Proceedings of ASME Turbo Expo 2012
GT2012
June 11-15, 2012, Copenhagen, Denmark

GT2012-68398

TURBOMACHINES UNDER PERIODIC ADMISSION – AXIOMATIC PERFORMANCE PREDICTION

Mehdi Nakhjiri
Chair of Fluid Systems Technology
Technische Universität Darmstadt
Darmstadt, Germany
Email: Nakhjiri@fst.tu-darmstadt.de

Peter F. Pelz
Chair of Fluid Systems Technology
Technische Universität Darmstadt
Darmstadt, Germany
Email: Pelz@fst.tu-darmstadt.de

ABSTRACT

In calibration tasks under engine conditions, steady-state manufacturer performance maps are applied to periodic turbocharger operation. This procedure is exposed to considerable uncertainties. In this work the axiomatic form of the energy equation as well as Euler turbomachinery equation are used to generate a general form of the respective equations which allow for periodicity. Thus, the concept of apparent speed and apparent efficiency is introduced. The latter can attain values greater than unity.

NOMENCLATURE

b	blade height
c	velocity
E, e	internal energy, specific internal energy
h	specific enthalpy
K	kinetic energy
M	torque
$\dot{m}$	mass flow rate
P, p	power, pressure
$\dot{Q}$	heat flow
R, r	ideal gas constant, radius
s	specific entropy
T, t	temperature / cycle duration, time
u	circumferential speed
V	volume
y	technical work

Greek Symbols

α	absolute blade Angle
β	relative blade Angle
η	efficiency
λ	power coefficient
ρ	density
φ	flow coefficient
ψ	pressure coefficient
Ω	engine angular velocity
ω	turbocharger angular velocity

Subscripts

1	inflow
2	outflow
app	apparent
C	compressor
is	isentropic
m	meridional
Sh	shaft
T, t	turbine, total
Θ	circumferential

1. INTRODUCTION

The model based control strategies for turbochargers and engine calibration in automotive applications are based on the steady-state power balance and call for efficiency maps as the main physical parameter. Test rig and vehicle experiments and the subsequent offline analysis of the gained measurements is the standard way for calibrating these maps with the engine. This approach makes use of either manufacturer performance maps which are in general generated on hot gas test stands, or physical models to describe turbocharger performance as proposed in [1] for a compressor.

Both methods assume steady-state flow; however, under engine conditions the turbocharger operation is transient due to periodic valve operating mechanism of the engine which results in periodic admission of the turbine in the exhaust path as well as periodic suction in the intake path. Hence, the procedure of applying steady-state manufacturer performance maps to transient turbocharger operation in calibration tasks is exposed to considerable uncertainties. Wallace and Blair [2] indicate that a quasi-steady prediction method consistently underestimates the power output of the turbine under unsteady conditions up to 25%.

The scope of the presented model in this work is to cope with these uncertainties on the basis of a physically motivated scaling method. It is shown, that the mismatch between the two different physical situations can be solved by considering a more general form of the conservation law of energy and angular momentum (Euler turbomachinery equation).

We consider all unsteady quantities additively composed of a mean value and a periodic part as shown in Fig.1 for the mass flow rate. Thus, we model the periodic signals in a suitable way and redefine dimensionless key performance indicators of turbomachines which include new terms reflecting the influence of the periodic part of the unsteady quantities in addition to the term for steady-state cases. Hence, we introduce an axiomatic relation between quasi-steady and transient operation conditions based on equations of energy and angular momentum in average of time. Further, we define the concept of apparent efficiency while the ideal process results from isentropic change of state in average of time.

The approach can also be applied to conventional steady-state 0D/1D models. This allows prediction of performance maps under periodic admission in average of time.

2. STATE OF THE ART

Many attempts have been made, both experimental and analytical, in order to understand the difference in turbine performance between tests on a steady-state gas stand and measurements on the engine test rig. Baines [3] gives a comprehensive survey to the state of the art regarding turbocharger turbine pulse flow performance and modeling.

Dealing with periodic flow situation in turbochargers, we are confronted with two fundamental questions to choose an appropriate modeling method. First question refers to the physical origin of the pulsating flow. Several works have already been done e.g. by Winterbone and Pearson [4], Karamanis et al. [5], Ehrlich et al. [6] to indicate if the pulsating flow is a consequence of the convection of mass flow through the turbine or it is caused by the action of pressure waves. Ehrlich [7] shows, based on experimental results, that both effects are significant. Pressure fluctuations are related to wave propagation while temperature fluctuations are assumed to be a result of convection of hot gas. Szymko et al. [8] use two different modified Strouhal numbers to infer the onset of unsteadiness due to the gas and pressure wave velocities for various components of interest. Frequency limit of steadiness is then determined on the basis of assumptions for length scale as well as critical value of 0.1 for the Strouhal number. The results exhibit three modes of behavior for the turbine under pulsating flow. The authors propose quasi-steady approach being adequate for Strouhal numbers less than 0.1 which should be valid for the rotor blade passage over the entire operating range of the engine with regard to frequency limits determined in [8].

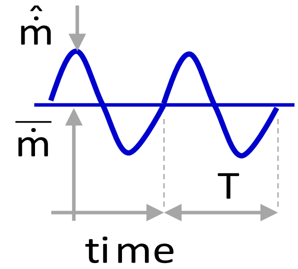

Figure 1: Periodic mass flow with time averaged and time varying part

This issue substantiates the significance of convection in pulsating flow and motivates a 0D analysis being an appropriate approach.

Second point refers to the general assumption of steady-state rotor modeling in 0D and 1D approaches. Hu and Lawless [9] and Costall et al. [10] present transient models, incorporating quasi-steady rotor models. This is due to lack of reliable correlations for transient flow in turbine rotors.

Further, Abidat et al. [11] investigate the impact of amplitude and frequency of the periodic inlet flow on pressure. The conclusion is that the phase difference between the flow rate and the pressure ratio shrinks when frequency decreases. Hence, the turbine operation approaches quasi-steady operation behavior for frequencies towards zero, which is expected (This has been observed in [8] as well). Experimental results show that, contrary to the amplitude, the effect of the frequency of the inlet flow on the mean mass flow rate is negligible. However, the mean turbine work output is dependent on both frequency and amplitude of the inlet flow.

Altogether, Abidat et al. prove considerable conformity between mean mass flow rate as well as turbine work at transient and quasi-steady operation. This motivates the assumption of quasi-steady rotor operation in this work. Thus, the fluctuations of the rotor speed are neglected. Transient operation of the turbocharger is considered at constant load points which can be analyzed under assumption of periodic admission. The goal is to provide an analytical guideline to generate an effective turbocharger transient performance map out of the steady-state manufacturer map based on a modified Euler work.

3. PERIODIC FLOW

Taking time dependent quantities into consideration, every quantity can be split into a time averaged part and a time varying part. Accordingly, the periodic mass flow rate and periodic total enthalpy can be rewritten as:

$$\dot{m}(t) = \bar{\dot{m}} + \tilde{\dot{m}}(t), \qquad (1)$$

$$h_{\mathrm{t}}(t) = \bar{h}_{\mathrm{t}} + \tilde{h}_{\mathrm{t}}(t). \qquad (2)$$

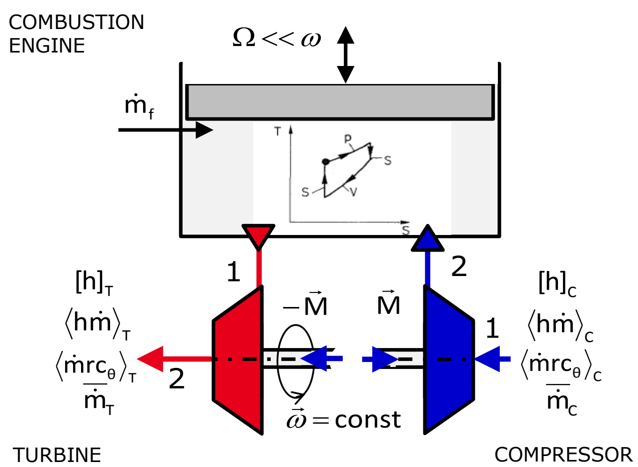

Figure 2: Sketch of turbocharged engine including relevant physical parameters

The time averaged part of Eq.1 is defined as:

$$\bar{\dot{m}} := \frac{1}{T}\int_0^T \dot{m}\,\mathrm{d}t. \tag{3}$$

Further, with the cycle duration $T = 2\pi/\Omega$ the time varying part in Eq.1 features:

$$\tilde{\dot{m}}(t) = \tilde{\dot{m}}(t+T). \tag{4}$$

The properties Eq.3 and Eq.4 are analogously applied to the time dependent total enthalpy in Eq.2.

4. THE ENERGY EQUATION

The most general and hence axiomatic form of the energy equation reads: the rate of change of kinetic energy K plus the rate of change of internal energy E is balanced by the total work done per unit time on the fluid P and the heat flux:

$$\frac{\mathrm{D}K}{\mathrm{D}t} + \frac{\mathrm{D}E}{\mathrm{D}t} = P + \dot{Q}\ ,$$

or

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_{V(t)}\left(\frac{c^2}{2} + e\right)\rho dV = P + \dot{Q}\ . \tag{5}$$

Here ρ denotes the density, e the specific internal energy and c the absolute velocity magnitude. As it is well known, for a steady-state flow this simplifies to:

$$\dot{m}(h_{t2} - h_{t1}) = P_{\mathrm{Sh}} + \dot{Q}, \tag{6}$$

which is most important for the steady-state behavior of a turbo machine. The brackets used in Eq.6 are introduced as a convenient apprehension for the enthalpy difference. 1 denotes the state upstream of the machine and 2 the state downstream of the machine. $P_{\mathrm{Sh}} = \vec{M}\cdot\vec{\omega}$ is only that part of P which is transmitted through the shaft of the machine. $\vec{M}$ denotes the transmitted torque and $\vec{\omega}$ the angular shaft frequency. The remaining part of the total power is, as usual, part of the enthalpy flux on the left side of Eq.5. The work done by the pressure p on the control surfaces 1 and 2 is part of the enthalpy flux, since the total enthalpy is by definition related to the pressure as:

$$h_{\mathrm{t}} := \frac{p}{\rho} + \frac{c^2}{2} + e = c_{\mathrm{p}}T + \frac{c^2}{2} + e_*. \tag{7}$$

e_* is an insignificant constant specifying the zero point.

4.1. TIME AVERAGED ENERGY EQUATION

In the context of this work we consider periodic flow situations, where the mass flow rate and the enthalpy are given by Eqs.1 and 2. For this flow situation the time average of the energy equation $\overline{\mathrm{D}(K+E)/\mathrm{D}t} = \bar{P} + \bar{\dot{Q}}$ gives the more general form of the energy equation:

$$\underbrace{\bar{\dot{m}}[h_{\mathrm{t}}]}_{\mathrm{Eq.6}} + \underbrace{\langle \dot{m}h_{\mathrm{t}}\rangle}_{\mathrm{new}} = \underbrace{\overline{P_{\mathrm{Sh}}} + \bar{\dot{Q}}}_{\mathrm{Eq.6}}. \tag{8}$$

Note that the term in the squared bracket refers to the difference of time averaged enthalpies while the expression in the round brackets stands for the difference of time varying products, according to the following definitions:

$$[h_{\mathrm{t}}] := \bar{h}_{\mathrm{t2}} - \bar{h}_{\mathrm{t1}}\ , \tag{9}$$

$$\langle \dot{m}h_{\mathrm{t}}\rangle := \frac{1}{T}\int_0^T \tilde{\dot{m}}_2\tilde{h}_{\mathrm{t2}}\,\mathrm{d}t - \frac{1}{T}\int_0^T \tilde{\dot{m}}_1\tilde{h}_{\mathrm{t1}}\,\mathrm{d}t\ . \tag{10}$$

Thus, we interpret Eq.8 as a steady-state flow characteristic with a superposed periodicity. The important Eq.8 is only valid for periodic thermodynamic state, since only in this case the volume integral on the left side of the energy equation $\overline{\mathrm{D}(K+E)/\mathrm{D}t}$ vanishes and only the flux terms remain:

$$\int_V \overline{\frac{\partial}{\partial t}\left(\frac{\rho c^2}{2} + \rho e\right)}\,\mathrm{d}V = \int_V \frac{1}{T}\left[\frac{\rho c^2}{2} + \rho e\right]_0^T \mathrm{d}V = 0\ . \tag{11}$$

In the more special case of steady-state the second term on the left hand side of Eq.8 vanishes and the energy equations Eq.8 and Eq.6 are identical.

4.2. EFFICIENCY AND THE CONCEPT OF APPARENT EFFICIENCY

The efficiency as a dimensionless measure of dissipative losses within a machine is defined as:

$$\eta^{\pm 1} := \frac{\dot{m}y_{\mathrm{t}}}{P_{\mathrm{Sh}}}. \tag{12}$$

The positive exponent stands for the compressor, the negative exponent for the turbine. The specific shaft work y_t in Eq.12 is defined as the difference of the Bernoulli's constant at isentropic flow:

$$y_\mathrm{t} = \left[\frac{c^2}{2}\right] + \left[\int \frac{\mathrm{d}p}{\rho}\right] = \left[\frac{c^2}{2}\right] + [h]_\mathrm{is} = [h_\mathrm{t}]_\mathrm{is}. \tag{13}$$

Here Gibb's relation $T\mathrm{d}s = \mathrm{d}h - \mathrm{d}p/\rho$ is used for the isentropic case indicated by the index "is". Hence, using the energy equation Eq.6 for stationary and adiabatic flow the efficiency Eq.15 is written in the equivalent form as:

$$\eta^{\pm 1} := \frac{\dot{m}[h_\mathrm{t}]_\mathrm{is}}{P_\mathrm{Sh}} \underbrace{=}_{\mathrm{Eq.6}} \frac{[h_\mathrm{t}]_\mathrm{is}}{[h_\mathrm{t}]} \approx \frac{h_{2,\mathrm{is}} - h_1}{h_2 - h_1}, \tag{14}$$

where at the end again the kinetic energy is neglected. The isentropic enthalpy results from isentropic temperature which is given by:

$$T_{2,\mathrm{is}} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}, \tag{15}$$

With the help of Eq.14 and Eq.15 the efficiency is determined by measured data either of the shaft torque and shaft speed or with the mass flux and the change of pressure and temperature.

Now the question arises: How does the efficiency for periodic flow look like?

Taking into consideration that the efficiency is a ratio of two powers, it is reasonable to define the efficiency for the periodic case in the very same way as for the steady-state case. Hence, we define the ***apparent efficiency*** as:

$$\eta_\mathrm{app}^{\pm 1} := \frac{\bar{\dot{m}}[h_\mathrm{t}]_\mathrm{is}}{\overline{P_\mathrm{Sh}}} \underbrace{=}_{\mathrm{Eq.8}} \frac{[h_\mathrm{t}]_\mathrm{is}}{[h_\mathrm{t}] + \frac{\langle \dot{m} h_\mathrm{t} \rangle}{\bar{\dot{m}}}}. \tag{16}$$

In Eq.16 we have replaced the shaft power by the energy equation Eq.8. The ideal specific work $[h_\mathrm{t}]_\mathrm{is}$ results from Eq.9 and Eq.15 applied to measured quantities in average of time. Provided that the measured quantities in average of time are equal to steady-state quantities, the apparent efficiency in Eq.16 differs from the efficiency in Eq.12 by:

$$\eta_\mathrm{app}^{\pm 1} = \frac{\eta^{\pm 1}}{1 + \frac{\langle \dot{m} h_\mathrm{t} \rangle}{\bar{\dot{m}}[h_\mathrm{t}]}}. \tag{17}$$

The name apparent is used to differ from the efficiency. The efficiency is a dimensionless measure of the losses within the machine. In contrast to that the apparent efficiency is not only a measure of the dissipation. In fact the apparent efficiency can be greater than one, as will be seen later in this paper. From Eq.16 the name apparent becomes clear, since only the time averaged quantities are taken to determine the apparent efficiency in the very same way as for the stationary case.

The term $\langle \dot{m} h_\mathrm{t} \rangle$ in the denominator of Eq.16 and Eq.17 characterizes damping in the machine. Considering negative damping, i.e. reducing the amplitude of the periodicity through the rotor blade passage, the apparent efficiency yields larger values than the efficiency $\eta_\mathrm{app} > \eta$. This is valid for power machines as well as working machines. Assuming aerodynamic ideal process, i.e. no entropy generation, the apparent efficiency can attain values greater than unity due to negative damping as it was already mentioned above.

5. EULER TURBOMACHINERY EQUATION

The Euler equation dates back to the year 1775 and reads: the rate of change of axial angular momentum is balanced by the axial torque component: $\overline{\frac{\mathrm{D}}{\mathrm{D}t} \int_{V(t)} \rho r c_\theta dV} = \bar{M}$.

5.1. TIME AVERAGED EULER EQUATION

We assume a constant rotational shaft speed ω, i.e. the periodicity in the rotational speed is assumed to be negligible. This is for example usually well fulfilled for a turbocharger. Thus Euler's equation is given for the periodic flow case:

$$\underbrace{\omega \bar{\dot{m}}[r c_\theta]}_{\text{Euler term}} + \underbrace{\omega \langle \dot{m} r c_\theta \rangle}_{\text{new}} = \overline{P_\mathrm{Sh}}, \tag{18}$$

where the brackets are used in the way as defined in Eq.9 and Eq.10. Further, in Eq.18 the first term is the Euler work for steady-state flow while the second term again implies the periodicity of the flow. Due to the postulated periodicity the volume integral vanishes:

$$\int_V \overline{\frac{\partial}{\partial t}(\rho r c_\theta)}\, \mathrm{d}V = 0\,. \tag{19}$$

5.2. ACTUAL SPEED AND THE CONCEPT OF APPARENT SPEED

Allowing for constant machine speed we bring Eq.8 and Eq.18 together introducing a relationship for the actual speed including the periodicity:

$$\omega = \frac{\bar{\dot{m}}[h_\mathrm{t}] + \langle \dot{m} h_\mathrm{t} \rangle}{\bar{\dot{m}}[r c_\theta] + \langle \dot{m} r c_\theta \rangle} \lessgtr \frac{\bar{\dot{m}}[h_\mathrm{t}]}{\bar{\dot{m}}[r c_\theta]} := \omega_\mathrm{app}\,. \tag{20}$$

Further, in Eq.20 the ***apparent speed*** is defined based on time averaged quantities in a similar way as the ***apparent efficiency***. In case the flux of enthalpy fluctuation is smaller than the flux of fluctuation in angular momentum $\langle \dot{m} h_\mathrm{t} \rangle < \langle \dot{m} r c_\theta \rangle$, then the actual speed is smaller than the apparent speed. Otherwise the apparent speed would be smaller.

6. APPLICATIONS

In the following, the achieved conservation equations are applied to a radial turbine and a radial compressor representing a power machine and a work machine, respectively.

6.1. TURBINE

Assuming non-rotational outflow $c_{2\theta} = 0$ at the design point and regardless of parasitic losses, the specific angular momentum is given by:

$$rc_{1\theta} = rc_{1\mathrm{m}} \cot \alpha_1, \tag{21}$$

where the rotor absolute inflow angle α_1 is known, e.g. from conservation of angular momentum in the volute or given by the stator outflow angle. The meridional absolute velocity results from the mass conservation so that Eq.21 can be recasted as follows:

$$rc_{1\theta} = \left(\bar{m} + \tilde{m}\right) \frac{\cot \alpha_1}{\rho 2\pi b}. \tag{22}$$

From Eq.18 and Eq.22 we obtain the turbine work output:

$$\frac{\overline{P_{\mathrm{Sh}}}}{\bar{m}} = -\omega \bar{m} \frac{\cot \alpha_1}{2\pi b \bar{\rho}} \left(1 + \frac{1}{2}\frac{\bar{\rho}}{\hat{\rho}}\frac{\hat{m}^2}{\bar{m}^2}\right). \tag{23}$$

It is to note that $\hat{m}$ is the amplitude (see Fig.1) of the periodic mass flow rate (please see Appendix to reproduce the transformation). Further, we account for the adiabatic case only, knowing the importance of heat transfer consideration for the non-adiabatic case (see Nakhjiri et al. [12]). We apply the definition of apparent efficiency from Eq.16 to Eq.23 and normalize the equation by the square of turbine speed $r^2\omega^2$ in order to obtain dimensionless expressions:

$$\eta_{\mathrm{app}} \frac{[h_{\mathrm{t}}]_{\mathrm{is}}}{r^2\omega^2} = -\frac{\bar{m}}{2\pi b r^2 \omega \bar{\rho}} \cot \alpha_1 \left(1 + \frac{1}{2}\frac{\bar{\rho}}{\hat{\rho}}\frac{\hat{m}^2}{\bar{m}^2}\right), \tag{24}$$

$$\bar{\lambda} \coloneqq \eta_{\mathrm{app}} \bar{\psi} = -\underbrace{\bar{\varphi} \cot \alpha_1}_{\text{Euler term}} - \underbrace{\frac{1}{2}\frac{\hat{\rho}}{\bar{\rho}}\frac{\hat{\varphi}^2}{\bar{\varphi}} \cot \alpha_1}_{\text{new}}. \tag{25}$$

In the equation above we define the time averaged pressure coefficient:

$$\bar{\psi} \coloneqq \frac{[h_{\mathrm{t}}]_{\mathrm{is}}}{r^2\omega^2}, \tag{26}$$

and the time averaged flow coefficient:

$$\bar{\varphi} \coloneqq \frac{\bar{m}}{2\pi b r^2 \omega \bar{\rho}}. \tag{27}$$

Additionally, the dimensionless periodic part of the mass flow is introduced:

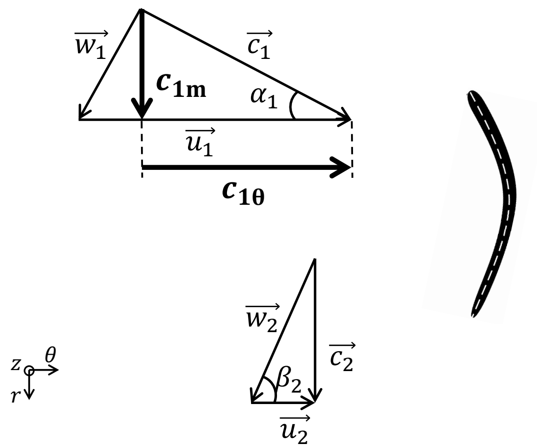

Figure 3: Turbine velocity triangle

$$\hat{\varphi} \coloneqq \frac{\hat{m}}{2\pi b r^2 \omega \hat{\rho}}. \tag{28}$$

Taking rotational outflow into account, Eq.25 can be rewritten:

$$\eta_{\mathrm{app}} \bar{\psi} = \underbrace{\frac{u_2^2}{u_1^2}\left(1 - \bar{\varphi}_2 \cot \beta_2\right) - \bar{\varphi}_1 \cot \alpha_1}_{\text{Euler term}} \underbrace{- \frac{1}{2}\frac{\hat{\rho}_1}{\bar{\rho}_1}\frac{\hat{\varphi}_1^2}{\bar{\varphi}_1} \cot \alpha_1 + \frac{u_2^2}{u_1^2}\frac{1}{2}\frac{\hat{\rho}_2}{\bar{\rho}_2}\frac{\hat{\varphi}_2^2}{\bar{\varphi}_2} \cot \beta_2}_{\text{new}}, \tag{29}$$

where $\bar{\varphi}_1$ and $\bar{\varphi}_2$ are determined according to Eq.27 with values at rotor inlet and outlet. In Eq.25 we recognize the Euler work for steady-state flow in the first term. As a fundamental result of the proposed averaging technique, the second term reflects the influence of the periodic part of the mass flow. Therefore, we obtain higher turbine work output under periodic flow which is in agreement with research works in literature of turbomachinery (see e.g. Wallace and Blair [2]). The contribution of the periodicity in addition to the Euler term can also be seen in Eq.29 for the more general case.

Considering the special case $\eta_{\mathrm{app}} \approx 1$, Fig.4 shows the variation of the theoretical pressure coefficient with the periodicity according to Eq.25 and Eq.29. Further, we assume that the second term of the additional periodic part in Eq.29 referring to rotor outlet is fully damped through the rotor passage. The continuous line refers to the Euler term equivalent to the steady-state case. It can be recorded that the impact of the dimensionless periodic part of the flow rises with decreasing averaged flow quotient. The pressure coefficient tends to infinity when the amplitude of the dimensionless periodic part of mass flow significantly exceeds the flow coefficient in average of time. The general message is that periodic admission leads to enhancement of work output. Further, the special case $\eta_{\mathrm{app}} \approx 1$ allows the conclusion of larger pressure coefficient compared to the Euler term.

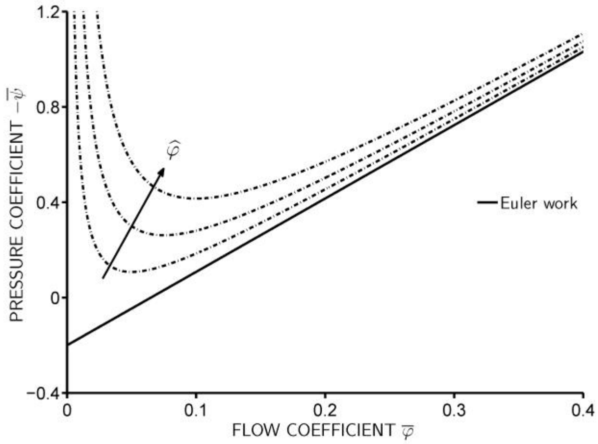

Figure 4: Theoretical pressure coefficient versus flow coefficient according to Eq.29, $\eta_{\text{app}} \approx 1$

As the pressure coefficient is by definition determined by the isentropic work, we obtain from Fig.4 that the ideal turbine work increases in average of time due to periodicity. This is in agreement with the work of Suresh et al. [13] who compare ideal turbine work based on averages of total pressure with instantaneous work.

It should be regarded that we obtain Fig.4 based on the assumption that the amplitude of the periodicity remains constant for the entire operating range while the time averaged mass flow increases when the flow rate is elevated. As the elevation of mass flow is achieved by increasing the engine speed and the excitation frequency, respectively, we can conclude from Fig.4 that the effect of convection is significant at low flow coefficient, i.e. at low frequency. This agrees with the works of Szymko et al. [8] and Abidat et al. [11].

Note that even in case of ideal process the apparent efficiency can vary from unity. However, we assume $\eta_{\text{app}} \approx 1$ based on the consideration that the quotient $\langle \dot{m} h_{\text{t}} \rangle / \overline{\dot{m}}[h_{\text{t}}]$ remains negligible. Conversely, the apparent efficiency can be greater than unity if the quotient $\langle \dot{m} h_{\text{t}} \rangle / \overline{\dot{m}}[h_{\text{t}}]$ attains significant values (negative values for working machines and positive values for power machines). In that case, the enhancement of pressure coefficient due to periodicity would be smaller. Generally, significant values for the quotient are reasonable considering that the periodicity is expected to be damped in the rotor.

6.2. COMPRESSOR

The devised approach can also be applied to the compressor operation. Indeed, pulsating flow is less significant for compressor operation since the unsteadiness is excited downstream of the compressor and mainly damped by the peripheral components in the air path of the engine like the charge air cooler on the one hand and the rotor inertia of the compressor itself on the other hand [15]. However, the respective equations are provided in the following.

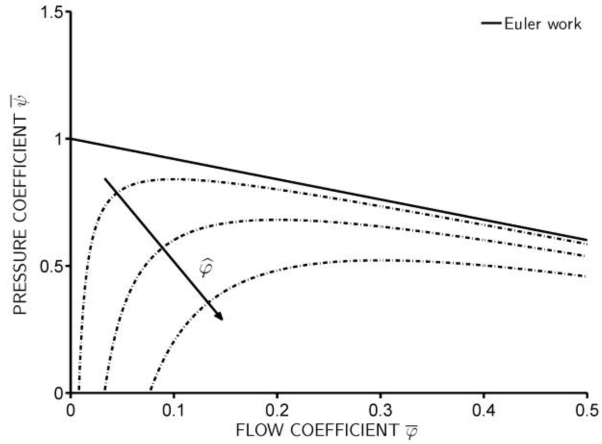

Figure 5: Theoretical pressure coefficient versus flow coefficient according to Eq.30, $\eta_{\text{app}} \approx 1$

Assuming non-rotational inflow and blade congruent outflow we obtain the following equation for the compressor:

$$\bar{\lambda} \coloneqq \eta_{\text{app}}^{-1} \bar{\psi} = 1 - \bar{\varphi} \cot \beta_2 - \frac{1}{2} \frac{\hat{\rho}}{\bar{\rho}} \frac{\hat{\varphi}^2}{\bar{\varphi}} \cot \beta_2 \,, \qquad (30)$$

analogous to turbine in Eq.29. As it can be obtained from Eq.30, the time averaged pressure coefficient deteriorates as a result of periodicity. Hence, periodicity has the inversed effect on the compressor operation which is illustrated in Fig.5.

7. CONCLUSION

In this work, a new averaging technique is presented and applied to the energy conservation equation and Euler turbomachinery equation. For this, the flow quantities are assumed to be additively composed of a mean value and a periodic part. The analysis contributes to fundamental modification of the Euler turbomachinery equation and a new definition of apparent efficiency. The proposed relations allow the mapping of quasi-steady performance providing that the steady-state performance is known.

The calibration procedure on the basis of manufacturers' steady-state performance maps is exposed to considerable uncertainties. In order to better meet the required accuracy in engine calibration tasks, the effect of pulsating flow needs to be taken into account beyond interpolation or extrapolation of steady-state maps.

The presented axiomatic connection between quasi-steady and periodic operation can be integrated in calibration procedures. On the one hand, the approach can be used on the basis of steady-state maps in order to generate modified turbine maps corresponding to different levels of unsteadiness. Different levels of unsteadiness can individually be abstracted from engine application. On the other hand, the approach can be applied to 0D/1D quasi-steady turbine models allowing for periodic flow. We consider energy conservation in average of time from rotor inlet to rotor exit, rather than turbine inlet to turbine exit. This is a reasonable approach under the assumption of quasi-steady rotor operation. This consideration implies higher available energy from the engine exhaust due to the pulse peaks. Thus, a more rigorous overall engine simulation can be provided.

The conditions in the volute can additionally be modeled in a 1D approach based on loss correlations for periodic flow. For example, Dimitrov and Pelz [14] show dependency of the dissipation rate on the Womersley number in internal periodic flows. Further, energy dissipation through the rotor passage under periodic flow admission of the turbine can be calibrated against appropriate test data equivalent to the steady-state case.

Many works introduce instantaneous efficiency as a function of time using instantaneous isentropic work output. Suresh et al. [13] introduce a definition that involves averages of total pressure and record differences up to 10% compared to instantaneous efficiency. However, there is currently no clear and accepted definition of unsteady turbine efficiency as it has been concluded in [3]. A model that can provide mass flow rate and power output for unsteady engine operation will satisfy all demands for engine simulation. The proposed approach in this work can contribute to establishing such a physically based model.

Beyond that, a measure of performance can support designers to distinguish between different turbine designs for a particular application. For this purpose, efficiency as a dimensionless measure of dissipation traditionally provides a simple indicator. The proposed concept of apparent efficiency in this work defines a mapping between steady-state and unsteady performance. The significance of this concept is that the compressor operation at constant load points under unsteady engine conditions can still be reliably predicted by steady-state maps [15].

Respective validation based on measurements should prove the reliability of the proposed approach in future work. For this we apply a new test rig setup in order to avoid the known difficulties and limitations of a standard engine test rig.

APPENDIX A

In the following, Eq.23 is derived step by step. For this purpose, based on Eq.18 and taking into account the specific angular momentum in Eq.22 the following terms can be rewritten for the left side of Eq.18:

$$\bar{\bar{m}}[rc_{1\theta}] = \bar{\bar{m}}^2 \frac{\cot\alpha_1}{2\pi b\bar{\rho}}\,, \qquad \text{(A.1)}$$

$$\langle \dot{m}rc_{1\theta}\rangle = \frac{1}{T}\int_0^T \tilde{\tilde{m}}^2 \frac{\cot\alpha_1}{2b\pi\tilde{\rho}}\,\mathrm{d}t\,. \qquad \text{(A.2)}$$

For simplification we consider a Fourier series expansion of the periodic signals and assume linear convergence. Further, we assume that there is no phase difference between the signals. Then, the integral in the equation above can be simplified as a result of the orthogonality of trigonometric functions. Note that the density is put outside the integral assuming that the periodicity vanishes due to the equation of state $\tilde{\rho} = \tilde{p}/R\tilde{T}$ and the amplitude of the density remains $\hat{\rho} = \hat{p}/R\hat{T}$. Hence, Eq.A.2 can be rewritten as:

$$\langle \dot{m}rc_{1\theta}\rangle = \hat{\hat{m}}^2 \frac{\cot\alpha_1}{4\pi b\hat{\rho}}. \qquad \text{(A.3)}$$

Then Eq.18 can be transformed into the following form:

$$\bar{\bar{m}}[rc_{1\theta}] + \langle \dot{m}rc_{1\theta}\rangle = \left(\bar{\bar{m}}^2 + \frac{1}{2}\frac{\bar{\rho}}{\hat{\rho}}\hat{\hat{m}}^2\right)\frac{\cot\alpha_1}{2\pi b\bar{\rho}}\,. \qquad \text{(A.4)}$$

By putting $\bar{\bar{m}}$ outside the brackets we obtain Eq.23:

$$\frac{\overline{P_{\mathrm{Sh}}}}{\bar{\bar{m}}} = -\omega\bar{\bar{m}}\frac{\cot\alpha_1}{2\pi b\bar{\rho}}\left(1 + \frac{1}{2}\frac{\bar{\rho}}{\hat{\rho}}\frac{\hat{\hat{m}}^2}{\bar{\bar{m}}^2}\right). \qquad \text{(A.5)}$$

REFERENCES

[1] Nakhjiri, M., Pelz, P., Matyschok, B., Horn, A. and Däubler, L., (2011): "Physical Modeling of Automotive Turbocharger Compressor: Analytical Approach and Validation", *SAE COMVEC paper 2011-01-2214*.

[2] Wallace F. J., and Blair, G. P., (1965), "The Pulsating-Flow Performance of Inward Radial-Flow Turbines", *Proceedings of Gas Turbine Conference and Products Show, ASME, Publication 65-GTP-21*.

[3] Baines, N. C., (2010), "Turbocharger Turbine Pulse Flow Performance and Modelling – 25 years on", *9th International Conference on Turbochargers and Turbocharging*, IMechE, London.

[4] Winterbone D. E. and Pearson R. J., (1998), "Turbocharger Turbine Performance under Unsteady Flow – a Review of Experimental Results and Proposed Models", *Turbocharging and Air Management Systems*, IMechE, pp. 193-206.

[5] Karamanis, N., Martinez-Botas, R. F. and Su, C. C., (2001), "Mixed Flow Turbines: Inlet and Exit Flow under Steady and Pulsating Conditions", *ASME Journal of Turbomachinery 123: 359-371*.

[6] Ehrlich, A. D., Lawless, P. B. and Fleeter, S., (1997), "On-Engine Turbocharger Turbine Inlet Flow Characterization", *SAE paper 971565*.

[7] Ehrlich, A. D., (1998), "Characterization of Unsteady on-Engine Turbocharger Turbine Performance", Ph.D. Dissertation, Purdue University, West Lafayette, IN.

[8] Szymko S., Martinez-Botas R. F., and Pullen, K. R., (2005), "Experimental Evaluation of Turbocharger Turbine

Performance under Pulsating Flow Conditions", *Proceedings of ASME Turbo Expo, GT2005-68878*.

[9] Hu, X. and Lawless, P. B., (2001), "Predictions of on-Engine Efficiency for the Radial Turbine of a Pulse Turbocharged Engine", *SAE paper 2001-01-1238*.

[10] Costall A., Szymko S., Martinez-Botas R. F., Filsionger D. and Ninkovic D., (2006), "Assessment of Unsteady Behavior in Turbocharger Turbines", *ASME paper GT2006-90348*.

[21] Abidat M., Hachemi M., Hamidou M. K. and Baines N. C. (1998), "Prediction of Steady and Non-steady Flow Performance of a Highly Loaded Mixed Flow Turbine", *Proc. IMechE, Part A: Journal of Power and Energy, Vol. 212, No. 3, 173-184*.

[12] Nakhjiri M., Pelz, P. F., Matyschok B., Däubler, L. and Horn A., (2012), "Apparent and Real Efficiency of Turbochargers under Influence of Heat Flow", *Isromac-14, Honolulu*.

[13] Suresh A., Hofer, D. and Tangirala V., (2009), "Turbine Efficiency for Unsteady Periodic Flows", *AIAA-2009-504*.

[14] Pelz, P. F., Dimitrov, M., (2011), "Verluste bei periodischen Strömungen – Beschreibung mittels spezifischer Dissipationsleistung, Theorie und Experiment.", *2. VDI-Fachtagung Schwingungsdämpfung, 16. - 17. November 2011, Leonberg*.

[15] Nakhjiri, M., (2012), " Virtueller Turbolader auf Basis Physikalischer Modelle", *unpublished research report, Technische Universität Darmstadt, 2012*.

