An Analytic Approach to Optimization of Tidal Turbine Fields

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Within the energy transition various technologies for harvesting of energy from renewable sources are developed. Hydrokinetic turbines get applied to surface watercourse or tidal flow to gain electrical energy. Since the available power for hydrokinetic turbines is proportional to the projected cross section area, fields of turbines are installed to scale shaft power. Each hydrokinetic turbine of a field can be considered as a disk actuator.
In [1], the first author derives the optimal operation point for hydropower in an open-channel. The present paper concerns about a 0-dimensional model of a disk-actuator in an open-channel flow with bypass, as a special case of [1]. Based on the energy equation, the continuity equation and the momentum balance an analytical approach is made to calculate the coefficient of performance for hydrokinetic turbines with bypass flow as function of the turbine head and the ratio of turbine width to channel width.

2 Optimal Operation Point for Open-Channel flows

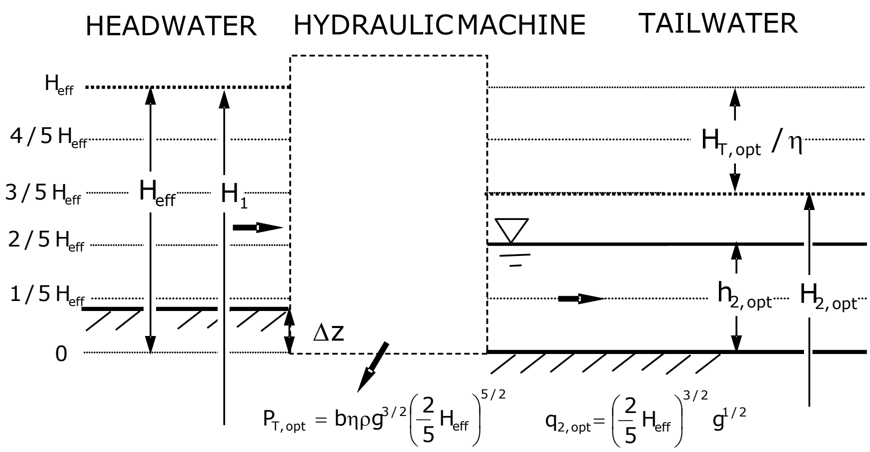

Fig. 1: Optimal operating point for hydropower [1].

The paper „Upper Limit for Hydropower in an Open-Channel Flow“ by the first author [1], deals with hydropower as an optimization problem. Based on strictly axiomatic derivation, the optimal flow rate and optimal head are determined, using the energy balance. Fig.1 shows the result of [1], i.e. the optimal operating point. The optimal head is $2\eta/5$ (being η the hydraulic efficiency) of the effective head $H_{eff} := \Delta z + E_1$ for given upstream energy height $E_1 := h_{10} + u_{10}^2/2g$. The coefficient of performance is defined as usual [1]

$$C_p := \frac{P_T}{P_{avail}}\,. \qquad (1)$$

P_T is the hydraulic power gained by the turbine

$$P_T = \eta\, Q\, \rho g H_T. \tag{2}$$

$q_2 = Q/b$ is the flow rate per depth unit for the tailwater width b of an open channel.

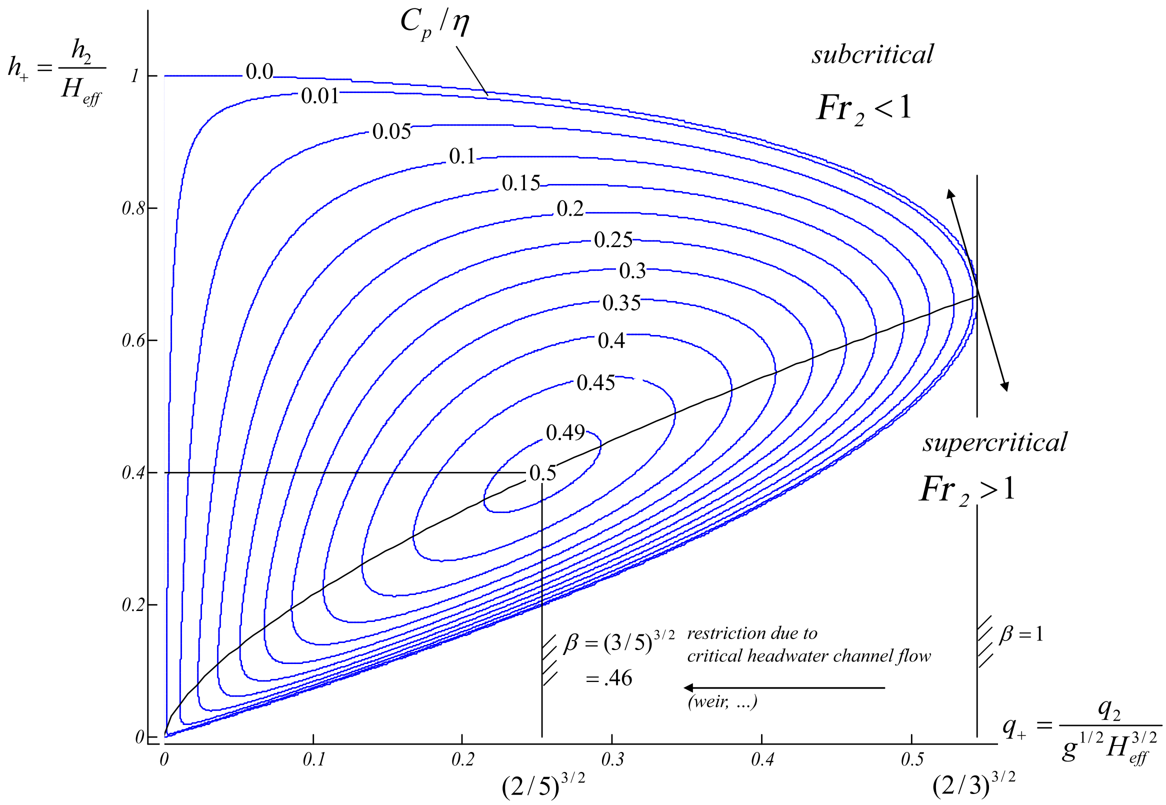

Fig. 2: Coefficient of performance as a function of dimensionless volume flow and head [1]

The available power P_{avail} represents the power that can be gained by a hypothetical ideal machine at its best operation point. It is defined as [1]

$$P_{avail} := 2\rho b \left(\frac{2}{5} H_{eff}\right)^{5/2} g^{3/2}. \tag{3}$$

With (1), (2) and (3) the coefficient of performance C_p can be written as a function of the operating point, with a maximum of $C_{p,max} = \eta/2$. I.e. even for a hydraulic efficiency of 100%, max. 50% of the available hydropower can be transformed into mechanical power in the best operation point sketched in Fig. 2.

Fig. 2 shows the coefficient of performance as function of the dimensionless water depth of the tailwater $h_+ := h_2/H_{eff}$ and the dimensionless specific volume flow rate $q_+ := Q/(b g^{1/2} H_{eff}^{3/2})$. The maximum of C_p/η determines the optimal operating point, with the optimal head

$$H_{T,opt} = \frac{2}{5}\eta H_{eff} \ , \tag{4}$$

the optimal volume flow rate [1]

$$Q_{opt} = b\left(\frac{2}{5}\right)^{3/2} g^{1/2} H_{eff}^{3/2} \quad , \tag{5}$$

and the optimal water depth level in the tailwater

$$h_{2,opt} = \frac{2}{5} H_{eff} \; . \tag{6}$$

Thus the optimal turbine power is

$$P_{T,opt} = \rho g H_{T,opt} Q_{opt} = \eta b \rho g^{3/2} \left(\frac{2}{5} H_{eff}\right)^{5/2} \; . \tag{7}$$

2 Application to Open-Channel with Bypass

Nowadays hydrokinetic turbines are applied to surface or tidal watercourses without damming the water. In most cases the width of the turbine is less than the width of the channel to which it gets applied. Thus there will be a part of the volume flow passing beside the turbine, which cannot be used for the gaining of energy and therefore has to be considered as leakage.

[2] examines the blockage effects of a tidal turbine by an analytic description of a disc actuator of infinite width.

Issue of the present paper, is a 0-dimensional Model for a disc actuator of infinite height, as shown in Fig. 3.A.

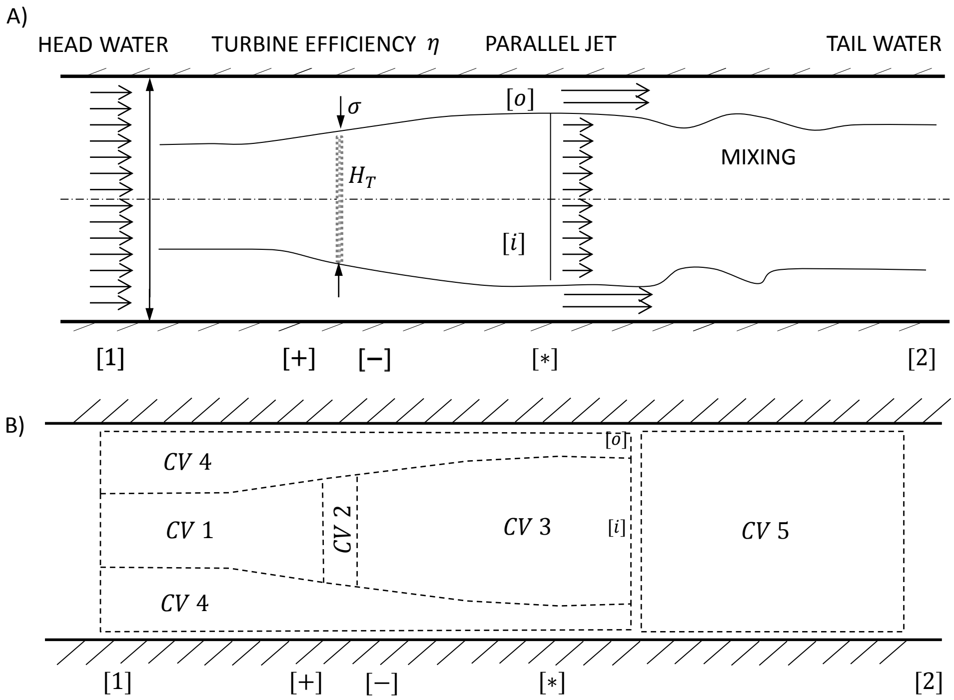

Fig. 3: 0-dimensional disc-actuator model for tidal turbines

The question to be answered is:
What is the influence of the turbine head H_T and the dimensionless turbine width σ on the coefficient of performance?

The 0-dimensional model considers the flow velocity and water level height at 6 points (see Fig.3, 1, +, −, o, i, 2) and the stream tube contraction ratio in the points 1 and i. Due to continuity of the stress tensor, the water level heights of the points i,o have to be of equal magnitude. This leads to the 13 unknowns $(u_1, h_1, u_+, h_+, u_-, h_-, u_i, u_o, h_{i,o}, u_2, h_2, \beta_1, \beta_{i,o})$.

By subdividing the model into the control-volumes shown in Fig. 3.B, one can derive five equations based on the conservation of energy law and 5 equations for the conservation of mass law, one for each control-volume. Furthermore three equations can be derived based on the conservation of momentum. This leads to the nonlinear equation system shown Table. 1.

To solve the equation system, an optimization algorithm was developed programmed in Matlab, based on the principle of least square methods, which leads to the results discussed in the following section.

Table 1: Nonlinear equation system for disk-actuator in open-channel with bypass

(1)	$CV\ 1{:}\quad u_1h_1\beta_+ - u_+h_+\sigma = 0$	CONSERVATION OF MASS
(2)	$CV\ 2{:}\quad u_+h_+ - u_-h_- = 0$	
(3)	$CV\ 3{:}\quad u_-h_-\sigma - u_ih\beta_- = 0$	
(4)	$CV\ 4{:}\quad u_1h_1(1-\beta_+) - u_oh(1-\beta_-) = 0$	
(5)	$CV\ 5{:}\quad u_oh(1-\beta_-) + u_ih\beta_- - u_2h_2 = 0$	
(6)	$CV\ 1{:}\quad H_1 - \left(h_+ + \frac{u_+^2}{2g}\right) = 0$	CONSERVATION OF ENERGY
(7)	$CV\ 2{:}\quad h_+ + \frac{u_+^2}{2g} - \left(h_- + \frac{u_-^2}{2g}\right) - \frac{H_T}{\eta} = 0$	
(8)	$CV\ 3{:}\quad h_- + \frac{u_-^2}{2g} - \left(h + \frac{u_i^2}{2g}\right) = 0$	
(9)	$CV\ 4{:}\quad H_1 - \left(h + \frac{u_o^2}{2g}\right) = 0$	
(10)	$CV\ 5{:}\quad u_ih\beta_-\left(h + \frac{u_i^2}{2g}\right) + u_oh(1-\beta_-)\left(h + \frac{u_o^2}{2g}\right) - u_2h_2\left(h_2 + \frac{u_2^2}{2g}\right) = 0$	
(11)	$[1]-[2]{:}\quad h_1\left(h_1 + \frac{u_1^2}{g}\right) - h_2\left(h_2 + \frac{u_2^2}{g}\right) - \frac{h_1H_T\beta_+}{\eta} = 0$	CONSERVATION OF MOMENTUM
(12)	$CV\ 5{:}\quad h(1-\beta_-)\left(h + \frac{u_o^2}{g}\right) + h\beta_-\left(h + \frac{u_i^2}{g}\right) - h_2\left(h_2 + \frac{u_2^2}{g}\right) = 0$	
(13)	$[1]{:}\quad H_1 - \left(h_1 + \frac{u_1^2}{2g}\right) = 0$	DEFINITION OF HEAD

3 Results

The answer to the question raised in the above section, can be read from Fig. 5. For $\sigma = 0$, or in other terms for an empty channel without any disk-actuator, there is no mechanical power gained. Hence the coefficient of performance vanishes. For $\sigma = 1$, or in other terms, a disk-actuator of the same width as the channel without bypass, the results are consistent to the results of [1] as expected. For $\mathrm{H_T}/\mathrm{H_1} = 0.4$ the optimum for the coefficient of performance $\mathrm{C_p}/\eta = 0.5$ is reached. For values of $0 < \mathrm{H_T}/\mathrm{H_1} < 0.4$ the maximum coefficient of performance is increasing from $\mathrm{C_p}/\eta = 0$ to $\mathrm{C_p}/\eta = 0.5$, for values of $0.4 < \mathrm{H_T}/\mathrm{H_1} < 1$ the maximum coefficient of performance is decreasing from $\mathrm{C_p}/\eta = 0.5$ to $\mathrm{C_p}/\eta = 0$. For values of $\mathrm{H_T}/\mathrm{H_1}$ and σ near 1 or zero the uncertainty of the solver rises. Hence for these in values of σ and $\mathrm{H_T}/\mathrm{H_1}$ the resolution of the optimization problem has to be improved. Yet the development of the optimization algorithm is in progress.

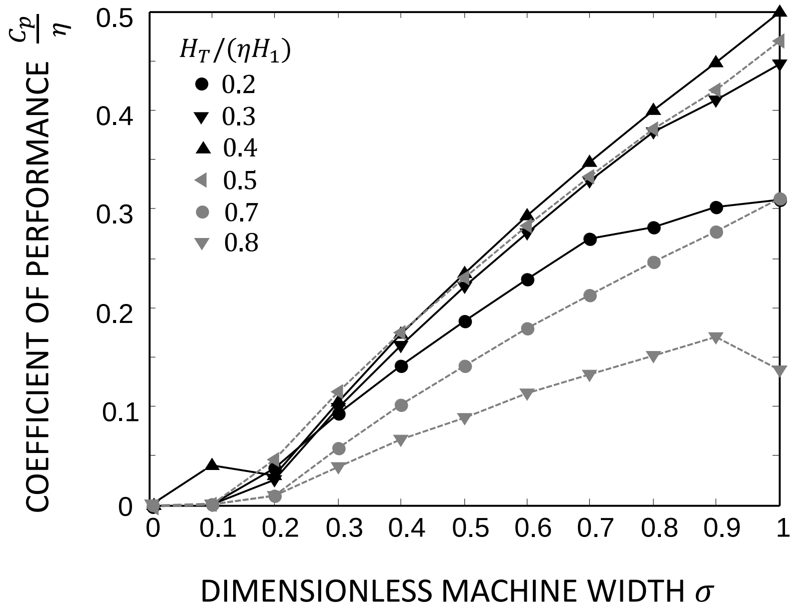

Fig. 5: Results of optimization algorithm

4 Conclusions

The nonlinear equation system describing a disk-actuator in an open channel flow can be solved by the developed optimization algorithm. Knowing the wake conditions of one disk-actuator, fields of turbines can be described with parallel or series connections of the disk-actuator model.

Since the optimization problem considered by [1] is valid independently of the machine type installed to an open-channel flow, the results of the disk-actuator with bypass have to be consistent to the results of Pelz. The 0-dimensional model accomplishes this consistence.

[1] Pelz, P: Upper Limit for Hydropower in an Open-Channel Flow, Journal of Hydraulic Engineering; ASCE / November (2011); DOI: 10.1061/(ASCE)HY.1943-7900.0000393

[2] Wheelan, J. et al.: A free-surface and blockage correction for tidal turbines, Journal of Fluid Mechanics vol. 624; Cambridge University Press (2009); DOI: 10.1017/S002211200900591