OPTIMIZATION OF POWER-SPECIFIC INVESTMENT COSTS FOR SMALL HYDROPOWER

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Abstract: The optimal head and volume flow rate for hydropower are determined through axiomatic and analytical considerations by Pelz in 2011. Based on these results the Cordier-diagram and its asymptotes are discussed. The technological progress causes that nowadays machines of high specific speed and hence small specific diameter, like Kaplan turbines, can be used, delivering the same power as the antiquated machines of low specific speed, like water wheels, did during the last centuries. Therefore the power-specific investment costs decrease. Within the legal borders, which are given by the legislative organs and due to the responsibility towards the environment, systems of robust technologies with minimal impact on nature at the same time are required. Considering all this, the present paper proposes an innovative concept for a fish permeable hydraulic machine for small hydropower.

1 Introduction

In article 35 paragraph 1 of the water resources law of the Federal Republic of Germany it is written: “The use of hydropower can only be approved if appropriate measures are taken for the protection of fish.” Yet these measures cause considerable edificial effort involved with increasing of the investment costs per installed capacity, which finally are determining for the long term success of a technology. Thus, for small hydropower plants economic profitable operation cannot always be accomplished. Furthermore the fish-protection-measurements do not always attain the effect provided for by statute.

In the year 1846 Ferdinand Redtenbacher, known as the originator of scientific engineering, wrote in his monograph about “Theory and Construction of Water Wheels” [9]: “A work on water wheels with horizontal axis at present is not a contemporary release. Those almost have become an antiquity, due to the rapid dissemination of the turbines. Although their relevance is minor than few years ago, they are and always will be power-machines of great utility, that never can be completely displaced by turbines”

Nowadays, 160 years after the writing down of these words, for the harvesting of hydro energy the use of turbines and therefore hydrodynamic machines is almost exclusively. Like the paddlewheel disappeared as driving mechanism for riverboats, so did the water wheel as power-machine. The present work investigates the reasons for that and deduces systematicaly, using the Cordier-diagram and a work of the first author [7], that deals with the subject of **optimal operating of hydropower**, an innovative hydropower plant that provides an integrated solution for fish protection with economically affordable operating at the same time.

2 Optimal Operating Conditions for Hydropower

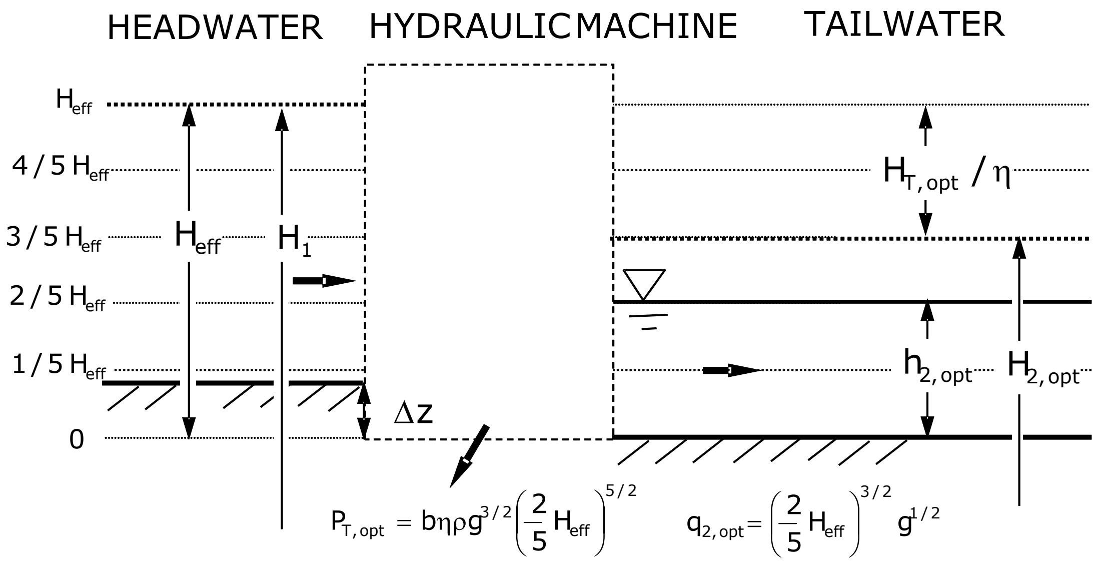

Fig. 1: Optimal operating point for hydropower [7].

The paper „*Upper Limit for Hydropower in an Open-Channel Flow*" by the first author [7], deals with hydropower as optimization problem. Based on strictly axiomatic derivation, the optimal volume flow rate and head are determined, using the energy equation. Fig.1 shows the result of [7], i.e. the optimal operating point. The optimal head is $2\eta/5$ (being η the hydraulic efficiency) of the effective head $H_{\text{eff}} = \Delta z + E_1$ for given upstream energy height $E_1 = h_{10} + u_{10}^2/2g$. The coefficient of performance is defined as usual [13]

$$C_p := \frac{\eta P_T}{P_{\text{avail}}}\,. \qquad (1)$$

P_T is the hydraulic power gained by the turbine

$$P_T = \eta\ bq_2\ \rho g H_T. \qquad (2)$$

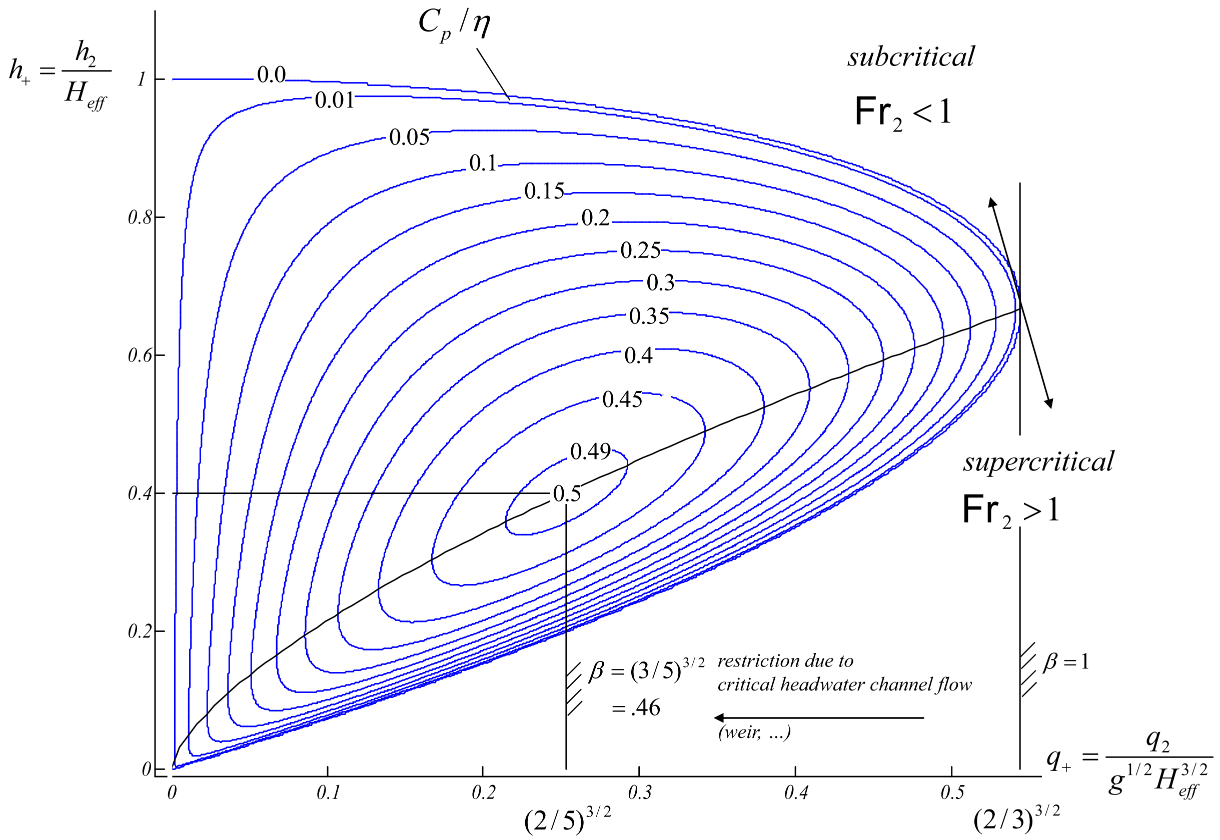

Fig. 2: Coefficient of performance as a function of dimensionless volume flow rate and head [7].

The available power P_{avail} represents the power that can be gained by the hypothetical ideal machine at its best operation point. It is defined as [7]

$$P_{avail} \coloneqq 2\rho b \left(\frac{2}{5} H_{\text{eff}}\right)^{5/2} g^{3/2}. \qquad (3)$$

With (1), (2) and (3) the coefficient of performance C_p can be written as function of the operating point, with a maximum of $C_{p,max} = \eta/2$. I.e. even for a hydraulic efficiency of one, max. 50% of the available hydropower can be transferred into mechanical power in the best operation point sketched in Fig. 1.

Fig. 2 shows the coefficient of performance as function of the dimensionless water depth of the tailwater $h_+ \coloneqq h_2/H_{\text{eff}}$ and the dimensionless specific volume flow rate $q_+ \coloneqq Q/(b_2 g^{1/2} H_{eff}^{3/2})$. The maximum of C_p/η determines the optimal operating point, with the optimal head [7]

$$H_{T,opt} = \eta \frac{2}{5} H_{\text{eff}} \qquad (4)$$

and the optimal volume flow rate [7]

$$Q_{opt} = b\left(\frac{2}{5}\right)^{3/2} g^{1/2} H_{eff}^{3/2} \quad . \tag{5}$$

Thus the optimal turbine power is

$$P_{T,opt} = H_{T,opt} Q_{opt} = \eta b \rho g^{3/2} \left(\frac{2}{5} H_{\text{eff}}\right)^{5/2} . \tag{6}$$

Science is about asking the right questions. The open question raised in the paper mentioned above [7] is:

> *„What is the maximum hydraulic power that can be transformed into mechanical energy by any possible machine in an open channel?“*

The present paper is dealing with the proximate question:

> *„Which machine can harvest the optimal power (6), providing high fish permeability, low power-specific investment costs and high robustness (for low production- and maintenance costs) at the same time?“*

3 Asymptotes in the Generalized Cordier-Diagram

The Cordier-diagram is of similar fascination for scientists like the famous Hertzsprung-Russel-diagram showing the distribution of stars along certain lines in a luminosity-temperature plane, beginning with the red dwarfs on the main sequence leading to our sun and beyond. Fluid flow machines are distributed along sequences of the $\sigma - \delta -$ plane of the Cordier-diagram, which puts the machines in a systematic order. In other words: The Cordier-diagram gives an overview of the economic and technical evolutionary selection of the past centuries engineering.

Using (4) and (5) the specific speed introduced by Keller [6] can be written as

$$\sigma := 2\sqrt{\pi}(2gH_T)^{-3/4} Q^{1/2} n \quad , \tag{7}$$

as well, as the specific diameter introduced by Baashuus [1]

$$\delta := \frac{\sqrt{\pi}}{2}(2gH_T)^{1/4}Q^{-1/2}d\ . \qquad (8)$$

Otto Cordier [3] ascertained, that all of turbo machines are distributed within narrow bands of the $\sigma-\delta-$plane:

$$\sigma_{opt} = \sigma_{opt}(\delta_{opt}). \qquad (9)$$

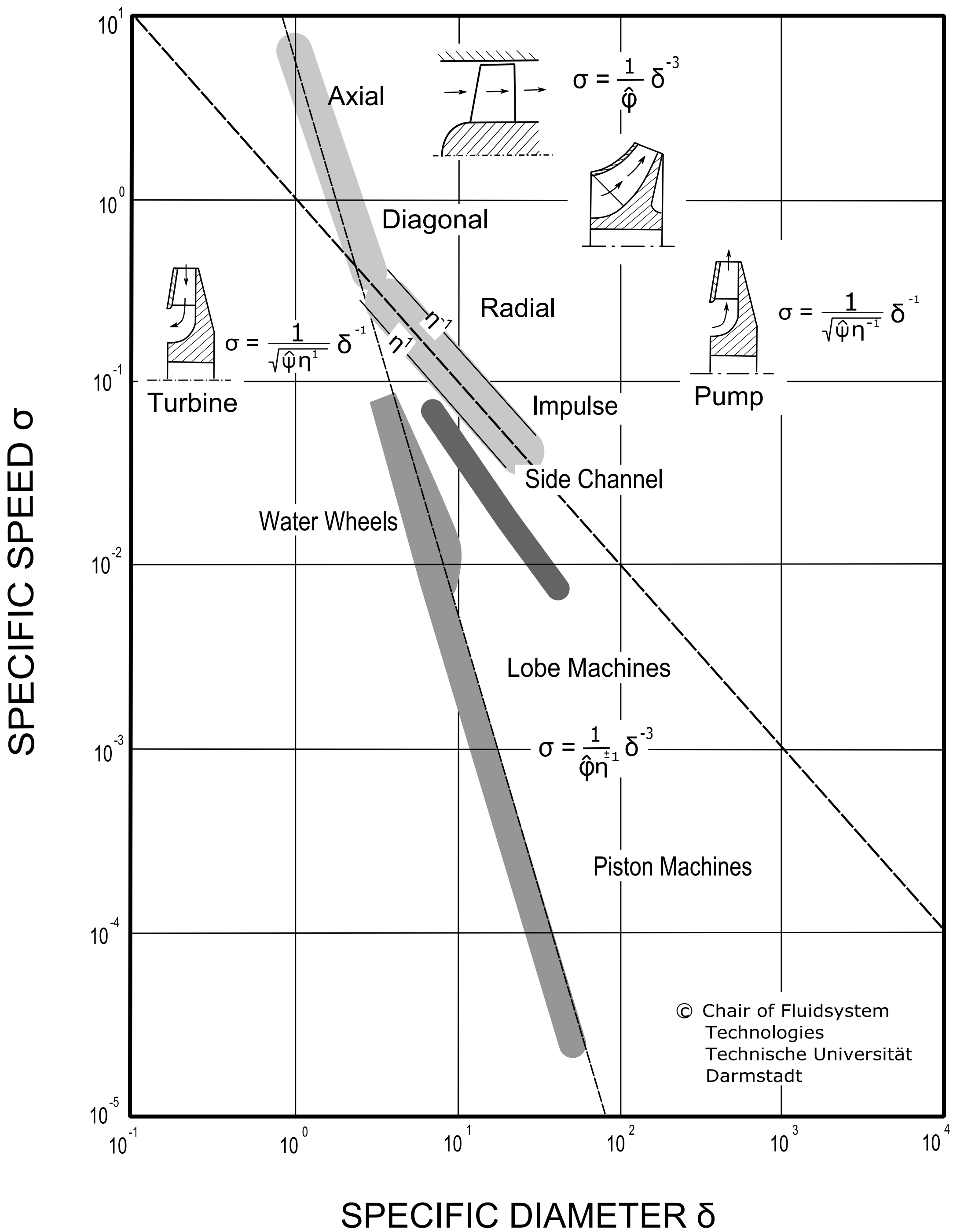

Fig. 3: Corider-diagram with machine typology and asymptotes.

Grabow [4] extends the range of the Cordier-diagram by hydrostatic i.e. positive displacement machines [2].

> *Here we discuss the asymptotes of the Cordier-diagram. We will use the derived three different asymptotic relations together with the results of Section 2 to deduce an optimal machine type for small hydro power in Section 4. Since our approach is axiomatic, the choice of the optimal machine type for small hydropower is hence out of the question once and for all.*

More common than specific speed and specific diameter are *flow number* φ and *pressure number* ψ which are defined as:

$$\varphi := \frac{1}{\delta^3 \sigma} = \frac{4Q}{n\pi^2 d^3} \ , \tag{10}$$

$$\psi := \frac{1}{\delta^2 \sigma^2} = \frac{2gH_T}{n^2\pi^2 d^2} \ . \tag{11}$$

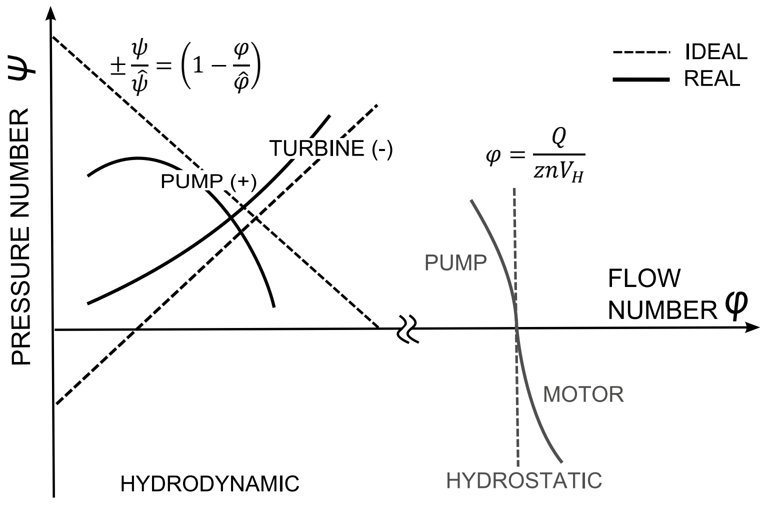

Fig. 4: Characteristics of pumps and turbines/motors.

The relation between pressure number and flow number for turbo machines (hydrodynamic machines) is given by the Euler-line (backgoing to the year 1775 Leonard Euler and his axiomatic derived conservation of angle momentum equation [11])

$$\pm\frac{\psi}{\hat{\psi}} = \eta^{\pm 1}\left(1-\frac{\varphi}{\hat{\varphi}}\right) \qquad \begin{matrix}\text{+1 for pumps}\\ \text{-1 for turbines}\end{matrix} \qquad (12)$$

Fig. 3 shows Equation (12) for pumps and fluid engines as well. The characteristics of hydrostatic machines is given by

$$\varphi = \eta^{\pm 1}\hat{\varphi}. \qquad \begin{matrix}\text{+1 for pumps}\\ \text{-1 for fluid engines}\end{matrix} \qquad (13)$$

For positive displacement machines the flow number $\varphi = Q/nzV_H$ depends on the displacement volume V_H of the machine, the rotational speed n and the number of chambers z. The *hydraulic efficiency* is conventional expressed as product of *mechanical and volumetric efficiency*

$$\eta = \frac{\Delta p_t Q}{\Omega M} = \underbrace{\frac{\Delta p_t V_H z}{M}}_{\eta_M} \underbrace{\frac{Q}{\Omega z V_H}}_{\eta_V} = \eta_M \eta_V \qquad (14)$$

being $\Delta p_t = \varrho g H_T$ the total pressure difference, η_M the mechanical efficiency and η_V the volumetric efficiency. Using (10), (13) the Cordier-asymptote for positive displacement machines can be written as:

$$\text{Asymptote} \qquad \sigma = \frac{\delta^{-3}}{\hat{\varphi}\eta^{\pm 1}} \qquad \text{for positive displacement machine.} \qquad (15)$$

For hydrodynamic machines the characteristics Equation (12) and Equations (10), (11) can be rearranged as quadratic equation:

$$0 = \sigma^2 - \frac{\sigma}{\hat{\varphi}\delta^3} - \frac{1}{\hat{\psi}\eta^{\pm 1}\delta^2} \quad . \qquad (16)$$

For large specific diameter (radial pump or turbine) the second term is negligible, which can easily be shown by solving the quadratic equation for σ. Hence Equation (16) reduces to:

$$\text{Asymptote} \qquad \sigma = \frac{\delta^{-1}}{\sqrt{\hat{\psi}\eta^{\pm 1}}} \qquad \text{for radial machine.} \qquad (17)$$

Due to different exponent (+1 for pumps and -1 for turbines) of efficiency η centrifugal pumps differ significant from radial turbines in the Cordier-diagram. In the first case the square root of the efficiency is in the denominator in the second case it is in the numerator (Fig. 2). For hydrodynamic machines of low specific diameter, hence axial turbines, the third term of (16) is negligible and the equation reduces to:

$$\text{Asymptote} \quad \sigma = \frac{\delta^{-3}}{\hat{\varphi}} \qquad \text{for axial machine.} \qquad (18)$$

The advantage of the asymptotes (15), (17), (18) is, that they provide an analytic approach to an easy use of the Cordier-diagram for machine design.

4 Power-Specific Investment Costs

The following section concerns how the optimized power $P_{T,opt}$ can be harvested with small power-specific investment costs k_p:

$$k_p := \frac{\text{IVESTMENT COSTS}}{\text{ELECTRIC POWER}} \; .$$

For the long term success of a power plant technology, the economic profitability is a decisive criterion, regardless of whether it is based on combustion of fossil fuels or on the use of renewable energy sources. Reducing power-specific investment costs k_p shortens the amortizations period and leads to larger overall profit over the lifecycle of any power plant.

> *Thus, to make hydropower more competitive, it is reasonable to concern about power-specific investment costs as optimization parameter.*

Since the energetic optimal head and volume flow rate are known due to (4) and (5) (cf. [7]), the Cordier-diagram discussed in the above section, can be used to select a machine type.

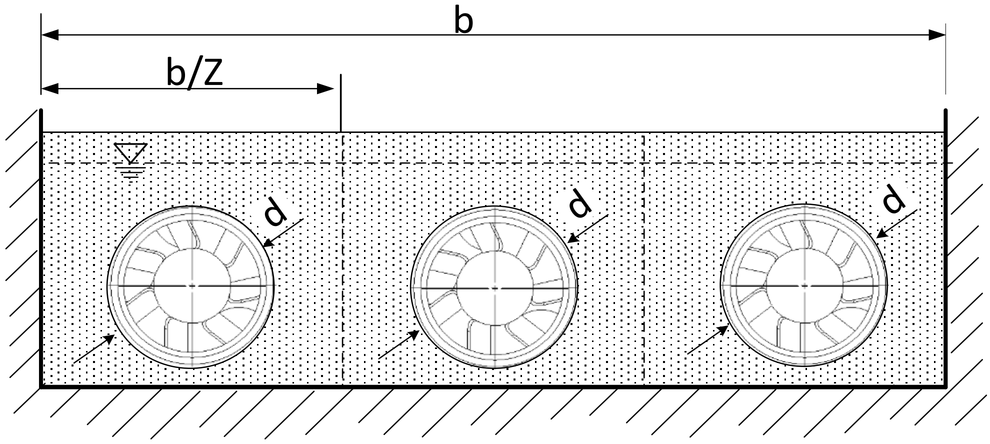

Fig. 5: Cross section through transverse structure of hydro energy plant with Z= 3 modules.

With (4), (5) used in (7) resp. (8) the specific speed for a hydro energy module at the energetic best operation point (cf. [7] and Section 2) is

$$\sigma_{opt} = 2\sqrt{\pi}\left(\frac{1}{2\eta}\right)^{3/4}\frac{n}{\sqrt{Zg/b}} \tag{19}$$

and the specific diameter is

$$\delta_{opt} = \frac{\sqrt{\pi}}{2}\left(\frac{25\eta}{2}\right)^{1/4}\frac{d}{\sqrt{bH_{\text{eff}}/Z}} \quad , \tag{20}$$

where Z is the number of modules, with the subdivision Q/Z of the entire volume flow rate of the open water course (Fig. 5). The relation between δ and σ is given by the asymptote (18) $\sigma = \delta^{-3}/\hat{\varphi}$. Using (19) and (20) in (18) the resulting equation is

$$\frac{\pi^2}{4}\frac{\hat{\varphi}\, Z\, nd^3}{b\left(\frac{2}{5}H_{\text{eff}}\right)^{3/2} g^{1/2}} = 1, \tag{21}$$

or, when solved for the machine volume which approximately is proportional to the investment costs

$$INVEST \sim Z\frac{\pi}{4}d^3 = \frac{2}{5}\,\frac{b\, H_{\text{eff}}^{3/2}\, g^{1/2}}{\pi n} \quad . \tag{22}$$

Using (6) in (22) the power-specific volume, which is for a first approach proportional to the power-specific investment costs

$$k_p \sim \frac{P_{T,opt}}{Z\,\pi/4\;d^3} = \frac{2\pi}{5}\;\hat{\varphi}\;\eta\;\rho g\;H_{\text{eff}}\;n\;, \tag{23}$$

with $\hat{\varphi} \approx 0.25 \ldots 0.5$.

The power-specific investment costs are proportional to the effective head H_{eff} [7] and rotational speed n. They are most important for the success of an energy plant technology. Since the invention of the electric generator, which has not such a limitation in rotational speed as a belt drive, there is today no need for water wheels or hydrostatic energy converter of low specific speed (15). It is surprising, that today is still funded research on such devices. The reason for the research and funding may be an unfamiliarity with the scientific fundament of fluid power machines.

5 A new Design for a Fish-Friendly and Robust Turbine

Like it is shown in the above section, the optimal machine for hydropower in open watercourse is an axial machine. Traditionally in this case of application Kaplan- or bulb turbines are used. The latter especially are used for open watercourse hydropower. Common disadvantages shared by all turbines that are state of the art are the high injury- and mortality rate for fish, passing the machine and high risk of blockage due to bed load or floating refuse. To avoid blockage, screens are used that cause blockage as well and need to be cleaned. To ensure fish permeability of turbines, bypasses with fish ladders are constructed. These cause high edificial effort and increasing of investment costs.

Furthermore the main part of the volume flow rate goes into direction of the turbine where fish get caught by the suction towards the screen and die. The main reasons for high mortality rates of fish in hydropower plants are high pressure gradients and chopping of the fish, by the rotating blades. The first reason is not relevant for open watercourse hydropower, where the pressure gradients are in a fish friendly range.

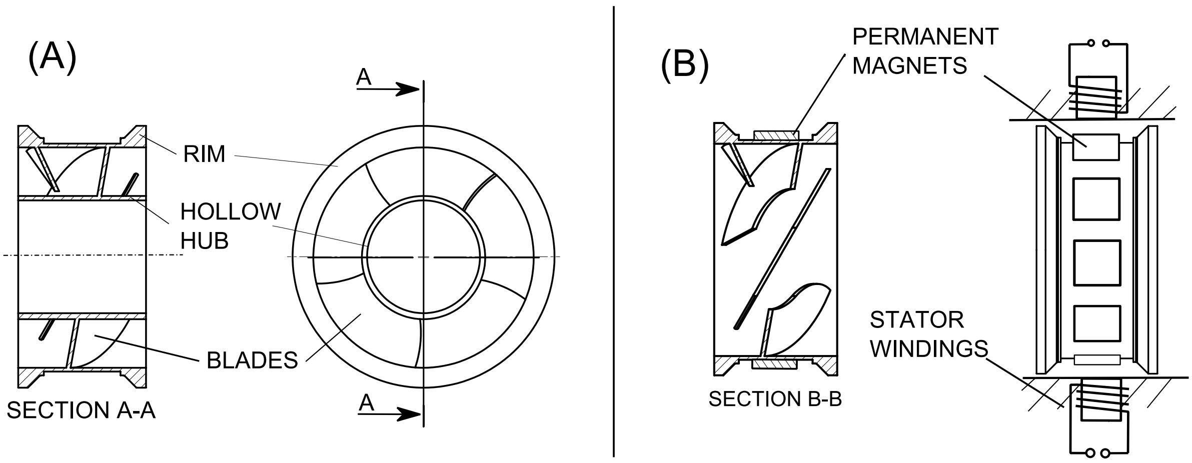

Fig. 6: Design of a fish friendly turbine [8].

The new machine design proposed by the Institute of Fluid Systems Technologies [8] is a hubless Kaplan turbine as shown in Fig. 6(A). The blades are attached to a cylindrical rim mounted to the casing of the machine. It can be operated as turbine or as pump as well. Instead of no hub there can be a hollow hub attached to the inner end of the blades as shown in Fig. 6(B).

For both machine types the cylindrical channel without rotating rigid bodies can potentially reduce the risks of blockage and fish mortality. The screen in front of the hydropower plant can be designed coarser to keep out floating refuse of the same size as the hollow hub only.

For economic reasons, it can be advantageous to construct the turbine-generator-unit as an integrated design. [12] concluded salient pole machines with permanent magnets to be the most profitable in this case of application, so that the permanent magnets must be attached to the outer rim of the turbine. The stator winding can be located in the casing of the turbine to avoid contact of the electricity conducting parts with water.

6 Summary

Based on the optimal operating point for any hydropower plant and Cordier-diagram it is shown, that for small hydro power axial turbines are preferable with respect to power-specific investment costs. A new concept of fish-friendly axial turbine is proposed.

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