Global System Optimization and Scaling for Turbo Systems and Machines

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Introduction

From a system point of view there are two distinct tasks to be fulfilled by turbo machinery systems.
First, the amount of energy needed to reach a functional goal should be minimized (compressor systems, pump systems, propulsion system…). Second, the power specific investment costs should be minimized for turbine systems (wind, water, wave, …). For the first case, mechanical power is applied to the fluid (gas, water, …), i.e. the shaft power is positive P>0, whereas in the second case power is extracted from the fluid and thus P<0.
Despite the task is formulated quite clearly, the purpose of the presented paper is to discuss the different ways to reach this task.
For both, pump or compressor systems (P>0) and turbine systems (P<0) a general approach for global system optimization is made. It shows, that the optimization method for both cases may differ. For the global optimization of pump and compressor systems the optimization has to be discrete, since the system topology is usually a priori unknown. The discrete optimization methods are part of our research area Technical Operation Research (TOR) at Technische Universität Darmstadt. In contrast, the optimization of a turbine system (wind or water turbine) is done by analysis since the performance is a continuous function of the operation point. As soon as it comes from the system to the machine level, scaling methods are needed. We give an overview of scaling methods developed recently to account for machine size, speed, roughness, gap width i.e. Reynolds and Mach number, but also for heat flux.
The examples to be explained in this paper are a hydropower plant and a pump system.

Continuous Optimization for Hydropower

For hydropower stations, the system to be optimized consists of three components:
(i) headwater, (ii) machine, and (iii) tailwater. The optimization objective consists of determining the operating point where the possible maximum of mechanical power output is generated from the available energy.

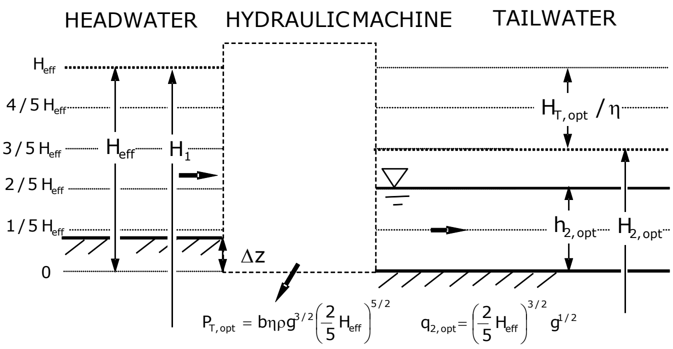

Fig. 1: Optimal Operation for Hydropower [Pelz, 2011]

For this purpose, Pelz took a fundamental view on the headwater-machine-tailwater system [Pelz, 2011], which is illustrated in Figure 1. He considered a channel with rectangular cross-section and width b. As with Betz, the conclusions that he has drawn from the model are independent of the type of machine being considered. He defined an effective height as

$$H_{eff} := \Delta z + h_1 + \frac{u_1^2}{2g} \qquad (1)$$

Here Δz describes the topological difference in elevation between the headwater and tailwater, h_1 represents the water level, and u_1 stands for the flow velocity at the headwater. The available energy can be determined, by applying Betz' thought experiment with a hypothetical, ideal machine, as

$$P_{avail} \coloneqq 2\rho g^{3/2} \left(\frac{2}{5} H_{eff}\right)^{5/2} b \qquad (2)$$

The difference between this case and the optimization task addressed by Betz is that the hydropower case requires to include gravitation as an additional parameter in the optimization equation.

Given the specific volume flow rate $q_2 = Q/b$ (volume flow Q, channel width in headwater b), we may express the mechanical power output P_T as a function of h_2 and q_2:

$$P_T = \eta \rho g q_2 b \left(H_{eff} - h_2 - \frac{q_2^2}{2gh_2^2}\right) \qquad (3)$$

Scaling for the available power (Eq. 2), we get the performance coefficient, whose maximum can be derived analytically as $C_{P,opt} = 0.5$. In words, this means that in an open channel flow at most half of the available energy can be converted into mechanical power. Furthermore, it is possible to derive the optimum operating point, i.e., the optimum volume flow rate and head, for a given energy supply. The result of doing so shows that the optimum operating state is achieved precisely when the head of the turbine H_T is exactly two fifths of the effective height H_{eff} and the Froude number at the tailwater attains the value 1. In algebraic terms, the result for the optimum operating point is:

$$q_{2,opt} = \left(\frac{2}{5} H_{eff}\right)^{3/2} g^{1/2}, \qquad (4)$$

$$H_{T,opt} = \frac{2}{5} H_{eff} \,. \qquad (5)$$

The optimal water level height in the tailwater then becomes

$$h_{2,opt} = \frac{2}{5}\, H_{eff}. \qquad (6)$$

and the tailwater's Froude number is

$$Fr_2 = \frac{q_{2,opt}}{g^{1/2} h_{2,opt}^{3/2}} = 1 \qquad (7)$$

Scaling for Optimal Operation

In the above section, the optimal operation point for a hydropower plant is determined based on axiomatic considerations. The optimization leads to the optimal tailwater volume flow per channel width and the optimal tailwater level height. In this section we discuss how a hydropower plant can be operated within these conditions.

At most of existing hydropower plants, especially river runoff stations, the possibilities to manipulate volume flow rate and effective head are poor. The headwater volume flow rate cannot be manipulated for one single hydropower plant, but is given by the duration curve of the river. Due to conservation of mass, the flow rate in the tailwater has to be equal to the flow rate in the headwater and must be considered as a naturally determined parameter as consequence:

$$Q = q_1 b_1 = q_2 b_2 \qquad (8)$$

For most hydropower plants the following simplifications can be assumed:

(i) The headwater level h_1 must be maintained within narrow bounds. Reasons for that can i.e. be the need of navigability of rivers, or the need of a certain water level for watering of surrounding areas.

(ii) The headwater Froude number for most hydropower plants, due to big headwater cross section area, is nearly zero. Hence the influence of flow velocity on the effective height is negligible.

This leads to the simplified form of the effective height

$$H_{eff} \approx h_1 = const. \qquad (9)$$

Since the effective head can be considered constant, the only way to adjust the optimal operation point is to change the tailwater condition. Precisely defined, the tailwater condition is the flow condition of the water at the outlet cross section of the machine. Immediately after leaving the machine the flow condition changes due to wall shear stress and can no longer be described by the optimization model of figure 1. Thus, the shape of the cross section at the outlet defines the tailwater condition. For most hydropower plants this is the section of the diffusor outlet.

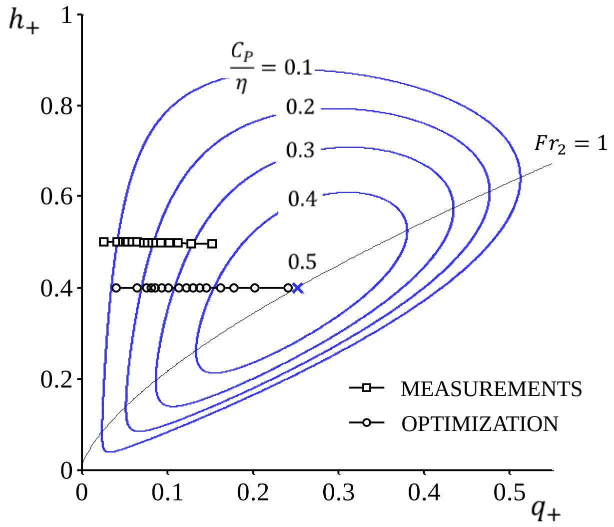

Fig. 2: Coefficient of Performance for Hydropower plant "Wässerung" (river Elsenz, Bammental, Germany)

In Figure 2, the quadratic shaped markers show measurements of a small hydropower plant at the river Elsenz in Bammental, Germany. It can be assumed, that the hydropower plant operates 20 days a year in each of these measurement points. The coefficient of performance as a field variable is plotted over the dimensionless volume flow rate, defined as $q_+ := q_2\, g^{-1/2} H_{eff}^{-3/2}$ and dimensionless tailwater level defined as $h_+ := h_2/H_{eff}$ [Pelz, 2011].

Since the natural volume flow rate of the Elsenz varies over the year, the dimensionless volume flow rate also varies within values $q_+ = [0.05, \dots, 0.32]$.

The dimensionless tailwater level h_+ is nearly constant, since the height of the outlet cross section is fixed and the effective headwater height is nearly constant (see. Eq. 9).

As a first step of optimization, the tailwater level can be adjusted to optimal height by changing the height of the outlet cross section to $h_{2.opt} = 2/5\ H_{eff}$.

As a second step, the width of the outlet cross section b_2 has to be adjusted. By increasing b_2, the measurement points of Figure 2 will expand over the q_+-range. By decreasing b_2, it the q_+-range of the measurement points will be narrowed. In this case a limit for the expansion in the q_+-range is set: The tailwater Froude number of $Fr_2 = 1$ should not be exceeded for reasons long life fatigue strength of the hydropower plant. Hence, the optimized operation points in Figure 2 are expanded to higher values of C_p until the operation point of the biggest volume flow rate reaches the $Fr_2 = 1$ curve.

The results concerning the coefficient of performance can be seen in the circular shaped marker in figure 2. The results concerning the gained electrical energy are discussed in the following section

Results

The optimization of [Pelz, 2011] applied to the hydropower plant "Wässerung" in Bammental as shown in Figure 2 can easily be translated to the gain of energy yield the optimized hydropower plant can feed into the grid.

Assuming that the hydraulic, mechanical and electrical efficiency are the same for both the measured and the optimized operation points, these efficiencies vanish from all equations. But due to higher coefficient of performance the energy yield can be increased significantly.

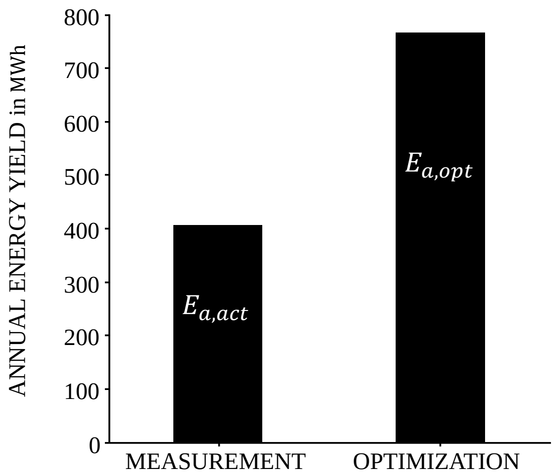

Fig. 3: Comparison of annual energy yields, actual system vs. optimized system

As mentioned in the above section the hydropower plant operates τ =20 days = 480 h a year in each of the operation points of Figure 2. Hence, the annual energy yield E_a calculates as

$$E_a = \tau \sum C_P \, P_{avail} \tag{10}$$

A comparison between the actual annual energy yield calculated with the measured operation points of Figure 2 and the predictions on base of the optimized operating points is illustrated in Figure 3. The increase in this case goes by the factor

$$E_{a,opt} \approx 1.88 \cdot E_{a,act} \tag{11}$$

System Design and Analysis with the help of Discrete Optimization

A major goal of Technical Operations Research is to establish quantitative methods for system synthesis in the engineering science. As an ideal serves the classical field of Operations Research that lead to huge improvements of economic processes in the areas logistics and other. In contrast to such traditional application fields, in Technical OR, modules are connected to systems, systematically. A typical domain is the optimal design of a pump booster station or the analysis of a heating station.

In order to make modern Mathematics like the mixed linear integer programming available for engineers, a methodology for system design tasks has been developed at the FST. In its core, it provides the following procedure, which is visualized in Fig. 4.

Fig. 4: The TOR pyramid.

1. The first step is always devoted to the question of the basic demanded function of the new system.
2. Thereafter, the question for subjective goals is asked, because a function can be fulfilled e.g. with small effort (minimal energy effort, minimal invest costs, minimal material demand, …) or with high availability (small probabilitiy of defects, long life time, small danger for plugging, low service, …). Possibly, a system designer’s answer will differ from a carrier’s or an owner’s one. The designer will have other emphasizes than a politician. In short, the determination of the optimization task makes the goal to a subjective value and depending on the goal, an optimization process delivers different answers. This does not affect the method itself. After all, the answer to “What is my goal?” is a political matter.
3. All the constraints that have to be considered in any implementation of the desired functionality must be collected and be agreed on. This has an analogy in game rules. In a game, like e.g. chess, the playing field and the rules are fixed. In analogy to this game metaphor, this must also be valid for the virtual design process. E.g., there should be a certain pre-selection of possibly usable components.
4. An optimization algorithm then decides which of them should be chosen, as well as the optimal control of these components and their topology. However, it is obvious that if there is a technical solution that is not part of the playing field, an algorithm will not be able to find it. We prefer, if ever possible, to model the system building task in form of a mixed integer linear model [Papadimitriou, Steiglitz 1998].
5. The proposed solution is validated in the fifth step with the help of physical-technical-economic models with concentrated parameters (so called 0-dimensional models [Nakhjiri et al.]). We use the modeling language Modelica [Modelica, Fritzson] for this purpose, which becomes increasingly accepted as the standard for coupled technical systems. The description of the components in algebraic differential equations as well as with the help of head curves allows a fine grain parameter optimization. Thus, the „missing link” is not step 5 but step 4 instead, i.e. the optimization via TOR in interaction with the physical-technical-economic system description in step

6. The validation takes place with state-of-the-art tools, i.e. three dimensional computational methods (CFD, FEM) or with the help of experiments.
7. Carrying out underlies the system builder as it is now.

Case Study Heating System

The darmstadium building is a green building with a hybrid ecologic heating system.

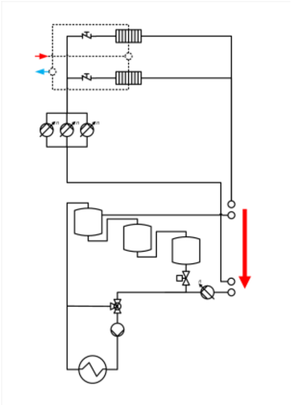

Fig. 5: The heating source side of the heating system in darmstadtium

This building possesses quite a large heating system which is responsible for the heating in summer and for cooling in the winter with the help of so called adiabatic cooling. The complete system can be divided into a destination subsystem and a source subsystem consisting of a community heating source (cf. Fig. 5 left upper corner), some heat storages (rectangle boxes in Fig. 5, left of the arrow), a woodchip boiler, and some minor pieces like valves and pumps. This paper deals with exactly the source subsystem. We measured the load

$$\dot{m}\Delta h = \varrho \dot{V}\, c(T_V - T_R)$$

over a complete year and derived a load profile from these data. For our optimization method, we call in question the topology of the system as well as the control of the volume flow.

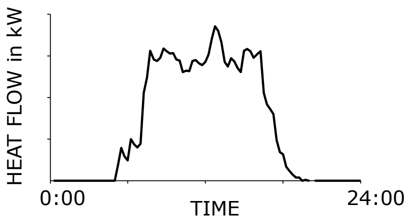

Fig. 6: Typical summer day profile

The demanded function that should be and is provided is heating (1). The subjective goal here is to fulfil the functionality with a minimum of energy costs, more precisely the task is to find out whether the three heat stores – cf. Fig. 5 left of the down-arrow – have any effect and if they can be better rearranged, considering the energy costs over a certain period (2). More complicated is the description of the "playing field" (3), the restrictions. First of all there are physical properties of fluid systems. Second, in order to provide an energy-low solution, a load profile of demands is required. Fig. 6 and Fig. 7 show how a typical summer day heating curve is transformed in an approximate relative load demand. The degrees of freedom that may be used to improve the heating system consist of more or less heating storages and the controlling of the system. Moreover, the 365 days of a year could be classified with the help of nine classes, which are summarized in Tab. 8. It shows how many of the days fall into which category, indicating how much heating power is desired. During nights the heat storages can be empty such that the days can be examined in a decoupled fashion. They form independent scenarios with a certain weight for the optimization, depending on their occurrence frequency. The various intra-day demands cannot be examined independently, because the heating storage couples different system states with each other.

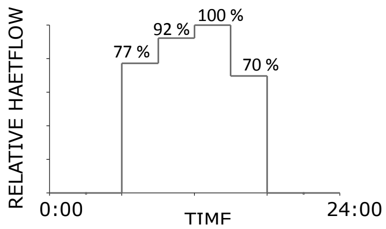

Fig. 7: Abstraction, relative daily

ID	LOAD CASE		POWER in KW								#DAYS
1	Summer	Design pt	0	0	1313	1567	1700	1185	0	0	1
2	Summer	Hot	0	0	596	712	772	538	0	0	18
3	Summer	Warm	0	0	282	337	366	255	0	0	79
4	Summer	Cool	0	0	91	109	118	83	0	0	79
5	Summer	Off	0	0	0	0	0	0	0	0	6
6	Winter	Very cold	337	367	541	649	603	554	418	361	6
7	Winter	Cold	219	238	350	420	391	359	271	234	84
8	Winter	Mild	89	97	143	172	159	147	111	95	84
9	Winter	Off	0	0	0	0	0	0	0	0	8

Maximallasten

Especially demanding is that a two stage problem has to be considered. In the first stage the topology must be decided under consideration of optimal control in the second stage.

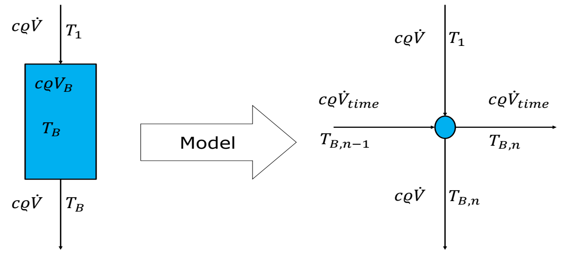

Fig. 8: The storage, classic representation and graph node

In order to handle this challenge, a mixed integer linear program (MIP) is developed. Components, i.e. pumps, valves, etc. are identified with their head curves. First stage variables allow to vary the topology of the system, i.e. represent different designs of the heat storage. Second stage variables represent control opportunities of the system topology which is defined by the first stage variables. A classical abstraction of a system or network is a graph [Grothklags et al. 2009]. Components like pumps are described with the help of edges, connection points are nodes. The evolution of storage changes in time can be expressed with the help of so called time expanded graphs [Köhler et al. 2002], which are encoded in the MIP. The major physical constraints consist of the energy conservation and the flow conservation. The energy conservation law is non-linear in its origin form and must therefore be linearized. More difficult is the model building for the heat storage. We model it (cf. Fig 8, left) as a node in the network graph. Heat can either be transported into the network from the storage or vice versa (Fig. 8, right, from top to bottom), or it can transport the heat through time (Fig. 8, left to right). This means that the volume of a storage becomes a volume flow through time at the same location.

Usually, the heat evolution in a storage is described as

$$c\varrho\dot V(T_1 - T_B) = \frac{dT_B}{dt} c\varrho V,$$

which is discretized into

$$c\varrho\dot V\left(T_1 - T_{B,n}\right) = \frac{T_{B,n} - T_{B,n-1}}{\Delta t} c\varrho V.$$

This formula leads to node rules for storage nodes as is presented in Fig. 8, as follows.

$$c\varrho\dot V T_1 + c\varrho\dot V_{time} T_{B,n-1} = c\varrho\dot V T_{B,n} + c\varrho\dot V_{time} T_{B,n}$$

$$c\varrho\dot V(T_1 - T_{B,n}) = c\varrho\dot V_{time}(T_{B,n} - T_{B,n-1})$$

As a consequence, the volume flow through time can be defined as

$$V_{time} := \frac{V_B}{\Delta t}$$

A solution of the optimization process in step (4) of the TOR pyramid is a proposal for a layout of an optimized heat system. The algorithmically deter-

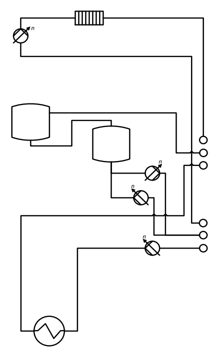

Fig. 9: Proposed solution

mined solution has then been validated with the help of a Modelica simulation as proposed in step (5).

Results

Our simulations indicate the possibility to save expenses of several thousand Euros per year with the topology shown in Fig. 9.

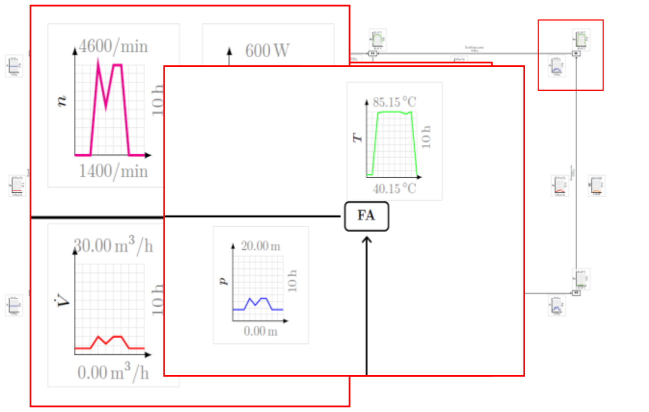

Fig. 10: Example for temperature and pressure at point FA over the period of a selected day.

Beside this design proposal, the optimization process also serves with a control strategy for all components. For the connection points of components, various parameters like temperature or pressure over time is delivered (cf. Fig. 10).

Conclusion

Depending on the nature of the synthesis task, either discrete optimization or continuous optimization is the right tool for laying out a system of components. Continuous optimization can be used when parameters can be varied, but the ground topology is already fixed. However, system design often requires decisions of the kind "do we consider this pump or another one?" (Fig. 11). In this case, discrete optimization techniques are the right choice.

For both kinds of tasks, we delivered an example. When examining the heating system in a building, discrete optimization has been used in order to deliver improvement proposals which save energy in the order of several thousand Euros per year. This has firstly be found out with the help of quite an abstract mathematical model and thereafter been validated in a more detailed Modelica simulation.

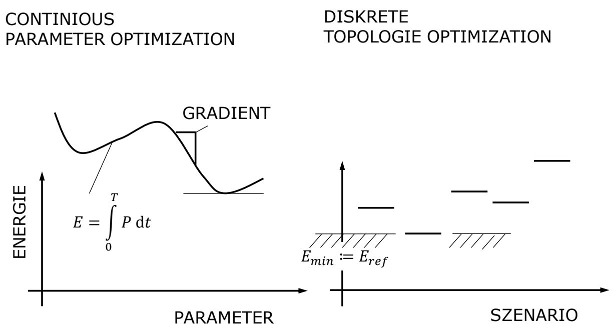

Fig. 11: Continuous vs. discrete optimization

The energy yield of hydropower plants can be increased significantly by continuous system optimization. Without changing the hydraulic efficiency, neither the efficiency of gearboxes and electrical machines, the energy yield of the discussed hydropower plant can be increased by the factor of 1.88.

Hence, for the system optimization of hydropower plants the optimization, and with it the coefficient of performance C_P has to be considered in the first place. Hydraulic and other efficiency caused by friction or heat losses are of secondary importance.

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