A PHYSICAL MODEL FOR THE TIP VORTEX LOSS – EXPERIMENTAL VALIDATION AND SCALING METHOD

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ABSTRACT  
Losses through secondary flows occur in every turbomachine. Between the rotating blades and the casing of a turbomachine there is a secondary flow through the tip clearance caused by the pressure difference between the pressure and the suction side of the blade. This tip leakage flow is not involved in the work done by the rotating blades hence it reduces the aerodynamic efficiency. The flow through the tip clearance rolls up to a spiral vortex on the suction side of the blade and induces drag. Size and circulation of this vortex, according to the Helmholtz vortex theorem, depend on the bound vortex and the width of the tip clearance. Examinations of this structure lead to an idea of describing the tip vortex loss with analytical methods. Therefore an analytical approach is made regarding mainly the circulation at the blade tips.

The method is discussed critically in the context of known loss models. It is shown to be a good summary of earlier methods. Since no explicit geometry data of the turbomachine is needed, it is much easier to use. The most important aspect is the excellent agreement with measurements performed at the Chair of Fluid Systems Technology. In total eleven different fan configurations are measured and analyzed in regard to their tip clearance losses. The measurements are performed at a test rig located at the laboratory of the Chair of Fluid Systems Technology at Technische Universität Darmstadt. Additionally further published measurement data is used to validate the method.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Aspect ratio</td>
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<tr>
<td>C</td>
<td>Machine typical dimensionless constant</td>
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<tr>
<td>D_{nd}</td>
<td>Induced drag</td>
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<td>D_{o}</td>
<td>Outer fan diameter</td>
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<td>L</td>
<td>Lift</td>
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<td>S</td>
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<td>\varepsilon</td>
<td>Glide ratio</td>
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<td>\delta_s^{<em>}, \delta_p^{</em>}</td>
<td>Boundary layer displacement thickness</td>
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<tr>
<td>\zeta</td>
<td>Loss coefficient</td>
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<tr>
<td>\eta</td>
<td>Efficiency</td>
</tr>
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<td>Hub-tip ratio, kinematic viscosity</td>
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\[ \varphi_A \]  
- Design Flow coefficient
\[ \psi \]  
- Pressure coefficient
\[ \psi_A \]  
- Design pressure coefficient
\[ \psi_{gap} \]  
- Tip clearance pressure loss coefficient
\[ \psi_{ideal} \]  
- Ideal pressure coefficient
\[ \psi_r \]  
- Remaining pressure loss coefficient

INTRODUCTION

For any turbomachine (compressor, turbine, fan, pump, etc.) there has to be some clearance between the rotating and the stationary part of the machine. Due to the volume flow through that gap, which is not involved in the work done by the rotating blades on the fluid, this clearance is always associated with energy loss. For machines with narrow spaced blades, which are covered by a ring, the loss can simply be determined by calculating the volume flow through that passage. This simple treatment fails for turbomachines with wide spaced blades like Kaplan turbines, wind turbines, propellers or fans.

In 1914 Betz [1] and later Prandtl [2] pointed out that the induced drag coefficient of an airfoil is a function proportional to the square of the lift coefficient and a constant geometry parameter. Betz [3] then showed the connection between an airfoil and the blade of a Kaplan turbine and specified the induced drag caused by the tip clearance by

\[ D_{ind} = \frac{L^2}{p_d l^2} k \quad \text{or} \quad c_{D_{ind}} = c_l^2 k, \quad (1) \]

in which \( L \) is the lift of the blade, \( p_d \) the dynamic pressure of the incoming flow, \( l \) the blade length and \( k \) a dimensionless constant which indicates the influence of the tip clearance.

Following this idea several tip clearance loss models dealing with the induced drag were developed. In 1951 Ainley and Mathieson [4] published a formula for the tip clearance losses as a part of their prediction method for turbine losses containing the inlet and outlet flow angles, a relative tip clearance and a constant, regarding shrouded \( k = 0.25 \) or unshrouded blades \( k = 0.5 \):

\[ \zeta_k = k \left( \frac{S}{h + S} \right) \left( \frac{C_l}{t/l} \right) \left( \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} \right) \quad (2) \]

This model was improved by Dunham and Came in 1970 [5] on the basis of experimental data, changing the exponent of the relative tip clearance \( / \) chord length to 0.78 and the constant to \( k = 0.47l/h \).

In 1982 a modified model based on Ainley and Mathieson for unshrouded blades was published by Kacker and Okapuu [6] containing the idealized total to total efficiency for zero tip clearance and the variation of the tip clearance.

A relative similar model to Dunham and Came was given by Lakshminarayana [7] in 1970

\[ \zeta_u = \left( 0.7 \frac{c_l^2}{A} \frac{S}{t} \right) \frac{t \left( \cos^2 \alpha_2 \right)}{\left( \cos^3 \alpha_m \right)} \quad (3) \]

where \( A \) denotes the aspect ratio \( h/l \) and 0.7 is an empirically determined value. Additionally the dissipation of energy due to viscous forces in the spanwise flow in the boundary layer is given by

\[ \zeta_w = \frac{\delta_x^* + \delta_p^*}{h/l} \frac{1}{C_p} \frac{c_l^{3/2} (S/t)^{3/2} V_m^3}{V_z V_1^2}. \quad (4) \]

\( \delta_x^* \) and \( \delta_p^* \) denote the boundary layer displacement thicknesses on the suction and pressure surface, \( V_m \) the mean velocity through the cascade, \( V_z \) the axial and \( V_1 \) the inlet velocity.

Another approach is given by Yaras and Sjolander [8], which is based on the models given earlier by Rains or Vavra [9].

\[ \zeta_{tip} = 2k \frac{l S}{C_d t \ h} \cos \alpha_2 \frac{c_l^{3/2}}{\cos^3 \alpha_m}. \quad (5) \]

\( k \) is a constant and \( C_d \) a discharge coefficient. The model is based on the approach that the kinetic energy carried by the normal component of the gap velocity is ultimately lost. It is one of three terms in the sum which result in the tip clearance loss coefficient.

Contrary to the models shown above, Denton [10] defines all losses by the development of entropy. He denotes three general modes of entropy origin, whereas the mixing of the tip clearance flow with the main passage flow is basically responsible for the tip clearance loss. He again distinguishes between shrouded and unshrouded blades. The entropy increased due to tip clearance flow is turned into a loss coefficient for unshrouded blades:

\[ \zeta = \frac{2C_d \ d l}{h t \ \cos \alpha_2} \int_0^l \left( \frac{V_2^2 + V_3^2}{V_2^2} \right) \left( 1 - \frac{V_2}{V_3} \right) \sqrt{(V_2^2 - V_p^2)} \frac{d\sigma}{l}. \quad (6) \]

\( V_p \) and \( V_3 \) are the surface velocities on pressure and suction side of the blade and \( V_2 \) the velocity behind the blade row. \( C_d \) in this case denotes a discharge coefficient, taking values from 0.7 – 0.8.

As a first conclusion of the literature review given above it can be stated that there are different physical approaches to predict the tip clearances losses – inter alia induced drag, losses of kinetic energy and mixing losses.

Of course all physical processes mentioned above have to be dissipative, i.e. being a source of entropy. Denton [10] argues that all loss models dealing with induced drag are
inviscid and hence cannot serve as a source of entropy. There is a need to discuss that thought. Prandtl outlined in his famous paper [11] that the viscosity should not be considered to be zero, but arbitrarily small. That is a difference since without viscous effects there would be no vortex. The magnitude of the viscosity of course does not scale the induced drag associated with the vortex. But nevertheless the vortex vanishes due to dissipation at the end. This argument is in analogy to the well-known Carnot loss. The magnitude of the Carnot loss does not depend on the viscosity and one might think it is an inviscid loss which of course is not true.

To conclude: Similar to physical mechanisms like mixing losses, induced drag models account for loss in efficiency. Indeed, the excellent agreement of the generalized induced drag losses, induced drag models account for loss in efficiency.

Taking up the idea first published by Betz the flow conditions at an airfoil are shown in Figure 1 with the change of the velocity triangles due to the induced drag by the bounded vortex. In the dimensionless form both, the drag and the lift are usually dimensionless with the dynamic pressure. A simple symmetry consideration already suggests the classical result (1): the lift coefficient is an odd function of the angle of attack and the drag coefficient is an even function of the angle of attack. Instead of the relation \( c_{\text{Ind}} \sim c_{\text{L}}^2 \), given first by Betz and Prandtl, the analog relation \( \psi_{\text{gap}} \sim \psi_{\text{ideal}}^2 \) given later by Pelz [12] is used. The relation given by Traupel in [13]

\[
\frac{\Delta c u}{c_{\infty}} = \frac{c_{\text{L}}}{2} \left(1 + e \cot \alpha_{\infty}\right),
\]

(7)

\[
\psi_{\text{ideal}} = \varphi \frac{c_{\text{L}}}{t} \left(1 + \varepsilon \cot \alpha_{\infty}\right),
\]

(8)

With \( c_{\text{L}} = f(\varphi, \psi_{\text{ideal}}) \) the formulas dealing with the lift coefficient given above can also be written as a function of \( \psi_{\text{ideal}} \) where the Euler characteristic of the machine is given by

\[
\psi_{\text{ideal}}(\varphi) = \frac{\psi(\varphi, \text{Re}, s)}{\eta(\varphi, \text{Re}, s)}
\]

(9)

The ideal machine performance can be determined by the relation:

\[
y_{\text{ideal}} = \frac{P}{m},
\]

(10)

with the shaft power \( P = 2\pi n M \), the mass flow rate \( m = \dot{V} \rho \) and the isentropic change in the specific work \( y = \Delta p/\rho \). Expanding the equation with \( 1/(2nD_o)^2 \) and \( \Delta p_1 \) leads to

\[
\frac{2\Delta p_{\text{ideal}}}{\rho(n\pi D_o)^2} = \frac{2Mn\pi}{V\Delta p_1} \frac{2\Delta p_1}{\rho(n\pi D_o)^2}.
\]

(11)

By definition, the first fraction is the ideal pressure coefficient

\[
\psi_1 := \frac{2\Delta p_{\text{ideal}}}{\rho(n\pi D_o)^2},
\]

(12)

the second fraction the inverse efficiency

\[
\eta := \frac{V\Delta p_1}{2Mn\pi},
\]

(13)

and the pressure coefficient \( \psi \)

\[
\psi := \frac{2\Delta p_1}{\rho(n\pi D_o)^2},
\]

(14)

which results in Equation (9). Defining the flow coefficient \( \varphi \)

\[
\varphi := \frac{4\dot{V}}{n\pi^2 D_o^3},
\]

(15)

If the relative tip clearance is denoted by \( s = S/D_o \) the approach proposed here reads as

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Figure 1: Flow Conditions at an Airfoil (non-rotational inlet flow \( \alpha = 90^\circ \))
The difference between the configurations is the hub-tip ratio which is varied between $\nu = 0.45$ and $\nu = 0.56$. The difference between the two hub-tip ratios is a result of the used rotational speeds during the measurements.

EXPERIMENTAL SETUP

The experiments are performed in a fan test rig according to ISO 5136 [14] located at the laboratory of the Chair of Fluid Systems Technology at Technische Universität Darmstadt. This standard defines the required setup to measure and compare the aerodynamic characteristics and emitted acoustic power of fans and other turbomachines. The measurements are performed with two different fan configurations. Both fans have nine skewed blades, 13 guide vanes and an outer fan diameter of $D_o = 0.63$ m. The differences between the configurations are the hub-tip ratio which is varied between $\nu = 0.45$ and $\nu = 0.56$ and the used blade design. Figure 2 shows the fan configuration with $\nu = 0.45$, the arrangement is representative for both configurations. The fan assembly is partitioned into rings. The ring around the rotor can easily be replaced by rings with different inner diameters to vary the tip clearance. The inner diameter of the flow path segments upstream and downstream of the rotor is designed in a way that even with the ring used for the largest tip clearance there is always a reduction in the cross-sectional area. This always leads to an accelerated flow, where round edges at the occurring steps additionally avoid any flow separation.

The tip clearances are $s = S/D_o = 0.1\%$, 0.2\%, 0.3\%, 0.5\% and 0.8\% of the outer fan diameter. The variation of the stagger angle ($\Delta \beta = 0^\circ, 6^\circ, 12^\circ, -6^\circ, -12^\circ$) for both hub-tip ratios and additionally $\Delta \beta = -18^\circ$ only for $\nu = 0.45$ was realized with different sets of blades for each configuration. Each set is trimmed separately in its mounted position in order to get a constant gap over the chord length. During the experiments the rotational speed is held constant to achieve a constant Reynolds number. The volume flow rate $\dot{V}$ is determined with a calibrated inlet nozzle and varied by a throttle on the pressure side end of the test rig by altering the counter pressure.

The total pressure rise over the fan stage is determined with the dynamic pressure by the known volume flow rate with negligible swirl and two planes of static pressure taps as shown in Figure 2. To determine the aerodynamic efficiencies of the fan an input power has to be obtained. Therefore a flying mounted torque measuring flange is installed between the driveshaft and the fan. Due to the direct installation at the rotor the torque $M$ is measured without any bearing friction torque. Because of the small surface, disc friction torque is negligible compared to the aerodynamic torque. The rotational speed is kept constant with a frequency inverter directly connected to the engine. All details of the experimental setup can be found in [15].

The maximal uncertainty in measurement of the pressure coefficient is $U(\psi_{\text{max}}) = 0.0023$ for $\nu = 0.45$ and $U(\psi_{\text{max}}) = 0.0045$ for $\nu = 0.56$. For the tip clearance times the ideal pressure coefficient, containing the measured torque the maximal uncertainty is $U(s\psi_{\text{ideal}})_{\text{max}} = 0.0172$ for $\nu = 0.45$ and $U(s\psi_{\text{ideal}})_{\text{max}} = 0.0225$ for $\nu = 0.56$. The difference between the two hub-tip ratios is a result of the used rotational speeds during the measurements.

RESULTS

In order to gain information about the influence of the tip clearance on the aerodynamic characteristics of the fan, five different tip clearances for each stagger angle are measured. Representative for all measured configurations, the most important aerodynamic parameters are shown in Figure 3. The influence of the tip clearance on the pressure coefficient and the stable operating range can clearly be seen by the shown characteristics - decreasing pressure coefficients and stable operating ranges for increasing tip clearance.

The characteristic of the ideal pressure coefficient for a fan configuration is calculated with Equation (9) for all measured operating points at the five different tip clearances. Assuming that due to the very small geometry changes of the casing the characteristic of the ideal pressure coefficient is the same for all measured tip clearances, it is determined with an interpolation of all points at all tip clearances. The measured data is analyzed at three different operating points for every fan configuration – the best efficiency point of the smallest tip clearance and two operating points $\pm 15\%$ into part- and overload. Hereby the

Figure 2: Fan Assembly

$$\psi_{\text{gap}} = f(s)\psi_{\text{ideal}}^2,$$  \hspace{1cm} (16)

with $f(0) = 0$ and $f(\infty) = \text{const.}$ and assuming only small relative tip clearances $s \ll 1$. A Taylor series expansion leads to

$$\psi_{\text{gap}} \approx Cs\psi_{\text{ideal}}^2$$  \hspace{1cm} (17)

where $C$ is a machine typical dimensionless constant. Instead of dealing with the exact flow conditions and geometry parameters as many of the mentioned methods this method is more practical in comparison and needs only the measured values for $\psi$ and $\eta$.

Equation (17) will be validated in the following and serves as the basis for a scaling method.
analyzed operating points are the same for all five tip clearances. The 15% are related to the common operating range of all tip clearance configurations. In order to quantify the losses in pressure coefficient caused by the tip clearance, an ideal tip clearance of $s_s/D_o = 0$ is extrapolated out of the measured values as shown in Figure 3. The difference between the ideal pressure coefficient and the value for the extrapolated zero tip clearance are the remaining losses $\psi_{\text{gap}}$ and the tip clearance losses $\psi_{\text{ideal}}$ are then the difference to the measured value with the correlating tip clearance.

$$\psi = \psi_{\text{ideal}} - (\psi_{\text{gap}} + \psi_s). \quad (18)$$

The results of the developed formula for three stagger angles ($\Delta \beta_s = -12^\circ, 0^\circ$ and $12^\circ$) and a hub-tip ratio $\nu = 0.56$ are shown in Figure 4. The measured tip clearance losses related to the extrapolated zero tip clearance value are plotted, motivated by the presented approach (16), versus the square of the ideal pressure coefficient multiplied with the relative tip clearance in percent. The measured tip clearance loss is plotted with a different marker for each operating point. The best fit line for each stagger angle is interpolated out of the measured data at the three operating points. The treatment of the measured points gathers them into a linear characteristic and therefore shows a very good agreement with the proposed approach. The values of $C$ for different stagger angles vary. The values of the constant are shown in Figure 4.

For the analysis of the stagger angle $\Delta \beta_s = -6^\circ, 6^\circ$ and $\nu = 0.56$ is shown in Figure 5. For $\Delta \beta_s = -6^\circ$ the measured and transformed values again show a very good agreement with the proposed approach. For $\Delta \beta_s = 6^\circ$ some values do not fit into the linear and show a more quadratic behavior. In Figure 6, results are plotted for $\Delta \beta_s = -12^\circ, 0^\circ$ and $12^\circ$ with a hub-tip ratio of $\nu = 0.45$. It can easily be seen that the tip clearance loss on the ordinate is smaller for the changed hub-tip ratio and therefore also the range of $s s_{\text{ideal}}^2$ on the abscissa also decreases. While regarding the same measured tip clearances, this effect depends only on the decreasing pressure coefficients. In this case $\Delta \beta_s = 0^\circ$ and $12^\circ$ again fit very well to the interpolated straight line. For $\Delta \beta_s = -12^\circ$ the prediction would fit for each operating point itself but the prediction for the tip clearance losses to be related to one dimensionless machine typical constant does not fit very well in this case.
The same behavior can be seen for $\Delta \beta_s = -18^\circ$, $-6^\circ$ and $6^\circ$ in Figure 7, again a prediction for one operating point would work, but the influence of the operating point cannot be predicted very well. $\Delta \beta_s = -6^\circ$ fits while $\Delta \beta_s = 6^\circ$ shows like $\Delta \beta_s = 0^\circ$ that the tip clearance loss is decreasing between $S/D_o = 0.1\%$ and $S/D_o = 0.2\%$. Several studies e.g. MacDougall et al. [16] or Freeman [18] also found that not mandatory the smallest or no tip clearance must have the best performance. Cumpsty [17] found with visualizations of the flow field in the tip region that with no tip clearance there is a major three dimensional separation in the end wall suction surface corner. The tip leakage flow for small tip clearances seemed to reduce this separation and had a positive effect. The smaller loss in the pressure coefficient for the relative tip clearance $S/D_o = 0.2\%$ could be led back to this phenomenon. The measurements in [15] also taken as a basis of this study, showed the best efficiency for $s = 0.2\%$ for $\Delta \beta_s = -6^\circ$, $6^\circ$ and parts of $0^\circ$ for $\nu = 0.45$.

Regarding these stagger angles it is obvious that the physical model for the tip clearance loss predicts the tendency of the losses but is not able to predict the bend to smaller losses for larger tip clearances. Comparing the determined dimensionless machine typical constants, they show a trend towards a higher $C$ and therefore higher tip clearance losses for decreasing stagger angles. This can be led back to the higher blade load for smaller stagger angles. Regarding the value of $C$ of the changed hub-tip ratio for each stagger angle remains in the same order of magnitude.

The behavior of this method for operating points in far off-design is shown in Figure 8 for a stagger angle $\Delta \beta_s = -6^\circ$, and a hub-tip ratio $\nu = 0.56$. A range of 90% of the stable operating range is covered, while $\varphi_{opt} - 30\%$ is very close to the last stable point in part load and $\varphi_{opt} + 60\%$ at high overload. The prediction works well for operating points with large tip clearance losses related on the uncertainty in measurement and fails for high overload, where the tip clearance losses are small. For crossing characteristics the extrapolation of a value for zero tip clearance fails and therefore the prediction gets wrong, which can be seen in Figure 8 for $\varphi_{opt} + 60\%$.

To broaden the experimental validation of the physically based approach, the method was also applied to data gained in other labs. In Figure 9 and Figure 10 the analysis of measurements out of a study by Brodersen [19] is plotted. Brodersen studied eight different blade configurations with a hub tip ratio $\nu = 0.55$ each with three different tip clearances $s = S/D_o = 0.1\%, 0.25\%, 0.5\%$. The plots show that the model works very well for one operating point but does not fit for the whole operating range. This can be led back to the fact that the measured influence of tip clearance in [19] is nearly constant over the complete operating range and is not growing.

Figure 7: Tip Clearance Loss for $\Delta \beta_s = -18^\circ$, $-6^\circ$ and $6^\circ$

Figure 8: Far Off-Design Application for $\Delta \beta_s = -6^\circ$

Figure 9: Tip Clearance Loss (data by Brodersen [19])

Figure 10: Tip Clearance Loss (data by Brodersen [19])
with increasing total pressure rise as expected. Regarding the analysis of five different types of blades only for the design point as shown in Figure 9 it can be seen that the model fits very well. In his study Brodersen also compared three different prediction methods for the tip clearance loss. The results are shown in Figure 11, the numbers on the left are the identifiers for the studied types of blades. The compared methods from Lakshminarayana [7] with constant $k = 3$, Eckert/Schnell [20] and Pfleiderer/Petermann [21] with the constant $k = 1.8$ are shown with the new approach (17) given here. The difference in efficiency with the model in this study was gained under the assumption of a constant ideal pressure coefficient. The typical machine constant $C$ was gained by an analysis of the two smaller tip clearances. The theoretical efficiency $\eta_{\text{theor}}$ was then calculated with the tip clearance pressure loss coefficient $\psi_{\text{gap}} = C_s \psi_{\text{ideal}}^2$. Considering the necessity of two measurements for determining the machine typical constant the procedure is rather a scaling method for the tip clearance loss than a prediction, which is shown in detail in the next section. Nevertheless the physical and easy to use model shows good results especially for the last 3 types of blades, which are blades for higher loads.

**APPLICATION OF THE METHOD FOR SCALING PURPOSES**

A possible application of the physical model is the prediction of efficiencies by means of a developed scaling method as shown by Pelz and Karstadt [22]. From Equation (9) with $\psi = \psi_{\text{ideal}} \cdot (\psi_{\text{gap}} + \psi_r)$, where the remaining losses besides the tip clearance losses are denoted by $\psi_r$. Hence the efficiency of a turbomachine can be calculated by

$$\eta = 1 - \frac{\psi_{\text{gap}} + \psi_r}{\psi_{\text{ideal}}}.$$  (19)

Introducing the new approach (16) this results in

$$\eta = 1 - C S \psi_{\text{ideal}}(\varphi) - \frac{\psi_r(\varphi, Re)}{\psi_{\text{ideal}}(\varphi)}.$$  (20)

Equation (20) gives the basis for a scaling method. Assuming that for $\varphi = \text{const.}$ and $Re = \text{const.}$ the efficiencies for two different relative tip clearances $s_1$ and $s_2$ are measured:

$$\eta_1 = 1 - C S \psi_{\text{ideal}} - \frac{\psi_r}{\psi_{\text{ideal}}},$$  (21)

$$\eta_2 = 1 - C S \psi_{\text{ideal}} - \frac{\psi_r(\varphi, Re)}{\psi_{\text{ideal}}}.$$  (22)

Solving this system of equations, the two unknowns $C$ and $\psi_r$ can be determined:

$$C = \frac{1}{\psi_{\text{ideal}}} \left( \frac{\eta_1 - \eta_2}{s_2 - s_1} \right),$$  (23)
\[ \psi_r(\varphi, Re) = \psi_{\text{ideal}} \left( 1 - \eta_l + s_1 \frac{\eta_2 - \eta_1}{s_2 - s_1} \right). \]

Hence the constant \( C \) and the function \( \psi_r(\varphi, Re) \) are known. If the new approach (Equation (17)) is correct, \( C \) should be constant, in particular independent of \( \varphi \) and \( Re \). Consequently the efficiency of a prototype can be calculated with Equation (20).

Six operating points (\( \varphi = 0.145, 0.17, 0.195, 0.22, 0.245, 0.26 \)) were regarded for the scaling. These operating points are equidistantly distributed over the complete operating range of this axial fan. For each operating point a separate scaling was performed. The results of the scaling formula with a variable \( C \) are shown in Figure 12. The unknown \( C \) and \( \psi_r \) are determined with the measured values for the relative tip clearances \( s = 0.1\% \) and \( s = 0.8\% \) for each operating point. It is therefore obvious that the measured and the calculated efficiency must fit for these points. Regarding the other tip clearances it can be seen that the calculation mostly fits very well.

To show that the machine typical constant \( C \) is independent of the flow coefficient \( \varphi \), the scaling is also performed with a constant \( C \) for all operating points. The calculated results in Figure 13 show good agreement with the measured results. The values for the relative tip clearances \( s = 0.1\% \) and \( s = 0.8\% \) do not fit exact anymore, because of the average value of the machine typical constant \( C \).

**CONCLUSION**

In this study a physically based model for the tip clearance loss, based on the circulation around the blades, was presented and validated for several different types of fans. It was developed on the basis of earlier studies taking the induced drag with explicit blade geometry into account. The model is very easy to use as you do not need explicit geometry data of the used blades. As acceptance tests are an essential part of any new developed blade design the influence of the tip clearance can then easily be estimated. A measurement with two different tip clearances is enough to extrapolate the losses for larger tip clearances. The approach shows good agreements with measurements made at the Chair of Fluid Systems Technology in Darmstadt. The model is able to predict the tip clearance losses even in off-design. This works excellent for part-load and light overload but fails for high overload as the tip clearance losses become too small. For some stagger angles the used model is not able to predict the influence of the operating point. For measurements with a different hub tip ratio performed at Technische Universität Braunschweig, the physically based model only fits for constant operating points. To show one possible application, a scaling method developed out of the physical model is presented. The results show a good agreement with the measurements for most operating points and tip clearances. The aim for the future is the validation with further experimental data, especially other types of turbomachinery.

**REFERENCES**


