TECHNICAL OPERATIONS RESEARCH (TOR) EXEMPLIFIED BY A HYDROSTATIC POWER TRANSMISSION SYSTEM

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The possibilities of fluid power system design include different components and control strategies for the same function. Thus the final topology is usually designed by the practical experience of an engineer and afterwards verified. “Technical Operations Research” (TOR) first encourages a phase of description and then uses mathematical optimization tools, known from Operations Research, to develop and structure a technical system. In contrast to parameter optimization, the topology of the system is not fully required, but can be created within the optimization process. The main advantage of this approach is the guarantee for global optimality within the model. We present an optimal topology for a hydrostatic power transmission system.

Keywords: Optimization, topology, system, power, efficiency.
Target audience: System designer, carrier.

1 Introduction

The final structure of a system always depends on its designer. Within several degrees of freedom different designers take different decisions, even if the requirements are the same. For power generation a maximum of harvestable energy is usually given /1/ /2/, but an achievable minimum for working or power supply systems is not available. The efficiency of a single component can be measured or computed, but a combination of components with high efficiency does not mandatorily lead to a highly efficient system. The question “Could it be better?” remains as an objective reference stays unknown.

Our aim is to create a decision support for designers, carriers or politicians, using an optimization setup that includes the generation of a reference. Note that an optimization of the system including binary decisions is different from a parameter optimization of individual components or control parameters /3/. While parameter optimization is already in widespread use (e.g. as meta-heuristics in the form of genetic algorithms) to optimize rotary speeds or valve adjustments, it is only a tool to improve an existing system design. Topology optimization starts with the creation of the system structure. The possibility of parameter optimization has to be considered within the structural design. Both processes are not exclusive, but can support each other.

Concept options like device selection, topology variation or control strategy, do not simply complement one another, but superpose each other. No wonder that a human designer cannot consider all possible options. In this paper we want to give an example for Technical Operations Research by the example of a hydrostatic power transmission system, i.e. a fluid power system. As a typical task, the rotational mechanical power of a pump shall be transformed into translational movement of a piston. Hydrostatic accumulators are applicable but not mandatory, pressure-level and size can be set by the optimization program.

2 Technical Operations Research

2.1 The TOR-Method

At the Chair of Fluid Systems, Technische Universität Darmstadt, an innovative method for system design is developed: Technical Operations Research (TOR) stands for objective design and optimal results. Compared to other design processes /4/ this one is divided into a phase of deciding and a phase of acting (Figure 1). Hence, it does not replace VDI 2221, but complements it. In the first three steps the information for the whole system is collected, while in the last four steps the system is designed with the help of different models.

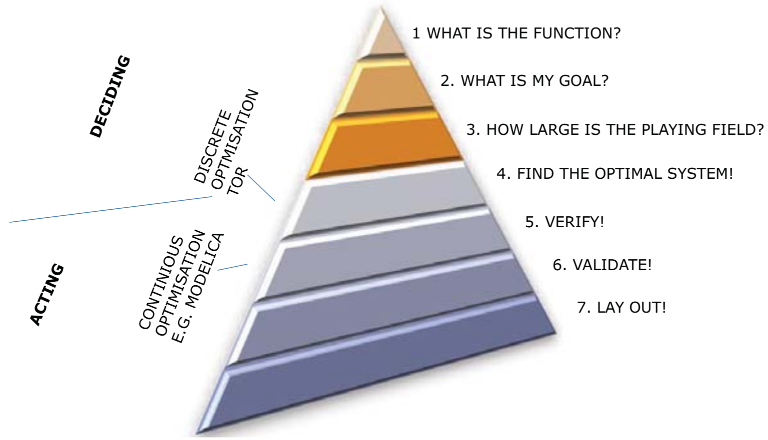

Figure 1: TOR-Pyramid /5/

The steps of TOR in detail are:

DECIDING

1. What is the function? The first step of design is always the question for the basic function of a system. If a system cannot fulfill its basic function, then it does not work and is totally useless. This function can be clearly determined, but might have uncertainties in the expected load history.

2. What is my goal? The function can be usually fulfilled in different ways. Minimal investment cost, maximal efficiency or long life time can only be examples for several different individual claims. The goal is always subjective and will differ, depending by who is asked. In contrast to the function, the goal is not absolutely necessary for the system to be working. A fluid power system, which has a poor system-efficiency, will still transfer energy. Nevertheless the answer to this question influences the design process: If the goal is changed, the outcoming system will change, too.

3. How large is the playing field? A common optimization task is the strategy of chess: The decision for the next move can be picked from a nearly endless number of possibilities, but the movement of any figure has to stay within the field. Staying in this metaphorical language, the rules for the optimization must be given: The variance of the system has to be described carefully. The playing field has to be described by the designer. It contains the full construction set, any parameter setting for the components, the rules for combining the components and the physical laws for the system behaviour, described in algebro-differential equations.

ACTING

4. Find the optimal system! Now, as any options and restrictions are defined, the system is varied to find the optimal solution. In an abstract formulation a suited graph is extracted and the optimization problem can be written as a Mixed Integer Linear Program (MILP). Using linear optimization guarantees a global optimal solution within the model considering the defined functions and goals. The optimization program replaces the intuition of the system designer.

5. Verify! The suggested solution needs to be reviewed and verified by the system designer. A model-system with concentrated parameters is used for the first verification. Modelica /6/ can be used for this scope. The model of this step stands in close interaction step with 4. The physical-technical-economic description used in this step is needed to build up the abstract model.

6. Validate! The validation of the system happens with the help of higher-dimensional computational models or experiments.

7. Lay Out! The last step of TOR is the realisation of the proposed and validated system.

2.2 Linear optimization

The innovation within TOR is the application of quantitative models and methods of Operations Research to technical-economic problems. OR methods are used in economical business, like logistics and led to enormous savings – within technical environments these methods are rarely known and uncommon, but can be applied as follows:

The mathematical description is divided into variables, parameters, constraints and objective function. Variables are that part of our mathematical program which will be filled by an algorithm. After the optimization process, the variables will present us the best possible solution. Parameters are input data. Constraints describe which features define a valid system and the objective function discriminates good from bad systems.

We formulate the problem with the help of linear constraints, resulting in so called deterministic equivalent programs /7/, which form large mixed-integer linear programs (MIP /8/) with a special block-structured coefficient matrix. With the help of this modeling formalism, the physical-technical constraints can be translated into a collection of linear constraints (equations or inequalities) and the life cycle costs can be estimated with a linear objective function, depending on different scenarios that may occur.

The main advantages of MIPs compared to other optimization approaches are /9/:

1. If a solution exists, the solver will find it within finite time.
2. The solution terminates with a verifiable global optimum.

3 Optimization of a fluid power system

In the following section a first example for the optimization of fluid power systems is given.

3.1 Task

The optimization task is the development of a hydrostatic power transmission system. The output of the system (load) is well defined by a repeated sequence of piston movements. The movements are specified in Table 1 by the force F and the velocity v of the plunger. The load cycle has to be repeated ten times, afterwards the piston has a break. Goal for the optimization is to generate a system with lowest possible costs (operation and purchase) during the lifecycle. Within the topology of the hydrostatic system accumulators of various sizes and pressure levels are available. The accumulator can be filled either while the piston is working or during the break. Figure 2 shows the incomplete system with load, supply and optional components.

The requested decisions to the optimization program are:

1. Which accumulator is necessary?
2. If needed: Which is the pressure level for the operation and which is the requested size?
3. What is the best control setup for covering the load?

ID	ACCUMULATED TIME t in s	PLUNGER VELOCITY u in m/s	APPLIED FORCE F in kN
1	0 - 20	0.1167	100
2	20 - 50	0.0667	500
3	50 - 57	0.0333	1000
4	57 - 62	-0.0333	-700
5	62 - 112	-0.0833	-100
6	112 - 132	0.0667	300
7	132 - 162	0.0833	400
8	162 - 202	-0.1000	-200

Table 1: Expected load cycle.

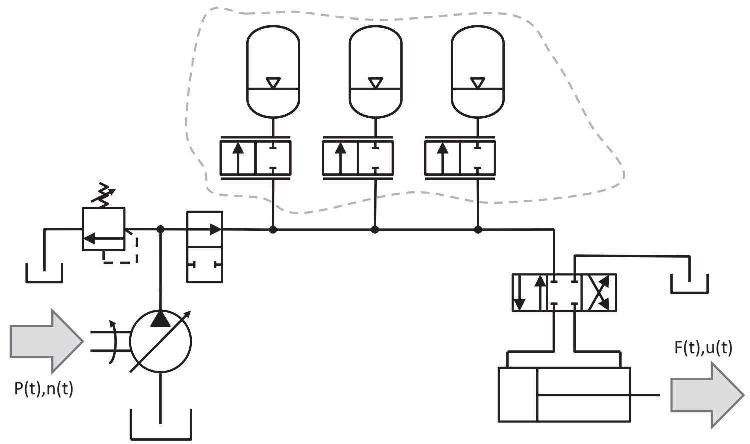

Figure 2: Power In- and Output

Generic characteristics of the construction kit describe the "playing field". Figure 3 illustrates the characteristic curves of the pump, the valves and the accumulator. The pump is speed-controlled within a range of 30 % to 100 % of its nominal speed. Its efficiency in any duty point depends on the rotational speed and the pressure head. Two types of 2,2-valves are taken into account: A switching valve and a proportional valve with a linear characteristic and zero lap.

Figure 3 shows the valve characteristics for the nominal pressure drop p_{nom}. For any other pressure drop the volume flow is calculated by the similarity law /10/:

$$\dot{V} = \dot{V}_{nom}\sqrt{\frac{\Delta p}{\Delta p_{nom}}}. \quad (1)$$

The objective of this research and the major task of an accumulator is "to enable a better energy utilization in the hydraulic system" /10/. The volume of the gas cushion of the hydraulic accumulator is assumed to be large compared to the fluid volume. Thus the accumulator-pressure p_{acc} is supposed constant and taken into account as an optimization assignment.

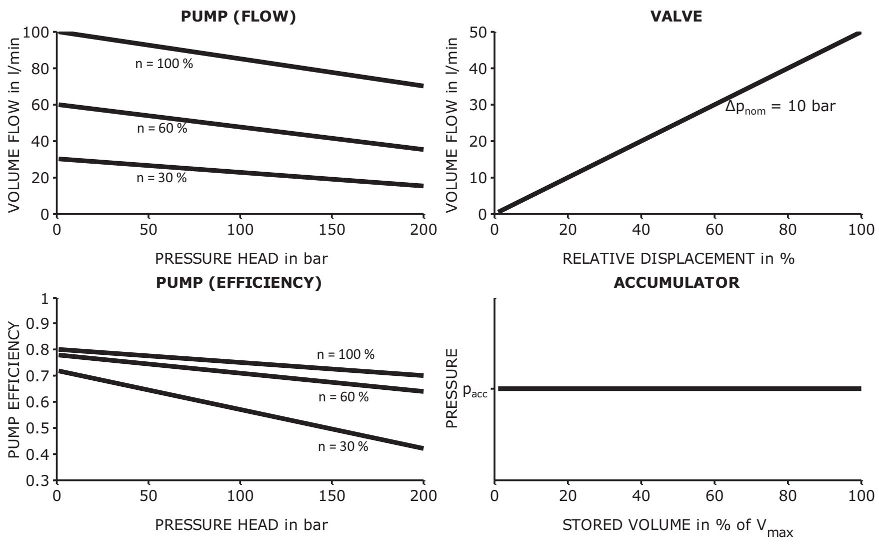

Figure 3: Characteristic curves of the construction set

The cost model for the optimization is also a simplified and generic one: It is assumed, that mechanical energy for the pump costs 0.10 € / kWh. The costs for an accumulator depend on its pressure-level and size. An accumulator with a maximum pressure of 80 bar and a size of 50 l costs precisely 1180 €. Any other component of the system is mandatory, so their costs have no influence on the optimization.

3.2 Mathematical Model

The problem can be stated as follows: Given a defined in- and output, a construction kit and a specification of load collectives: Compare all possible systems which satisfy every load and choose the one with the lowest expected lifecycle costs. In a first step, an interconnection of hydraulic components can be abstracted as a (mathematical) graph *G = (V,E)* with vertices *V* and edges *E*. A vertex or an edge represents a component from the kit, e.g. a connection, a switchable connection (switching valve) an adjustable connection (proportional valve) or a pressure rise (pump). The decisions of the optimization problem are described by variables: First and second stage variables. In the first stage, the optimization program has to decide, if a component is needed and thus bought. In the second stage, a bought component can be turned on/off and controlled due to cover a specific load.

An accumulator is a PT_1-Element in the whole circuit that cannot be simply added to a stationary graph. The graph needs to become "time expanded" /11/. The whole graph must be duplicated for every time step (Figure 4) and a stored volume of a certain pressure is transported as a volume flow via an edge through time back to its own origin (red edges).

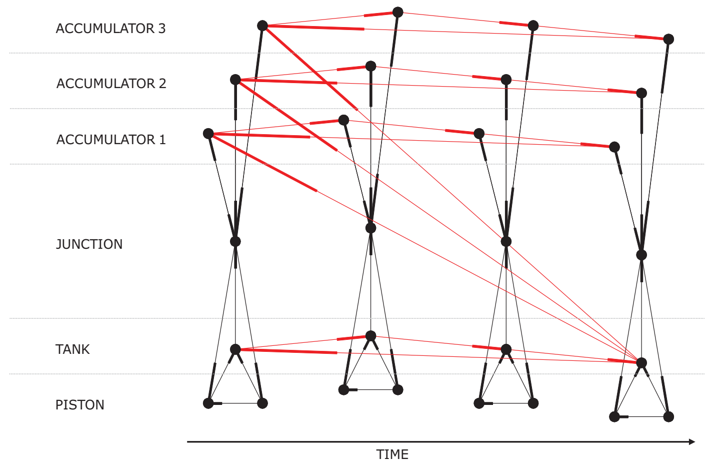

Figure 4: Mathematical graph of the hydrostatic power transmission system

The first-stage decision (i.e., the investment) is given by a set of binary variables (indicators) y_v for each vertex V and $y_{i;j}$ for each edge $(i;j)$ of the graph. In the objective function, these indicators are weighed with the purchase costs of their respective components. After the investment has been made, the load and so the control settings will change over the system's life time. The system with all topological options can be modelled as a graph of the construction kit. Any technical reasonable solutions for covering the load now are subgraphs. For each possible system, i.e. for each subgraph defined by a purchase decision, the technical specification implies a set of scenarios: The system must be able to satisfy every prospected load.

Additionally, we have to guarantee that each working point lies on the component's characteristic curve. We do this by generating a sufficient number of points on this curve and forcing the model variables on the linearized curves defined by these base points. Equation (1) is not linear, but has the form

$$y = c\sqrt{x} \qquad (2)$$

and thus must be reformulated by piecewise linearization. Figure 5 shows the linearization of Equation (2) in the range of [1,100] using secants. The distribution of the interpolation points hat major influence on the accurateness of the linearization an hence on the correctness of the result: On the left the linearization points are distributed equal-distant (1) and on the right logarithmic (2). The model formulation is straight-forward and can be looked up in literature /12/. The task now is to find a specific subgraph of the complete graph that describes a well defined system for which the purchase costs plus the expected energy costs over the given load scenarios are minimal.

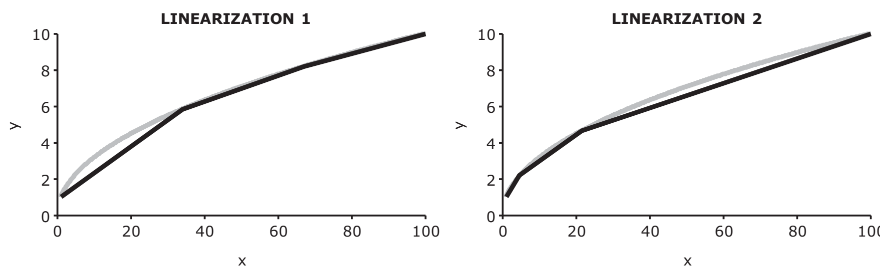

Figure 5: Linearization examples

3.3 Result

The result of the optimization includes a topology and thus a buying decision for the hydraulic accumulators and the controller rules for any point of the load. Without any accumulators the estimated energy costs for the whole lifecycle are 18040 €. By investing 210 € in a buffer, the lifecycle costs can be lowered to 17420 €. Thus the savings are 410 €. The topology of the optimal solution is shown in Figure 6.

The controlling strategy for the optimal solution is as follows: The accumulator is filled during time step 6 and used during time step 8. In the break the accumulator does not need to be refilled. Although using the accumulator generates losses, it enables the pump to run in a better operating point and store the energy.

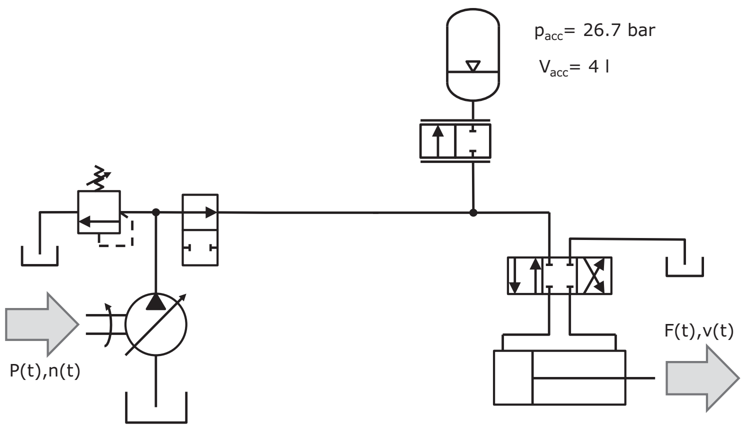

Figure 6: Topology of the optimal System designed by TOR

Applying the TOR-method and optimization program to a technical task leads to not necessarily obvious solutions. The addition of topological options to the playing field is much easier than the simulation of every system configuration one could think of. Given other load cycles or larger variance one could find other optimization problems. For instance more accumulators could be useful, if another piston is added to the system. Additionally the number and type of the pump could be taken into account for playing field.

4 Summary and Conclusion

Technical Operations Research (TOR) facilitates the optimization of technical-economical systems. The new methodical approach provides a system design divided in the phases of deciding and acting. In the decision-phase the system is described by its function, the subjective goal of the builder and the playing field which contains all options and constraints. The acting phase covers the mathematical optimization, the verification, validation and the layout of the system. Linear optimization allows to find and guarantee for a global optimal solution.

TOR can be applied to every fluid power system. An abstracted model of the hydraulic components on a mathematical graph is linearized and computed with the goal of lowering the working costs. The example shows the development of an optimal accumulation strategy for hydrostatic energy. Higher investment costs for buying an accumulator pay back in lower energy costs over the lifecycle. This information can only be extracted by considering all possible system designs in the given playing field. Questioning a whole system instead of improving only one component pays back.

Nomenclature

Variable	*Description*	*Unit*
c	Constant	[1]
i	Index	[1]
j	Index	[1]
n	Relative Revolution speed	[1]
p	Pressure	[bar]
p_{acc}	Accumulator pressure level	[bar]
t	Time	[s]
u	Velocity	[m/s]
x	Variable	[1]
y	Variable	[1]
E	Edges of G	[1]
F	Force	[kN]
G	Graph	
P	Power	[kW]
V	Vertices of G	[V]
V_{acc}	Accumulator volume	[l]
$\dot{V}$	Volume flow	[l/min]
$\dot{V}_{nom}$	Nominal Volume flow	[l/min]
Δp	Pressure drop	[bar]
Δp_{nom}	Nominal pressure drop	[bar]

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