MAXIMUM YIELD AT MINIMAL COST - NEW THINKING IN SMALL-SCALE HYDROPOWER GENERATION

At the Department of Fluid Systems Technology at the Technical University of Darmstadt, new concepts for optimising environmentally sound hydropower facilities are being developed under the direction of Univ. Prof. Dr.-Ing. Peter F. Pelz. The requirements imposed on hydropower generation by today's Water Management and Environment Protection Acts call for solid, economically viable machine concepts.

Adolf Pfarr, who in 1897 was the first to assume the Chair of Fluid Systems Technology at the Technische Universität Darmstadt, was also the first to develop a controlled hydropower turbine. It was Pfarr's designs that the trailblazing Voith turbine controller of 1879 was based on. Following in the footsteps of Adolf Pfarr, we specialise in pioneering new ideas when it comes to developing environmentally compatible turbine concepts with a high level of profitability.

Where the long-term success of a technology in the energy sector is concerned, output-indexed investment costs are a crucial factor. The initial electricity production costs of small-scale hydropower stations are determined chiefly by the investment costs, with maintenance and operating costs typically ranging much lower in scale [Wissel et al., 2008]. Low output-indexed investment costs help enable short amortisation times for power stations and ensure a high level of profitability in the long run. For this reason, lowering these investment costs is a focus of research at the Institute for Fluid Systems Technology.

IS EFFICIENCY A SUITABLE MEASURE FOR COMPARING HYDROPOWER STATIONS?

To anticipate the answer to this question: no, it is not!

In technology, as in public discourse, there are terms that are either used imprecisely or (e.g. for marketing reasons) misleadingly. “Energy efficiency” is one such term. For an unbiased discussion of the matter - which today concerns all areas of engineering - it is necessary to understand the path to energy efficiency as a two-stage process, which consists first and foremost in optimising the system, and secondarily in scaling up e.g. the efficiency level of individual system modules. Efficiency issues are always part of the scaling tasks of a module. A much more important issue - and therefore of higher priority - is the optimisation task of the system.

For wind turbines Albert Betz already discovered in 1920 that the theoretical energy maximum that can be generated from any wind-powered machine is 16/27 of the total wind energy used. This means that even an ideal, lossless machine (with hydraulic efficiency $\eta = 1$) cannot transform more than 16/27 of the available amount of wind energy into mechanical power. Conversely, it is possible that an unfavourably set operating point of a machine with the hydraulic efficiency $\eta = 1$ may have a mechanical power output that is considerably below the 16/27 mark.

Most studies on the topic of energy efficiency for hydropower installations focus on the hydraulic efficiency $\eta$ of the machine as an overall quality criterion for the hydropower station. This is usually defined as the ratio of the mechanical power $P_T = M \Omega$ (M: torque; $\Omega$: angular frequency) of the machine to its hydraulic power output $P_H = \rho g H T Q$ (H_T: turbine head, Q: volume flow rate) of the volume flow passing through the machine.

$$\eta = \frac{M \Omega}{\rho g H T Q} = \frac{P_T}{P_H}$$

However, all the hydraulic power ratio tells us about the machine is which fraction of the hydraulic power the machine is capable of gene-
rating at the selected operating point. The question one must ask oneself when it comes to optimising the system is: at which operating point does the ratio of the available energy transformed into mechanical power attain its maximum (i.e., optimum)? This ratio is described by the coefficient of performance (or “yield factor”)

\[ C_p = \frac{P_e}{P_{\text{available}}} \]

To highlight the difference between the hydraulic output efficiency and the performance coefficient, Figure 1 indicates the measurements of these factors in an undershot waterwheel by M. Troger [Troger, 2008], plotted above the dimensionless volume flow and the (also dimensionless) tailwater level. These factors, which we shall deal with in more detail in the next section, describe the system’s operating point. However, the following is already obvious from the graphs: the red arrows mark the maximum levels of hydraulic output efficiency (left graph) and performance coefficient (right graph). At the point where the performance coefficient attains its maximum \( (C_p = 0.315) \), the hydraulic output efficiency is \( \eta = 0.85 \), i.e., around 15 per cent below the optimum hydraulic output efficiency of \( \eta = 0.999 \). Yet, the energy yield at maximum performance is higher by a factor of around 2.

Our conclusions, viz. (i) that the hydraulic output efficiency is not an appropriate measure for comparing small-scale hydropower stations, and (ii) that the performance coefficient depends on the operating condition of the system, lead to the following question:

\[ k_p = \frac{\text{INVESTMENT COSTS}}{\text{ELECTRICAL POWER}} \]

**WHAT IS THE OPTIMUM OPERATING POINT OF THE HEADWATER – MACHINE – TAILWATER SYSTEM?**

For hydropower stations, the system to be optimised consists of the three components of headwater, machine, and tailwater. The optimisation objective consists in determining the operating point where the possible maximum of mechanical power output is generated from the available energy. For this purpose, Pelz took a fundamental view on the headwater-machine-tailwater system [Pelz, 2011], which is illustrated in Figure 2. Let us consider a channel with rectangular cross-section and width \( b \). As with Betz, the conclusions that can be drawn from the model as it stands are independent of the type of machine being considered. The “effective height” is defined as

\[ H_{\text{eff}} = \Delta z + h_1 + \frac{u_1^2}{2g} \]

Here \( \Delta z \) describes the topological difference in elevation between the headwater and tailwater, \( h_1 \) represents the water level, and \( u_1 \) stands for the flow velocity at the headwater.

Here, the available energy can be determined, by applying Betz’ thought experiment with a hypothetical, ideal machine, as

\[ P_{\text{avail}} = 2pg \left(\frac{H_{\text{eff}}}{5} \right)^{5/2} \]

The difference between this case and the optimisation task addressed by Betz is that the hydropower case requires that we have to include gravitation as an additional parameter in our optimisation equation.
Given the specific volume flow rate \( q_2 = Q/b \) (volume flow \( Q \), water level at tailwater \( h_3 \)), we may express the mechanical power output \( P_T \) as a function over \( h_1 \) and \( q_2 \):

\[
P_T(b_2, q_2) = \eta \rho g q_2 (H_{\text{eff}} - h_2 - \frac{q_2^2}{2gh_2^2})
\]

Scaling for the available power, we get the performance coefficient, whose maximum we can calculate analytically as \( C_{\text{p, opt}} = 0.5 \). In words, this means that in an open channel flow at most half of the available energy can be converted into mechanical power. Furthermore, it is possible to derive the optimum operating point, i.e., the optimum volume flow rate and head, for a given energy supply. The result of doing so shows that the optimum operating state is achieved precisely when the head \( H_T \) is exactly two fifths of the effective height \( H_{\text{eff}} \) and the Froude number at the tailwater attains the value 1. In algebraic terms, the answer to the search for the optimum operating point is:

\[
q_{2, \text{opt}} = \left(\frac{2}{5} H_{\text{eff}}\right)^{3/2} \sqrt{g}, \quad H_{\text{opt}} = \frac{2}{5} H_{\text{eff}}.
\]

Table 1 shows a comparison of key factors at the optimum operating point for hydropower and wind power.

### WHAT IS THE IMPACT OF MACHINE TYPE AND DESIGN ON THE HYDROPOWER STATION’S INVESTMENT COSTS?

Using the method introduced in the previous section, we are now able to optimise the energy output and, consequently, the financial yield in the form of the power station’s operating income. In the following, we shall introduce a possible approach to the systematic minimisation of investment costs. A useful tool that we may employ for this purpose is the Cordier diagram (Figure 3). This diagram shows the tip speed ratio \([\text{Keller, 1934}]\) against the diametre size \([\text{Baashus, 1906}]\). The Cordier diagram exerts a similar fascination on the research engineer as the Hertzsprung-Russell diagram. This shows the distribution of the stars along certain lines on the brightness/temperature plane, starting with the red dwarfs at the main sequence, then proceeding to stars like our sun and beyond.

Fluid energy machines align in similar linear fashion with respect to their tip specific speed. This enables a systematic typography of machines. During the planning stage, engineers only need to look at the Cordier diagram to determine at a glance, for example, the machine diameter for each type of machine at a given rotational speed, head and volume flow. Or alternatively, they may want to determine the required rotational speed of the machine at a given installation space, head and volume flow. The practical applications of the Codier diagram are very varied indeed. Having calculated the known optimum operating point (as per \([\text{Pelz, 2011}]\)), one can use the Cordier diagram to select a machine and estimate its diameter and rotational speed. For example, when using a synchronous machine as a generator, it is possible to estimate the required rotational speed based on the commercial frequency and the number of pole pairs, and the diameter number can be directly read off the Cordier diagram.

Figure 3 shows that the diameter number for displacement machines - which includes waterwheels, Archimedean screws, etc. - lies in the range \( 2 < \sigma < 100 \). For high-speed axial machines, it is possible to achieve diameter numbers of \( \sigma = 1 \). The diameter number is proportional to the actual geometric diameter size. This means that when using an axial machine as opposed to a displacement machine, the machine diameter can be reduced by at least 50%.

The crucial information from this is that in small-scale hydropower stations, axial turbo machines always have the lowest performance-specific investment costs, thanks to their high power density. This is because an initial approximate calculation shows that the investment costs are proportional to the cubed diameter. Waterwheels, Archimedean screws and similar displacement machines, on the other hand, incur high investment costs at relatively low power yield. This settles the question of which machine to select in terms of investment costs. This only leaves the question of how the actual design of the machine impacts the investment costs. In his paper “Design and Optimization of Small Hydropower Type Series for Surface Watercourse” \([\text{Metzler, 2012}]\) the author examines the influence of modular construction designs on investment costs. Specifically, he addresses the question whether choosing several less powerful machi-
nes over a single machine may be advantageous with respect to investment costs. It can be shown analytically that the investment costs are indirectly proportional to the square root of the number of modules, i.e., that they decrease when the number of modules increases:

\[ k_p \sim \frac{1}{\sqrt{Z}} \]

Also, modular arrangements have the benefit of allowing a larger operational area to be served at a high performance coefficient during times of naturally fluctuating volume flows by activating or deactivating individual machine units.

So, to answer the question as to the impact of the type and design of the machine on the investment costs, we can state that axial machines achieve a cost optimum in small-scale hydropower operations, with modular arrangements holding an additional cost cutting potential.

References:


