Proceedings of the ASME Turbo Expo 2014 Conference
Turbo Expo 2014
June 16-20, 2014, Duesseldorf, Germany

GT2014-25403

INTRODUCTION OF AN UNIVERSAL SCALE-UP METHOD FOR THE EFFICIENCY OF AXIAL AND CENTRIFUGAL FANS

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ABSTRACT

Acceptance tests on large fans to prove the performance (efficiency and total pressure rise) to the customer are expensive and sometimes even impossible to perform. Hence there is a need for the manufacturer to reliably predict the performance of fans from measurements on down-scaled test fans. The commonly used scale-up formulas give satisfactorily results only near the design point, where inertia losses are small in comparison to frictional losses. At part- and overload the inertia losses are dominant and the scale-up formulas used so far fail. In 2013 Pelz and Stonjek introduced a new scaling method which fullfills the demands ([1], [2]). This method considers the influence of surface roughness and geometric variations on the performance. It consists basically of two steps: Initially, the efficiency is scaled. Efficiency scaling is derived analytically from the definition of the total efficiency. With the total derivative it can be shown that the change of friction coefficient is inversely proportional to the change of efficiency of a fan. The second step is shifting the performance characteristic to a higher value of flow coefficient. It is the task of this work to improve the scaling method which was previously introduced by Pelz and Stonjek by treating the rotor/impeller and volute/stator separately. The validation of the improved scale-up method is performed with test data from two axial fans with a diameter of 1000 mm / 250 mm and three centrifugal fans with 2240 mm / 896 mm / 224 mm diameter. The predicted performance characteristics show a good agreement to test data.

NOMENCLATURE

Latin Symbols

a	Speed of sound
A_{B}	Wetted Area
b	Width
B	Constant
c	Absolute velocity
c_{f}	Friction coefficient
c_{d}	Drag coefficient
C	Constant
C_{s}	Constant for tip clearance / gap width
D	Diameter
f	Weight factor
F	Force
h	Enthalpy
k	Relative roughness height
K	Roughness height
l	Chord length
L	Characteristic length
Ma	Mach number
n	Rotational speed
p	Pressure
P	Power
$quality$	Quality
r	Radius
Re	Reynolds number
s	Relative tip clearance / gap width

S	Tip clearance / gap width
shape	Shape
u	Circumferential velocity
U	Circumference
w	Relative velocity
V	Loss distribution factor
$\dot{V}$	Volume flow rate
x	Coordinate
y	Coordinate
Y	Specific work
z	Number of blades

Greek Symbols

a	Speed of sound
α	Blasius constant
β	Stagger angle
γ	Isentropic exponent
ε	Inefficiency
η	Efficiency
κ	Ratio of lengths
λ	Power coefficient (compressors)
ν	Kinematic viscosity, hub tip ratio
ρ	Density
σ	Specific speed
φ	Flow coefficient
ψ	Pressure coefficient
Ω	Angular velocity

Subscripts

a	Outer
f	Frictional
i	Inner
imp	Impeller
lam	Laminar
l	Loss
m	Model, meridional
P	Plate
ref	Reference
rotor	Rotor
spi	Spiral casing / volute
stator	Stator
t	Total, tangential
turb	turbulent
0	Design
1	Inlet impeller
2	Outlet impeller
3	Inlet stator
4	Outlet stator

THE IMPORTANCE OF EFFICIENCY SCALING

In addition to sound intensity, reliability and robustness, the efficiency of a turbo machine belongs to the most important properties of the machine. The efficiency is for all turbomachines and hence for all fans a measure for the quality of the transformation of mechanical energy supplied by the shaft to utilizable fluid energy which is impressed to the flow inside the machine. By raising the efficiency, the same amount of utilizable fluid energy can be transmitted to the flow with a lesser energy consumption.

The present paper continues the previous work of the authors [2] which deals with a new analytical scaling method. For the purpose of readability we introduce the present paper similar as we did it in the past.

Specific work $Y = Y(\dot{V}, \Omega, D, \nu, a, K, S, shape)$ and efficiency $\eta = \eta(\dot{V}, \Omega, D, \nu, \rho, a, K, S, shape)$ of a turbomachine change as the following physical quantities are varied:

- machine size, given by the rotor/impeller diameter D,
- rotational speed $\Omega = 2\pi n$,
- kinematic viscosity ν,
- density ρ,
- compressibility measured by the speed of sound a,
- typical roughness height K and
- gap width (centrifugal fans: gap between shroud and inlet) or tip clearance S (axial fans).

By means of dimensional analysis the number of independent parameters can be reduced by 3. This yields to $\eta = \eta(\varphi, Re, Ma, k, s, shape)$, $\psi = \psi(\varphi, Re, Ma, k, s, shape)$. The dimensionless products are

- flow coefficient $\varphi = 4\dot{V}/uD^2\pi$ with the circumferential speed given by $u = \Omega D/2$,
- Reynolds number $Re = uD/\nu$,
- Mach number $Ma = u/a$,
- relative roughness $k = K/L$ with the characteristic length L,
- relative gap width or tip clearance $s = S/L$,
- efficiency $\eta = 1 - P_l/P$ wherein P is the applied power and P_l is the sum of dissipative losses and
- pressure coefficient $\psi = 2Y/u^2$.

Scaling methods serve to predict the change in efficiency and pressure coefficient with the change of one or more of the listed dimensionless parameters for a flow coefficient φ. The shape of the machine is described by a finite number of dimensionless parameters, such as the ratio of chord length to the rotor/impeller diameter $\kappa_l = l/D$ for example.

SCALING METHODS

For several reasons, reliable, easy to apply and general valid scaling laws are needed for design but also for application engineers. The scaling laws are needed for the purposes of

- calculating the behavior of a full-scale (smaller or greater) machine from model test data obtained from a scaled machine,
- knowing the characteristic of one machine family containing machines of different scale by measuring only one machine,
- predicting the change in efficiency with changed rotational speed for the same machine,
- predicting the loss of efficiency for increased surface roughness

and other reasons. The most important point is the first one: As the experimental evaluation of the operational behavior and particularly the efficiency of large machines, e. g. fans to ventilate tunnels and mines or to conduct combustion air and smoke gas in power plants, cannot be carried out with original sized machines on test stands due to geometric and power limitations, down scaled models are used which however usually do not indicate complete analogy in flow and shape. As a result, the efficiency as well as the dimensionless pressure rise of model and original sized machine is not identical. On account of this a calculation of the higher efficiency compared to the efficiency of the model machine is necessary (i. e. efficiency scaling).

A scaling method should

- be physically based and (hence) reliable,
- understandable and easy to apply,
- universal, i.e. applicable to centrifugal as well as to axial machines of different specific speed,
- account for the aerodynamic quality of the machine,
- be reliable not only at the best point but also at off peak condition.

Although many scaling methods were developed in the past, only the physically based ones are mentioned in this work for conciseness reasons. The first physically based scaling method can be traced back to Pfleiderer in the year 1946 [3]. He was guided by the thought, that the inefficiency $1-\eta$ is proportional to the friction coefficient c_f. For hydraulically smooth surface $c_\mathrm{f} \sim Re^\alpha$ the ratio of inefficiencies from full scale to model (subscript 'm') yields

$$\frac{1-\eta}{1-\eta_\mathrm{m}} = \left(\frac{Re}{Re_\mathrm{m}}\right)^\alpha . \tag{1}$$

According to pipe flow analogy with turbulent flow and hydraulically smooth wall α was set to -0.25 ... -0.1. Ackeret in 1948 (published by Mühlemann [4]) improved the method of Pfleiderer, by taking inertia losses into account

$$\frac{1-\eta}{1-\eta_\mathrm{m}} = V\left[1+\left(\frac{Re}{Re_\mathrm{m}}\right)^\alpha\right] , \tag{2}$$

where the loss distribution factor V was arbitrarily set to $1/2$. Hess and Pelz [5] considered the loss distribution factor depending on the flow coefficient $V(\varphi)$. This takes the increase of inertia losses at off design operation into account.

Casey and Robinson [6] published a semi empirical scaling method where the difference in efficiency is given by

$$\Delta\eta = \eta - \eta_\mathrm{m} = -B_\mathrm{ref}(\sigma)\frac{\Delta c_\mathrm{f}}{c_\mathrm{f,ref}} . \tag{3}$$

B_ref is an empirically determined function of specific speed. The disadvantage of Eqn. (3) and Eqn. (2) is that both methods need empirical functions which depend on specific machine. Hence, there is always an uncertainty in applying these methods.

In 2013 Pelz and Stonjek published an new method of scaling the efficiency [1], [2]. The method is based on the assumption, that the inertia losses in model machine and original sized machine remain unaltered. The method consists mainly of two steps: At first, the efficiency is scaled up in the point of best efficiency of the model machine. Secondly the efficiency characteristic is shifted to higher values of flow coefficient.

The new developed scaling method is based on the total derivative of the inefficiency $\varepsilon = 1-\eta =: c_\mathrm{d}/\lambda$:

$$\frac{\mathrm{d}\varepsilon}{\varepsilon} = \frac{\mathrm{d}c_\mathrm{d}}{c_\mathrm{d}} - \frac{\mathrm{d}\lambda}{\lambda} . \tag{4}$$

Eqn. (4) is exact. As shown in [1], for the differential Eqn. (4) follows the difference as:

$$\Delta\varepsilon = \varepsilon(quality, \sigma)\left[\frac{\Delta c_\mathrm{f}}{c_\mathrm{f}} + \Theta\frac{\Delta s}{\varepsilon\lambda}\right] , \tag{5}$$

with

$$\Theta = \begin{cases} C_s\lambda^2, & \text{axial fan} \\ 4\mu d_s \psi^{3/2}/\varphi & \text{centrifugal fan} \end{cases} . \tag{6}$$

Principally an altered gap width from model machine to original sized machine is contained therein. In this context, models for tip clearance loss related to Karstadt and Pelz for axial fans [7] and gap loss related to Pfleiderer for centrifugal fans [3] were integrated. As the focus of this paper are not gap or tip clearance losses we set $\Delta s = 0$ in the following. This yields

$$\Delta\varepsilon = \varepsilon(quality, \sigma)\frac{\Delta c_\mathrm{f}}{c_\mathrm{f}} . \tag{7}$$

Eqn. (7) has two exeptional advantages: Firstly, there are almost no empirical constants in this formula. It should be always the task to omit as far as possible any empirical relation to gain a truly universal, physically based scaling method. Secondly, the shape of the machine, given by the specific speed σ, and the aerodynamic quality are implicitly considered by the inefficiency of the model machine.

The shift of the point of best efficiency to higher values of the flow coefficient is founded by a changed displacement thickness of the boundary layer at the blades and is calculated by

$$\Delta\varphi = -\frac{1}{C}\Delta c_{\mathrm{f,P,rotor/imp}}\,. \tag{8}$$

We gave a physically explanation for the shift in flow rate in [1]. But calculating the displacement thickness of a plate with pressure gradient and surface curvature with analytic equations is not a trivial task and do not fullfill industrial requirements of application. In fact, we have determined the constant C from measurement data of the axial ($C = 0.25$) and centrifugal fans ($C = 0.1$), we have installed in our laboratory. Therefore we have only one free empirical parameter left, which is a great advantage compared to other methods.

DIFFERENT EFFECT OF IMPELLER AND VOLUTE ON THE PERFORMANCE

In a preceding CFD study we examined the loss distribution of impeller and volute for a specific centrifugal fan ($D_2 = 896\,\mathrm{mm}$, see Tab. 1). Fig. 1 shows the importance of the volute with respect to power loss for the fan we considered. It has to be mentioned that this type of loss analysis cannot lead to a statement about the frictional losses that are relevant for efficiency scaling. Nevertheless it gives a strong hint that both the rotor/impeller and the volute/stator have to be considered separately in a physically based scaling method. In this context it becomes obvious that a scaling method taking only the rotor/impeller into account would always be misleading. That means, that the friction coefficient in Eqn. (7) has to be determined seperately for rotor/impeller and stator/volute. We show how to do this in the next section.

DETERMINING OF WEIGHT FACTOR AND REYNOLDS NUMBER

In [1] we have defined the friction coefficient universally by

$$c_{\mathrm{f}} =: \frac{2h_{\mathrm{l,f}}}{u_2^2}\,. \tag{9}$$

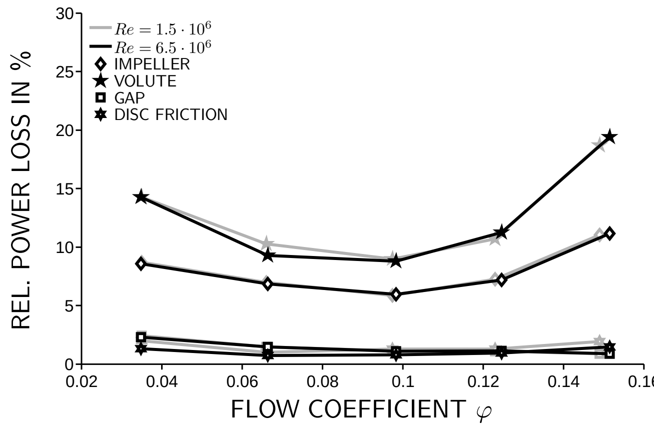

FIGURE 1. RELATIVE POWER LOSS BY CFD COMPUTATION

Defining it for the i-th component of the machine we get

$$c_{\mathrm{f},i} =: \frac{2h_{\mathrm{l,f},i}}{u_2^2}\,. \tag{10}$$

In contrast the definition of the friction coefficient for a flow along a flat plate is

$$c_{\mathrm{f,P},i} =: \frac{2W_i}{A_{\mathrm{B},i}\rho w_i^2}\,, \tag{11}$$

wherein W_i is the drag of the plate and $A_{\mathrm{B},i}$ is the wetted area. Equalizing the resulting power loss from both equations

$$\dot{m}h_{\mathrm{l,f},i} = W_i w_i \tag{12}$$

yields to

$$c_{\mathrm{f},i} = f_i(shape,\varphi)c_{\mathrm{f,P},i} \tag{13}$$

with

$$f_i(shape,\varphi) = \frac{4A_{\mathrm{B},i}}{D_2^2\pi}\left(\frac{w_i}{u_2}\right)^3\frac{1}{\varphi}\,. \tag{14}$$

Thus we write for the overall friction coefficient of the machine

$$c_{\mathrm{f}} = \sum_{i=1}^{N} f_i(\varphi,shape)c_{\mathrm{f,P},i}(Re_i,k_i)\,. \tag{15}$$

In the same way, we have to act with the Reynolds number for each component Re_i which is needed to determine the friction coefficient $c_{\text{f,P},i}$ in Eqn. (15). Since the Reynolds number is a measure of friction we define the i-th Reynolds number by the characteristic relative velocity $w_{\text{char},i}$, a characteristic length L_i and the kinematic viscosity of the flow ν:

$$Re_i = \frac{w_{\text{char},i} L_i}{\nu}\,. \tag{16}$$

As already mentioned in context with turbo machines the definition of the Reynolds number with the diameter of the impeller D_2 as characteristic length and the circumferential velocity u_2 at the outer diameter of the impeller as characteristic velocity is reasonable, because the diameter of the impeller is a concise length of the machine and the circumferential velocity dominates under normal conditions:

$$Re = \frac{u_2 D_2}{\nu}\,. \tag{17}$$

Eqn. (16) and Eqn. (17) yield to

$$Re_i = \frac{w_{\text{char},i}}{u_2}\frac{L_i}{D_2} Re\,. \tag{18}$$

The application of Eqn. (14) and Eqn. (18) is shown in the next section.

IMPELLER

CENTRIFUGAL FAN Concerning the velocity triangles the relative velocity at the inlet and the outlet of the impeller is

$$w_1 = \frac{D_2^2}{4 D_1 b_1 \sin\beta_1} u_2 \varphi \tag{19}$$

and

$$w_2 = \frac{D_2^2}{4 D_2 b_2 \sin\beta_2} u_2 \varphi\,. \tag{20}$$

With Eqn. (18) and Eqn. (19) we get the Reynolds number for the channel of the impeller

$$Re_{\text{imp}} = \frac{D_2 l_{\text{imp}}}{4 D_1 b_1 \sin\beta_1} Re \varphi\,. \tag{21}$$

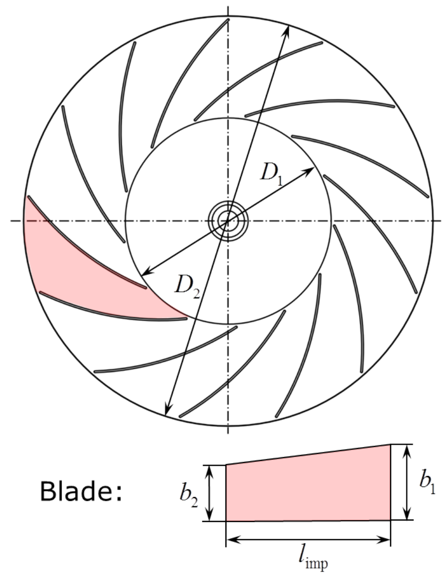

FIGURE 2. SHAPE OF THE IMPELLER

The wetted surface area of the impeller channels is calculated with the surface area of the blades, hub and shroud (Fig. 2):

$$A_{\text{B,imp}} = \frac{1}{2}(D_2^2 - D_1^2)\pi + (b_1 + b_2) l_{\text{imp}} z\,. \tag{22}$$

With the average velocity in the impeller channel

$$\overline{w} = \frac{w_1 + w_2}{2} \tag{23}$$

and the wetted surface area $A_{\text{B,imp}}$ we get the weight factor

$$f(shape,\varphi) = \frac{A_{\text{B,imp}} D_2^4}{128\pi}\left(\frac{1}{D_1 b_1 \sin\beta_1} + \frac{1}{D_2 b_2 \sin\beta_2}\right)^3 \varphi^2 \tag{24}$$

for the impeller of the centrifugal machine.

AXIAL FAN A comparison of the shape of axial and centrifugal fans makes clear, that the calculation of the Reynolds number and the weight factor have to differ from that of centrifugal machine. The velocity at the inlet and outlet of the rotor is

$$w_1 = \frac{1}{(1-\nu^2)\sin\beta_1} u_2 \varphi \tag{25}$$

and

$$w_2 = \frac{1}{(1-\nu^2)\sin\beta_2} u_2 \varphi \,, \tag{26}$$

wherein ν is the hub tip ratio of the rotor. With Eqn. (18) and Eqn. (25) we get the Reynolds number for the channel of the rotor

$$Re_{\text{rotor}} = \frac{l_{\text{rotor}}}{D_{\text{a}}(1-\nu^2)\sin\beta_1} Re\varphi \,. \tag{27}$$

The wetted surface area is determined by the stagger angle of the rotor blades $\beta_{0,\text{rotor}}$, the length of the blades l_{rotor}, the hub-tip-ratio ν and the outer diameter of the rotor D_{a}:

$$A_{\text{B,rotor}} = l_{\text{rotor}} D_{\text{a}}(1-\nu)z + D_{\text{a}}(1+\nu)\pi l_{\text{rotor}} \sin\beta_{0,\text{rotor}} \,. \tag{28}$$

Similar to the centrifugal machine we use the average velocity Eqn. (23) of the rotor channel and the wetted surface area A_{B} following Eqn. (28) for determining the weight factor for the rotor of axial machines

$$f(shape,\varphi) = \frac{A_{\text{B,rotor}}}{2D_2^2\pi(1-\nu^2)^3} \left(\frac{1}{\sin\beta_1} + \frac{1}{\sin\beta_2}\right)^3 \varphi^2 \,. \tag{29}$$

VOLUTE/STATOR

CENTRIFUGAL FAN To calculate the Reynolds number of the volute, a characteristic length and velocity is necessary. The characteristic length is assumed to be the rolling of the side wall of the spiral casing U. To calculate this easily, the average value of the circumferential length of the inner and the outer envelope U_{m} is calculated (see Fig. 3):

$$U_{\text{m}} = \frac{1}{2}\left(D_{\text{a,spi}} + D_{\text{i,spi}}\right)\pi \,. \tag{30}$$

The characteristic velocity is calculated with the circumferential velocity component at the outlet of the impeller, which is reduced because of the conservation of angular momentum $c_u r = \text{const}$:

$$c_{u,\text{spi}} = \frac{D_2}{D_{\text{m,spi}}} u_2 \left(1 - \varphi\frac{D_2}{4b_2}\cot\beta_2\right) \,. \tag{31}$$

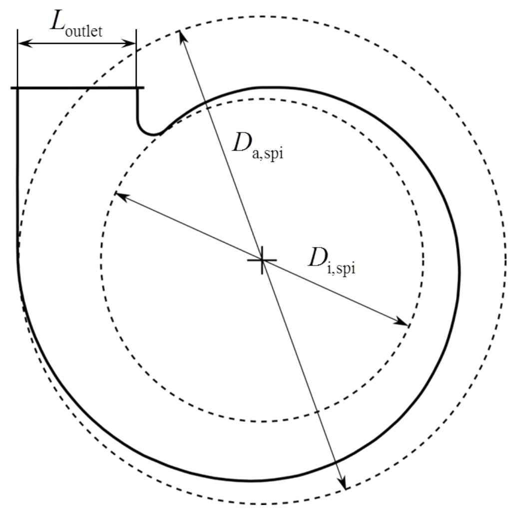

FIGURE 3. SHAPE OF THE VOLUTE

This yields with Eqn. (18) to the Reynolds number of the volute

$$Re_{\text{spi}} = \pi\left(1 - \varphi\frac{D_2}{4b_2}\cot\beta_2\right) Re \,. \tag{32}$$

The wetted surface area is calculated by the side lengths L_{outlet} and B_{outlet} and the rolling of the side wall U_{m}:

$$A_{\text{B,spi}} = (2L_{\text{outlet}} + B_{\text{outlet}})U_{\text{m}} \,. \tag{33}$$

The velocity in Eqn. (14) which is important for the dissipation power is assumed to be the same as the characteristic velocity for the Reynolds number Eqn. (31). This yields to the weight factor

$$f(shape,\varphi) = \frac{4A_{\text{B,spi}}\pi^2}{U_{\text{m}}^3}\left(1 - \varphi\frac{D_2}{4b_2}\cot\beta_2\right)^3 \frac{1}{\varphi} \,. \tag{34}$$

AXIAL FAN The velocity of the flow against the guide vanes results from the velocity triangles at the outlet of the rotor:

$$c_3 = \frac{1}{(1-\nu^2)\sin\beta_3} u_2\varphi \,. \tag{35}$$

This leads to the characteristic Reynolds number of the stator

$$Re_{\text{stator}} = \frac{l_{\text{stator}}}{D_2(1-\nu^2)\sin\beta_3} Re\varphi \,. \tag{36}$$

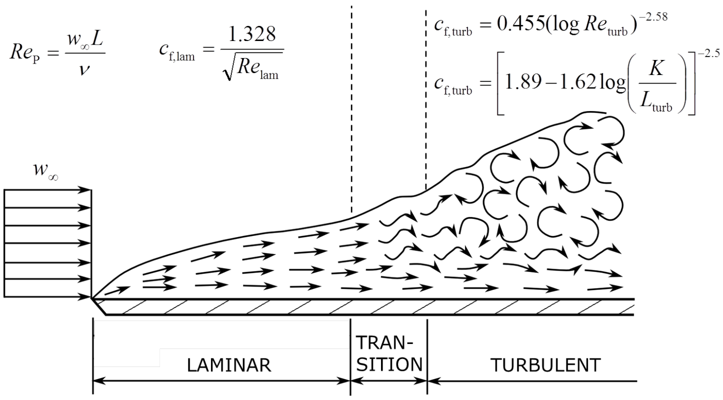

FIGURE 4. FLOW ALONG A FLAT PLATE

The wetted surface area of the stator is calculated by

$$A_{B,stator} = l_{stator} D_a (1-\nu) z + D_a (1+\nu) \pi l_{stator} \sin\beta_{0,stator} \, . \quad (37)$$

The velocity in Eqn. (14) is the average velocity at inlet and outlet of the stator

$$\bar{c} = \frac{c_3 + c_4}{2} \quad (38)$$

with c_3 following Eqn. (35) and

$$c_4 = \frac{1}{(1-\nu^2)\sin\beta_4} u_2 \varphi \, . \quad (39)$$

We get a weight factor for the stator of axial machines

$$f(shape, \varphi) = \frac{A_{B,stator}}{2D_2^2 \pi (1-\nu^2)^3} \left(\frac{1}{\sin\beta_3} + \frac{1}{\sin\beta_4} \right)^3 \varphi^2 \, . \quad (40)$$

DETERMINING OF THE FRICTION COEFFICIENT

The friction coefficient which is an integral part of the scaling method Eqn. (7) is determined using the analogy of the flow along a flat plate. Three flow regimes can be found in a longitudinal flow against a plate (see Fig. 4). At first there will be a laminar flow. After that follows a transient region, where the flow is partly laminar and partly turbulent. After the region of transition the flow changes to fully turbulent flow. In the following we assume that the region of transition is small compared to the laminar and the turbulent region and therefore we devide the total plate length L into a plate length of laminar flow L_{lam} and a plate length of turbulent flow $L_{turb} = L - L_{lam}$. For determing the overall friction coefficient for the plate with respect to laminar/turbulent transition a separate calculation of the drag for the laminar part

$$F_{lam} = \frac{1}{2} \rho L_{lam} w_\infty^2 c_{f,lam} \quad (41)$$

and for the turbulent part of the plate

$$F_{turb} = \frac{1}{2} \rho L_{turb} w_\infty^2 c_{f,turb} \quad (42)$$

is necessary. The definition of the friction coefficient for the whole plate

$$c_{f,P} = \frac{F}{\frac{\rho}{2} w_\infty^2 L} = \frac{F_{lam} + F_{turb}}{\frac{\rho}{2} w_\infty^2 L} \quad (43)$$

yields to the overall friction coefficient $c_{f,P}$ composed of the friction coefficient for laminar and the turbulent flow

$$c_{f,P} = c_{f,lam} \frac{L_{lam}}{L} + c_{f,turb} \left(1 - \frac{L_{lam}}{L} \right) . \quad (44)$$

The friction coefficient for laminar flow is [8]

$$c_{f,lam} = \frac{1.328}{\sqrt{Re_{lam}}} \, . \quad (45)$$

with the Reynolds number

$$Re_{lam} = \frac{w_\infty L_{lam}}{\nu} \, . \quad (46)$$

The turbulent region can be subdivided into hydraulically smooth and hydraulically rough flow. There is no single formula, that covers the whole region from hydraulically smooth to hydraulically rough flow. The interpolation formula for the friction coefficient for hydraulically smooth flow is

$$c_{f,turb} = 0.455 (\log_{10} Re_{turb})^{-2.58} \quad (47)$$

and for hydraulically rough flow

$$c_{f,turb} = \left[1.89 - 1.62 \log_{10} \left(\frac{K}{L_{turb}} \right) \right]^{-2.5} , \quad (48)$$

that are both given by Schlichting [8]. The Reynolds number of turbulent flow is defined by

$$Re_{\text{turb}} = \frac{w_\infty L_{\text{turb}}}{\nu} = \frac{w_\infty (L - L_{\text{lam}})}{\nu}. \tag{49}$$

The differentiation between the flow regimes is done by the relative roughness of the plate surface:

$$K/L_{\text{turb}} = \begin{cases} \leq 100/Re_{\text{turb}} \text{ hydraulically smooth} \\ > 100/Re_{\text{turb}} \text{ hydraulically rough} \end{cases}. \tag{50}$$

The Reynolds number of transition Re_{lam} depends strongly on the intensity of turbulence and the relative roughness and there is no reliable information about the value of Re_{lam}. Schlichting indicates a value for flat plates of from $2.2 \cdot 10^5$ to $3 \cdot 10^6$. In contrast Gülich indicates for pumps values of $2 \cdot 10^4$ till $2 \cdot 10^6$. In this work we assume a average value of $Re_{\text{lam}} = 1 \cdot 10^5$. The overall Reynolds number of the plate is defined with the length of the plate L and the undisturbed flow velocity w_∞:

$$Re_{\text{P}} = \frac{w_\infty L}{\nu}. \tag{51}$$

Because of

$$\frac{L_{\text{lam}}}{L} = \frac{Re_{\text{lam}}}{Re_{\text{P}}} \tag{52}$$

Eqn. (44) yields to

$$c_{\text{f,P}}(Re_{\text{P}}, K/L_{\text{turb}}) = c_{\text{f,lam}} \frac{Re_{\text{lam}}}{Re_{\text{P}}} + c_{\text{f,turb}} \left(1 - \frac{Re_{\text{lam}}}{Re_{\text{P}}}\right). \tag{53}$$

Eqn. (53) is shown in Fig. 5.

A much easier way to determine the friction coefficient is to assume fully turbulent flow in each part of a turbomachine. In many relevant cases the laminar flow is neglicable indeed and it is possible to do this for practical use but with lower accuracy in scaling the efficiency. In case of running a model machine with very low Reynolds numbers this simplification is not applicable.

VALIDATION

EXAMINED FANS

For validation of the scaling method there are measurement data of axial and centrifugal fans available, which have been collected by the Chair of Fluid Systems and Technology and by a fan

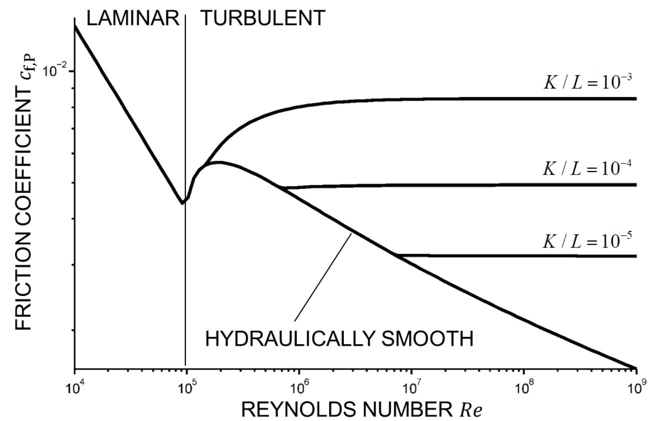

FIGURE 5. FRICTION COEFFICIENT AGAINST REYNOLDS NUMBER

manufacturer under the guidance of a research assistant (centrifugal fan with $D_2 = 2.240\,\text{m}$). The test rigs are built conforming to the DIN24163 [9] respectively ISO5801 [10] standard. The shaft power of the fans is measured by a flying mount torque metering shaft without measuring any mechanical losses in bearings and gaskets. A detailed fan description including the measurement equipment can be found in [11], [12] and [13]. The variation of the Reynolds number within one machine is achieved by changing the rotational speed. The measurement data were evaluated complying the VDI guideline VDI2044 [14] and the international standard ISO5801 in compressible way. The test data of the lowest Reynolds number have an uncertainty below 2.3 % points of total efficiency due to accuracy of torque measurement. The uncertainty of test data with highest Reynolds number is quite lower (below 0.5 % points). An overview about all fans is given in Tab. 1.

CENTRIFUGAL FANS

Fig. 6 shows measured performance characteristics from a centrifugal fan (large model) and predicted characteristics using different scaling methods mentioned in the introduction. The scaling is performed with test data from the large model machine with the lowest Reynolds number represented by open dots. The desired characteristic is the test data of the large model machine with the highest Reynolds number represented by black dots. The predicted performance characteristics using the new method (Pelz and Stonjek) show a good agreement to the test data. Remarkable are deviations from the desired and the predicted performance characteristic in off design region. A possible statement could be the influence of the higher Mach number with the measurement data with the highest Reynolds number. Implementing a Mach number correction could improve the prediction in off-design operation. But this is not done yet.

TABLE 1. OVERVIEW ABOUT THE ANALYSED FANS

	centrifugal fans			axial fans	
Impeller/rotor diameter D_2 in mm	224	896	2240	250	1000
Rotational speed n in $1/\mathrm{min}$	2500 ... 10000	625 ... 2500	400 ... 980	2800 ... 6900	1240 ... 2475
Reynolds number Re $1/10^6$	0.5 ... 1.6	1.6 ... 6.5	6.5 ... 16	0.6 ... 1.5	2.0 ... 8.6
Mach number Ma	0.09 ... 0.33	0.09 ... 0.33	0.16 ... 0.33	0.11 ... 0.26	0.19 ... 0.38
R_z of impeller/rotor in μm	4	7	40	9	12
R_z of volute/stator in μm	6	25	35	80 (estimated)	10 (estimated)
Gap width/tip clearance in mm	0.35	1.4	3.5	0.25	1
Shaft power in kW	4.4	70	440	1	37

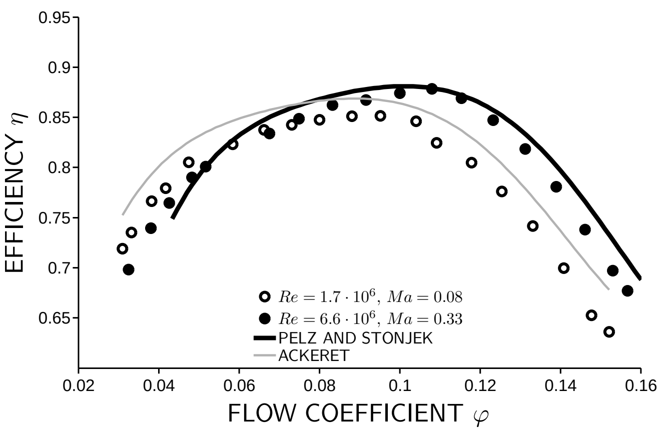

FIGURE 6. LARGE MODEL (CENTRIFUGAL FAN)

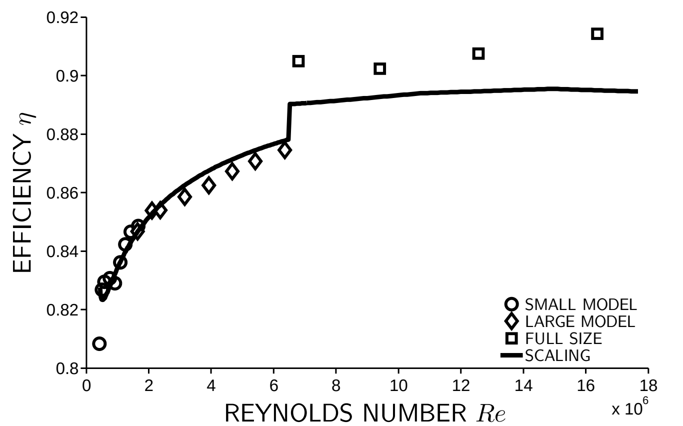

FIGURE 7. BEST EFFICIENCY POINTS AGAINST REYNOLDS NUMBER (CENTRIFUGAL FAN)

The scaling from the small model to full scale machine is shown in Fig. 7. The agreement is quite good for the machines tested in our laboratory. The full scale machine was tested at the fan manufacturer and has an unexpected high efficiency. We assume, that the full scale machine ran in conditions with another intensity of turbulence which leads to another value of Re_{lam}. By changing the value of Re_{lam} when determining the friction coefficient we test if it is possible to achieve the high efficiency of the full scale machine (see the discontinuity Fig. 7). Because this is a very important point in scale-up formulas, the difference between test data and scale-up method should be closely checked and verified in future work.

AXIAL FANS

Fig. 8 and Fig. 9 show the measured data and the predicted performance characteristics of the axial fans. The scaling is performed with the test data of lowest Reynolds number of the small model machine (sm) (open dots) and the desired characteristic is the test data of the highest Reynolds number of the large model machine (lm) (black dots). To obtain different geometries on the same test rig, the stagger angle of the axial fan was varied. $\Delta\beta_0$ is the stagger angle of the design point. It was varied from $\Delta\beta_0 = -6°$ to $\Delta\beta_0 = 0°$. The prediction of the characteristic with the method introduced in this work is good for small variations from design point stagger angle. Fig. 10 and Fig. 11 show the best efficiency points plotted against the Reynolds number and the performance prediction. Similar to the validation of the new scaling method with centrifugal fans (Fig. 7) the agreement of the prediction ist quite good. It has to be pointed out, that there are test data of two machines in these figures. Therefore small discontinuities are present between both machines.

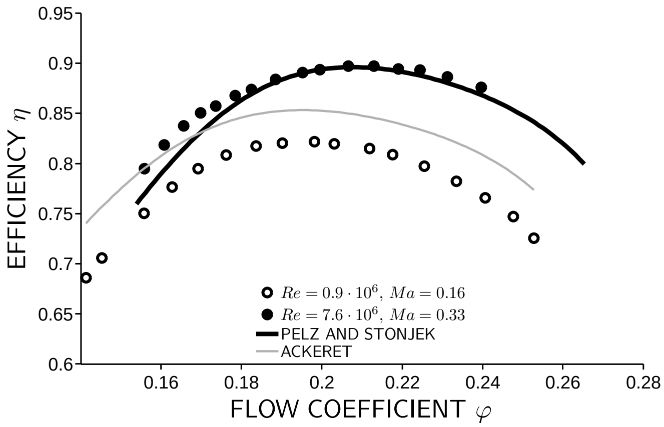

FIGURE 8. EFFICIENCY SCALING WITH AXIAL FAN $\Delta\beta_0 = 0°$

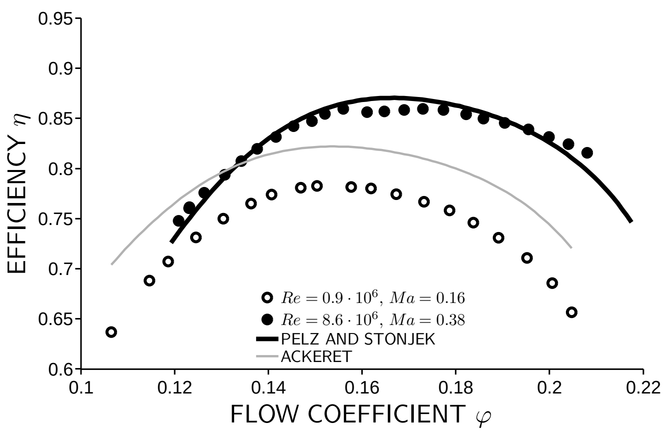

FIGURE 9. EFFICIENCY SCALING WITH AXIAL FAN $\Delta\beta_0 = -6°$

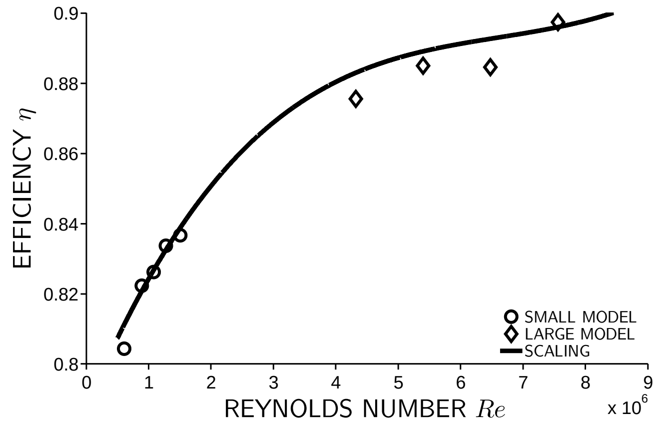

FIGURE 10. EFFICIENCY SCALING WITH AXIAL FAN $\Delta\beta_0 = 0°$

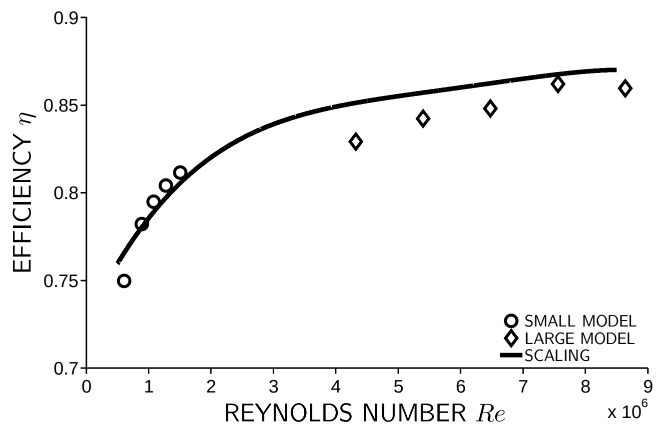

FIGURE 11. EFFICIENCY SCALING WITH AXIAL FAN $\Delta\beta_0 = -6°$

CONCLUSION

The new scaling method presently contains the influence of Reynolds number, surface roughness and the gap width. An essential point of this method is the consideration of aerodynamic quality and shape (given by the specific speed) which are implicitly considered by the inefficiency of the model machine. In addition, the fact that the spiral casing has a greater impact than the impeller/rotor on the efficiency scaling has to date not been taken into account in other scaling methods. The influence of the Mach number could easily be integrated by finding out the coefficient of friction dependency on Mach number. The use of measured data from three geometrically similar centrifugal fans and two axial fans is a strong validation for this work which continuously provides better results than prior used methods like the well-known and often used equation of Ackeret.

The new scaling method has essential advantages compared to previous introduced scaling methods

- simple application,
- physical motivation for the scaling effect with seperate consideration of impeller/rotor and stator/volute and
- good results.

More work has to be done in regard to the determination of the constant for the shift in flow rate performed by Eqn. (8). Furthermore, measurement data of machines running at high values of Mach number show that the shape of the master curve alters with rising Mach number. Up to now the introduced scaling method is valid for subsonic flows below $Ma = 0.35$ where compressibility effects are deemed of negligible impact on efficiency scaling. Nevertheless, the method shows very good agreement to test data within the scope of the fans analyzed at the Chair of Fluid Systems and Technology.

ACKNOWLEDGMENT

The authors would like to thank the Arbeitsgemeinschaft industrieller Forschungsvereinigungen 'Otto von Guericke' e. V. (AiF), the Bundesministerium für Wirtschaft und Technologie (BMWi), and the Forschungsvereinigung für Luft- und Trocknungstechnik (FLT) e. V. whose support made this work possible. Additional thanks to the members of the supervising working group of the FLT, in particular to Christian Rohdich.

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