g} - z_2 - h_L \ . \qquad (9)$$

All of losses due to dissipation in between head water and tail water summarized in the head loss h_L. The common explanations for the functions of draft tubes with diffuser nozzles [7], [6], [5], [8], based on equations (8) and (9) assume, that the velocity at the diffuser's outlet can be lowered without a change of h_2. The definition of $k = u_2^2/2g(H_1 - H_2)$ as a loss includes this statement. It is in contradiction to the first law of thermodynamics as equation (10) shows.

$$H_2 = H_{eff} - H_T - h_L \ . \tag{10}$$

Equation (10) represents the first law of thermodynamics for an open channel-flow. It shows that the total head in the tail water of a hydropower plant is constant for a given operating point of the hydraulic machine. Regardless of the tail water flow speed the same amount of energy leaves the plant with the tail water flow. Also there is no explanation for a pressure drop at the rotor outlet in the first law of thermodynamics. The conversion from kinetic energy into pressure energy but only changes its quantity.

The wrong assumption u_2 could be reduced, while keeping h_2 constant, has its origin in the fact, that the Bernoulli Equation represents conserved quantities along a streamline. This streamline, in case of hydro power plants, usually starts at the surface of the headwater and ends at the surface of the tail water. Since the atmospheric pressure vanishes, the algebraic description is easier this way. Since the level h_2 itself is necessary to describe the endpoint of the streamline, it is constant for all considerations based on Bernoulli's Equation.

Consideration of Free Surface

To explain the function of the diffuser in an adequate manner, the effects of the free surface have to be included in the analytic considerations. Therefore the first law of thermodynamics and mass conservation has to be used. The first law of thermodynamics for the considered case is equal to Equation (9). The main difference compared to the Bernoulli Equation is, that first law of thermodynamics is not only valid along a streamline, but as conservation equation for the integral of particular parameters in any cross section of an open channel. This fact becomes obvious when mass conservation is added to the analytic description of the flow.

For stationary operating of a hydro power plant, volume flow rate Q can be considered as constant. We assume that the tail water flow is incompressible and inside a rectangular channel of width b_2. Mass conservation than writes

$$Q = u_2 h_2 b_2 \ . \tag{11}$$

All statements of this paper are valid for any kind of nonrectangular channels as well. The rectangular channel is only chosen for reasons of simple analytic description. Equation (11) is an integration of the flow velocity over a cross section of the channel and obviously not valid for a streamline.

Solving Equation (11) for u_2 the tail water velocity is

$$u_2 = \frac{Q}{b_2 h_2} \ . \tag{12}$$

Equation (12) substituted in Equation (9) leads to

$$H_T = H_{eff} - h_2 - \frac{Q^2}{2g\, b_2^2 h_2^2} - h_L \ . \tag{13}$$

Figure 4 shows an example for the possible states of an open channel-flow, i.e. the possible water depths (pressure head in terms of pressure) for each total head. It is valid for a rectangular channel of constant width for a given volume flow rate Q. Both, total head and water depth are divided by H_{eff} for a dimensionless description of the flow. The state variables at the channel cross sections are shown in figure 1 (1, i, i‘, a). At cross section 1 the total head equals the effective head. The function of the diffuser is discussed for two exemplary cases. Case I is a draft tube without diffuser (see figure 5). Case II is a draft tube with diffuser (see figure 1).

Case I: Draft tube without diffuser (see Figure 5)

In this case the index i‘ represents the position at the outlet of the draft tube without diffuser. The velocity head at the turbine outlet $u_{i'}^2/2g$ is comparatively high, while the pressure head $h_{i'}$ is low.

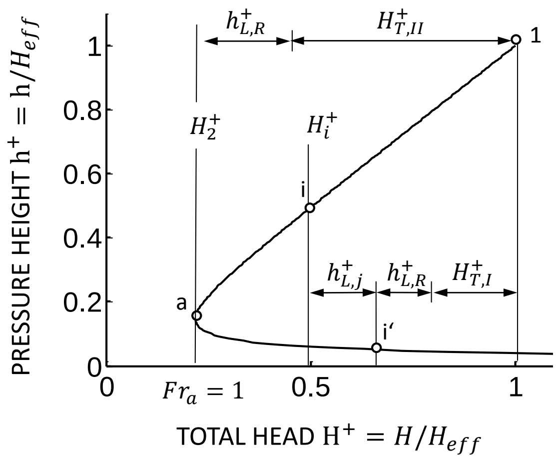

Figure 4: State diagram for open channel-flow

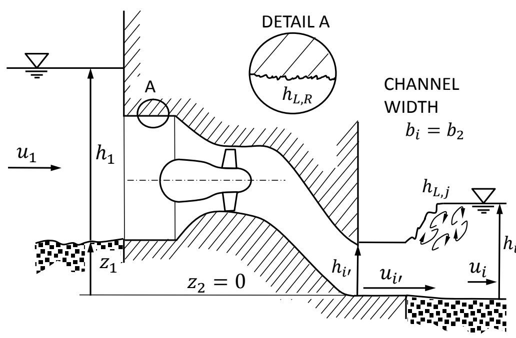

Figure 5: Draft tube without diffuser (Case I)

Point i in Figure 4 is the tail water state of case I. The transition from state i‘ to i is known as hydraulic jump. For each total head within the codomain of the total head for a given volume flow rate Q and $Fr = u/\sqrt{gH} \neq 1$ there are two possible water depths. One of them with $Fr_{i'} > 1$ is called supercritical, the other with $Fr_i < 1$ is subcritical.

The biggest possible difference in total Head between Headwater and tail water occurs when the tail water flow is critical ($Fr_2 = 1$ (s. Figure 4)). This statement has been established by the second author [1].
In case I, the flow is supercritical at the draft tube outlet ($Fr = u_{i'}/\sqrt{gh_{i'}}$>1). A hydraulic jump occurs in between draft tube outlet and tail water. The total head in the tail water H_i/H_{eff} is comparatively high. The shaft power is low due to an inefficient operation point and high losses.

$$\frac{H_{T,I}}{H_{eff}} = 1 - \frac{h_i}{H_{eff}} - \frac{Q^2}{2g\, b_2^2 h_i^2 H_{eff}} - \frac{h_{L,I}}{H_{eff}}. \qquad (14)$$

The dissipative loss $h_{L,I} = h_{L,R} + h_{L,j}$ is the sum of the losses inside the power plant $h_{L,R}$ (wall friction and others, see figure 5) and the losses that are related to the hydraulic jump from state i‘ to state i, $h_{L,j}$.
For all of the cases considered in the present paper, the width of the tail water is assumed to be the same ($b_i = b_a = b_2$).

Case II: Draft Tube with Diffusor (see Figure 1)
In case II the flow is converted from state i‘ to state a within the diffuser. As a consequence of this conversion the total head of the tail water decreases. Also there are no hydraulic jump losses. Hence the convertible fraction of the effective head H_{eff} increases

$$\frac{H_{T,II}}{H_{eff}} = 1 - \frac{h_a}{H_{eff}} - \frac{Q^2}{2g\, b_2^2 h_a^2 H_{eff}} - \frac{h_{L,II}}{H_{eff}}. \qquad (15)$$

The increase of turbine head then is

$$\Delta H_{T,I,II} = H_{T,II} - H_{T,I}\ . \qquad (16)$$

The pressure drop at the rotor outlet can be explained by equation (16). The diffuser enhances the coefficient of performance C_P (see figure 2) but the optimal coefficient of performance is not attained for case II. A simple design strategy for a diffuser operating at $C_P/\eta = 0.5$ is proposed in the following section.

Diffusor Design for Optimal Operation
As it is mentioned in the first section the optimal operating point is established by the second author [1]. A diffuser should be designed to work in this operating point. For a rectangular cross section at the outlet, the optimal height of the diffusor is the optimal tail water height $h_+ = 2/5$ (see figure 2), hence $h_{D,opt} = h_{2,opt} = \frac{2}{5}H_{eff}$. The optimal volume flow rate per width unit $q_{2,opt} = g^{1/2}\left(\frac{2}{5}H_{eff}\right)^{3/2}$ can be adjusted by variation of the width of the diffuser. For optimal operation the width calculates

$$b_{D,opt} = b_{2,opt} = \frac{Q}{q_{2,opt}}\ . \qquad (17)$$

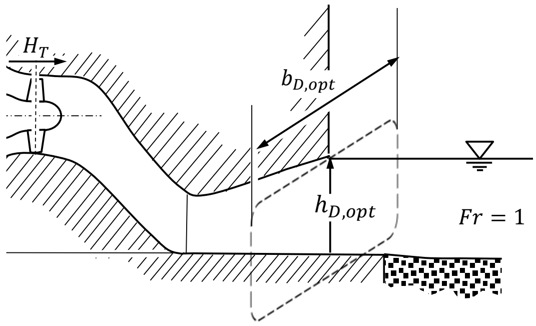

Figure 6: Rectangular diffuser for optimal operation

Conclusion
For an adequate discussion of efficiency in hydropower, the coefficient of performance is the measure to be discussed. Limitation on the discussion of hydraulic efficiency can be misleading.
For an adequate explanation of the function of a draft tube, effects of the free surface have to be considered. These effects cannot be described by Bernoulli Equation but by the first law of thermodynamics. The geometry of the diffuser can contribute to achieve the optimal coefficient of performance $C_P/\eta = 0.5$.

References

[1] P. Pelz, „Upper Limit for Hydropower in an Open-Channel Flow,“ Journal of Hydraulic Engineering ASCE/; DOI: 10.1061/(ASCE) HY.1943-7900.0000393, November 2011.

[2] E. Becker, Technische Strömungslehre, Stuttgart: Teubner, 1986.

[3] O. Cordier, „Ähnlichkeitsbedingungen für Strömungsmaschinen,“ Brennstoff-Wärme-Kraft, p. 337 ff., 1953.

[4] P. Pelz und M. Metzler, „OPtimization of Power-Specific Investment Costs for Small Hydropower,“ in 17. Internationales Seminar Wasserkraftanlagen, Wien, 2012.

[5] S. Deniz, in Saugrohre bei Flusskraftwerken, Zürich, Prof. Dr. Vischer, 1990, p. 68 ff..

[6] J. Raabe, in Hydro Power, Düsseldorf, VDI Verlag, 1989, p. 203 ff..

[7] J. Giesecke und E. Mosonyi, in Wasserkraftanlagen - Planung, Bau und Betrieb, Heidelberg, Springer Verlag, 2009, p. 539 ff..

[8] H. Sigloch, in Strömungsmaschinen: Grundlagen und Anwendungen, München, Hanser Verlag, 2013, p. 207 ff..