Technical Paper
(max. 8-10 pages including abstract)

Session Name: (to be defined later and completed by us)

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Designing Pump Systems by Discrete Mathematical Topology Optimization: The Artificial Fluid Systems Designer (AFSD)

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Summary

The development of sophisticated pump systems is a hard challenge in the rotating equipment industry. There are many high-quality software tools which aid designers to simulate, validate and engineer their systems. However, the right design decisions in early stages of development are still a matter of human intuition.

In this paper we present a system approach, which may serve the system designer or the pump manufacturer as an ***artificial fluid system designer****.* Its purpose is to automatically find the optimal pump system design for a prospected field of application. The artificial, or algorithmic, system designer is comparable to a chess player in the sense that we use solution techniques similar to well-proven chess-playing algorithms to solve a special quantified mixed-integer linear optimization problem. Our goal is to support the human designer right from the beginning of his development efforts by providing him with forsighted design ideas.

Introduction

When a rotating equipment company or a plant designer accepts an order for a custom pump system like a booster station and has no appropriate solution in stock, a new pump system must be developed from scratch, satisfying the customer's needs. In a first step, customer and designer have to agree on a technical specification which is a fair balance of function, costs and availability (cf. Fig. 1) of the prospected system. The designer then has to combine available components (pumps, pipes, valves and so on) to an initial system design based on experience and creativity. We call this phenomenon the system design miracle. Afterwards, he implements a physical model and enhances the design until it meets the requirements.

In the following, we sketch a new approach on how to overcome the creative human process with the help of a mathematically and algorithmically driven artificial system design process. The main idea is to model the set of all possible designs and their physical interrelations with a rough, phenomenological or physical model, which can be optimized with the help of mathematical software.

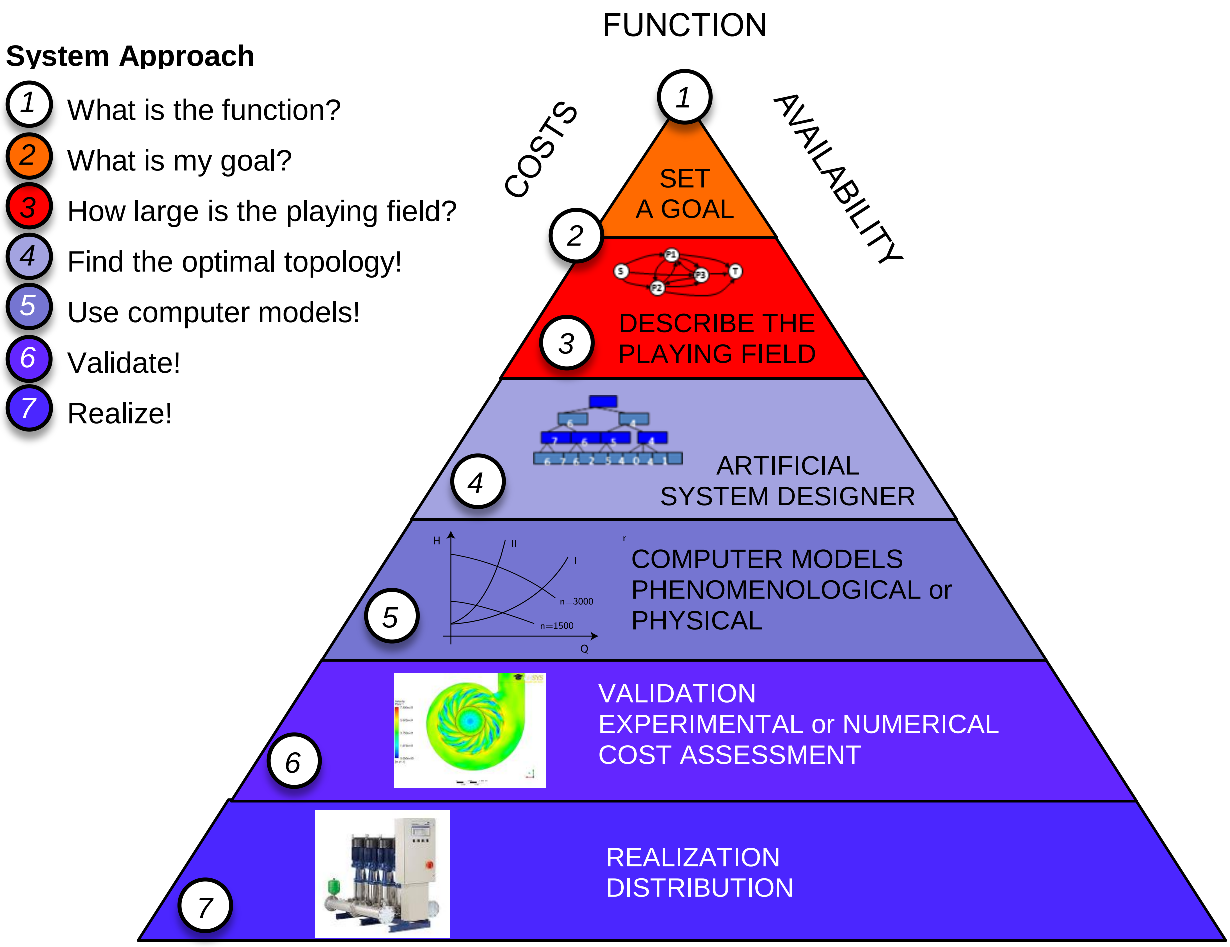

Figure 1: The fluid system development procedure illustrated by a Pelz/Lorenz pyramid

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Figure 1 illustrates the system design process as proposed by the authors. The designer together with the customer still has to balance his desired system function (1) with limiting factors such as costs or availability, but now he has to formally describe his goal (2). Then, he models a "playing field" by describing the variety of components he would traditionally have chosen from himself (3). The actual step of designing the initial topology is now done by an ***artificial system designer*** based on discrete mathematical topology optimization (4). The following steps of simulation, validation and construction of the system (5-7) remain as before.

Note that an ***optimization of the system topology*** is different from a ***parameter optimization*** of individual components or control parameters. While parameter optimization is already in widespread use (e.g. as metaheuristics in the form of genetic algorithms or simulated annealing) to optimize rotary speeds or valve adjustments, it is only a tool to *improve* an existing system design. It relates to step 5 of the pyramid (cf. Fig. 1) and cannot miraculously cure a bad initial topology. In contrast, an artificial system designer begins one step earlier and finds the best topology in view of the designer's goal. Since both approaches enter the development process at different times, they are *not* mutually exclusive, but can complement each other.

Problem Statement

To implement an algorithmic system design process, two questions have to be answered beforehand: "Which kind of components can be brought together in order to form a valid system and how are the physical interconnections between the components defined?" and "When is one system *better* for a specific purpose than another system?"

The first question can be answered quite generally: If a fluid can enter at any component and come out somewhere else without violating the laws of physics, this combination will be considered a reasonable pump system. We assume that all kinds of components are known in advance, i.e. we can construct the desired system out of standard components. This given pump construction kit consists of speed-fixed or speed-controlled pumps, for which all characteristic curves $H(Q)$ and $P(Q)$ or respectively $H(Q,n)$ and $P(Q,n)$ are known. To simplify the model, we assume pipes and valves interconnecting the pumps to have no properties except their price.
The answer on the second question strongly depends on the individual goal of the human designer. In this paper, we balance purchase costs for the components and energy costs over a given period of time. *Our aim is to minimize the purchase costs plus the expected energy costs*.

Finally, the desired load-collective has to be incorporated. We assume that the transition times and transition costs between such load-changes are negligible, compared to the total costs. Thus, we inspect a quasi-stationary model. In each situation, the pump system has to satisfy the demanded load, i.e., it has to generate a specific pressure head difference at a given volume flow rate. One specific load of the load collective is called a *scenario*, and we assign probabilities to the load scenarios, which depict the fraction of the pump system's life time they are prospected to occur.

Model Formulation

The problem can be stated as follows: Given a construction kit of rotary pumps and a technical specification of load collectives, compare all possible pump systems which satisfy every load and choose the one with the lowest expected life cycle costs. In a first step, an interconnection of pumps can be abstracted as a (mathematical) graph $G = (V,E)$ with vertices V being pumps and edges E being pipes.

A pump system has two additional vertices, namely a water supply and a water outlet, with corresponding pipes. Therefore, all possible topologies of our pump system can be modelled as a complete graph of the pump construction kit and two water plugs. A (binary) decision variable for each pump and pipe will indicate if the component needs to be purchased or not.
For each possible pump system, i.e. for each subgraph defined by a purchase decision of pumps and pipes, the technical specification implies a set of scenarios: The pump system must be able to satisfy every prospected load, which is given by a volume flow and the pressure head difference. The task now is to find a specific subgraph of the complete graph that describes a well defined pump system for which the purchase costs plus the expected energy costs over the given load scenarios are minimal.

We formulate the problem with the help of linear constraints, resulting in so called deterministic equivalent programs [1], which form large mixed-integer linear programs (MIP, cf. [5]) with a special block-structured coefficient matrix. With the help of this modeling formalism, we describe the possible topologies and their physical properties as a collection of linear constraints (equations or inequalities) and estimate the life cycle costs with a linear objective function, depending on different scenarios that may occur.

The mathematical description is divided into variables, parameters, constraints and objective function. Variables are that part of our mathematical program which will be filled by an algorithm. After the optimization process, the variables will present us the best possible solution. Parameters are input data. Constraints describe what defines a valid pump system and the objective function discriminates good from bad pump systems.

The following table presents the set of variables. Let *R* be the set of available rotary pumps and *J* the set of two plugs, i.e. water source and water outlet. Let $V = R \cup J$ be the set of nodes of the graph and $E = V \times V$ the set of edges. In the table, indices specify vertices or edges of the topology graph $G = (V, E)$. The type abbreviations stand for 'binary' ($x \in \{0,1\}$), 'continuous' ($x \in [l,r]$, l and r being some rational numbers) and 'semi-continuous' ($x \in [l,r] \cup \{0\}$).

variable	type	description
y_r	B	y_r= 1 if and only if pump r shall be purchased
$y_{i,j}$	B	purchase decision for pipe $e = (i, j)$
x_r^s	B	x_r=1 if and only if pump *r* is active in scenario *s*
$x_{i,j}^s$	B	operating decision for pipe *e=(i,j)* in scenario *s*, i.e. a valve is open if x_{ij}=1 and closed if x_{ij} = 0.
n_r^s	SC	rotational target speed of pump *r* in a certain scenario s
q_r^s	SC	target volume flow through pump *r* in a certain scenario s
h_r^s	SC	target pumping head of pump *r* in a certain scenario s
p_r^s	SC	target power consumption of pump *r* in a certain scenario s
$q_{i,j}^s$	SC	target volume flow through pipe *e=(i,j)* in a certain scenario
$h_{*,r}^s$	SC	target pressure head before pump *r* in a certain scenario
$h_{r,*}^s$	SC	target pressure head after pump *r* in a certain scenario

The problem input is taken in form of the following parameters:

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parameter	description
C_r^{pump}	purchase cost of each pump r, given for each available pump
C^{pipe}	purchase cost of a pipe, given for all pipes
$C^{\text{kW}\cdot\text{h}}$	price per kilowatt hour
W_s	probability (weight) of scenario s
Q_s	demanded volume flow in scenario s
H_s^{source}	incoming pressure head in scenario s
H_s^{outlet}	demanded outgoing pressure head in scenario s
$Q^{\min}, Q^{\max}$	minimal and maximal volume flow
$H^{\min}, H^{\max}$	minimal and maximal pressure head

Given these variables and parameters, we can express the investment costs as the sum over all purchased pumps and pipes:

$$c_{\text{investment}}(y) = \sum_{r \in R} C_r^{\text{pump}} \cdot y_r + \sum_{(i,j) \in E} C^{\text{pipe}} \cdot y_{i,j}$$

The energy cost for each scenario (supposed it would last the whole life time T) is:

$$c_{\text{energy}}^s(p) = C^{\text{kW}\cdot\text{h}} \cdot \sum_{r \in R} p_r^s \cdot T$$

Finally, the life cycle costs comprise the investment costs and the expected energy costs:

$$c_{\text{total}}(y,p) = c_{\text{investment}}(y) + \sum_{s \in S} W_s \cdot c_{\text{energy}}^s(p)$$

This objective function results in a pump being purchased either if it is necessary to fulfill the specification or if the energy savings due to operational flexibility justify its redundancy.

The physical properties of the pump system are provided by the following list of constraints:

	purchase decision
$y_{i,j} + y_{j,i} \leq 1$	Between each pair of two components (pumps or plugs) there is at most one pipe.
$x_r^s \leq y_r$	A pump can only operate if it was purchased.
$x_{i,j}^s \leq y_{i,j}$	A pipe can only operate if it was purchased.
$\sum_{(i,j) \in E} x_{i,j}^s \geq 1$	In each scenario at least one pipe must be operational.
	operational bounds

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$q_r^s \geq Q^{\min} \cdot x_r^s$ $h_r^s \geq H^{\min} \cdot x_r^s$ $h_{*,r}^s \geq H^{\min} \cdot x_r^s$ $h_{r,*}^s \geq H^{\min} \cdot x_r^s$	$q_r^s \leq Q^{\max} \cdot x_r^s$ $h_r^s \leq H^{\max} \cdot x_r^s$ $h_{*,r}^s \leq H^{\max} \cdot x_r^s$ $h_{r,*}^s \leq H^{\max} \cdot x_r^s$	If a pump is operational, its volume flow, pumping head and adjacent pressure heads are reasonable. Otherwise they vanish.
$q_{i,j}^s \geq Q^{\min} \cdot x_{i,j}^s$	$q_{i,j}^s \leq Q^{\max} \cdot x_{i,j}^s$	If a pipe is operational, its volume flow is reasonable. Otherwise it vanishes.
		<u>continuity equation</u>
$Q_s = \sum_{(\text{source},j) \in E} q_{\text{source},j}^s$	$\sum_{(i,\text{outlet}) \in E} q_{i,\text{outlet}}^s = Q_s$	The flow rates at source and outlet are equal.
$q_r^s = \sum_{(r,j) \in E} q_{r,j}^s$	$\sum_{(i,r) \in E} q_{i,r}^s = q_r^s$	The flow rate through every pump is conserved.
		<u>Bernoulli equation</u>
$h_{*,r}^s + h_r^s = h_{r,*}^s$		Pumping head increases pressure head.
$+(H_s^{\text{source}} - H_s^{\text{outlet}}) \leq H^{\max} \cdot (1 - x_{\text{source,outlet}}^s)$ $-(H_s^{\text{source}} - H_s^{\text{outlet}}) \leq H^{\max} \cdot (1 - x_{\text{source,outlet}}^s)$		Water source and water outlet can only be connected, if no pressure head difference is required.
$+(H_s^{\text{source}} - h_{*,j}^s) \leq H^{\max} \cdot (1 - x_{\text{source},j}^s)$ $-(H_s^{\text{source}} - h_{*,j}^s) \leq H^{\max} \cdot (1 - x_{\text{source},j}^s)$ $+(h_{i,*}^s - H_s^{\text{outlet}}) \leq H^{\max} \cdot (1 - x_{i,\text{outlet}}^s)$ $-(h_{i,*}^s - H_s^{\text{outlet}}) \leq H^{\max} \cdot (1 - x_{i,\text{outlet}}^s)$		If a pump is connected with the water source or the water outlet, the pressure head propagates through the pipe.
$+(h_{i,*}^s - h_{*,j}^s) \leq H^{\max} \cdot (1 - x_{\text{source},j}^s)$ $-(h_{i,*}^s - h_{*,j}^s) \leq H^{\max} \cdot (1 - x_{\text{source},j}^s)$		If two pumps are interconnected, the pressure head propagates through the pipe.

Additionally, we have to guarantee that the working point of each pump lies on its characteristic curve. We do this by generating a sufficient number of points from the empirically known *H(Q,n)* and *P(Q,n)* functions and forcing the model variables on the linearized curves defined by these base points. For the purpose of clarity and due to limited space, we do not present these equations here. However, the model formulation is straight-forward and can be looked up in literature [6].

A simple demonstrator example

Table 1 shows a possible load collective for a desired system. Our interpretation of the collective is that e.g. the first scenario S1 occurs with a probability of 44%. As we assume a quasi-static model, such that dynamic switching behaviour of the system between various states is not considered, this is equivalent to the assumption that in 44% of the entire life time, the system will have a demand of 20 m^3/h and an outgoing head of 38.13 m, at an incoming head of 0 m.

Table 1: Example of a technical specification

scenario	probability	volume flow	incoming head	outgoing head
S1	44%	20	0	38.13
S2	35%	40	0	45.75
S3	15%	60	0	53.38
S4	6%	80	0	61.00

Table 2: Example of a construction kit

<u>speed-controlled</u>

1) n = 730-2920 (€4554)			*2) n = 350-1400 (€6080)*			*3) n = 725-2900 (€11362)*		
Q	H	P	Q	H	P	Q	H	P
0	58	2500	0	11	560	0	65	3400
10	58	4000	10	11	680	35	50	7500
26	40	6000	28	9	1250	60	0	5800

<u>speed-fixed</u>

4) n = 2900 (€2864)			*5) n = 2900 (€3550)*			*6) n = 2900 (€5504)*		
Q	H	P	Q	H	P	Q	H	P
0	43	2800	0	65	6000	0	70	5000
30	42	5500	40	64	10500	40	66	12500
70	30	8000	90	56	17000	120	45	23000

Table 2 shows the three-point discretization of the head curves of six examplary pumps, three of which are speed-controlled and three are not. The units are given in $[Q] = \mathrm{m}^3/\mathrm{h}$, $[H] = \mathrm{m}$, $[P] = \mathrm{W}$ and $[n] = \mathrm{rpm}$. For the speed-controlled pumps, the base points of the characteristic curve are given for the largest rotation speed and must be scaled down with affinity laws, i.e. according to $Q \sim n$, $H \sim n^2$ and $P \sim n^3$. We further assume a life time of 5 years (of uninterrupted work) with an average energy cost of 4 ct/kWh. Therefore, purchase and average energy costs over the life time are of the same dimension and neither is negligible for the optimization.

The result (i.e. the output of the optimization process) is easily understandable with the help of an image. Fig. 2 is separated into four sections for the four scenarios and a statistical summary at the bottom. In each scenario, the small rectangles filled with text represent pumps or plugs, and lines represent pipes. Black pumps and pipes are operational in this scenario, whereas grey pumps are off and grey pipes are unused (both can be interpreted as closed valves). Note that in all scenarios, the same pumps and pipes are shown, since the investment is a one-time decision the model has to make with knowledge of the scenario possibilities. But then, the purchased components may be activated or deactivated depending on the individual scenario. Each scenario reports the energy costs for the imaginary case that this scenario would occur with 100% probability, that is non-stop for the life time of 5 years. The resulting mean value (the expected energy costs), the investment costs and the expected life cycle costs are shown at the bottom of the figure. Note that one speed-controlled pump operates with different speeds in all scenarios.

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In principle, the presented mathematical model can serve as input for a mixed integer program solver, as e.g. CPLEX [7] or Gurobi [8], which will find a solution. With the help of the branch and cut algorithm [5] these solvers are able to provide an optimal or near-optimal solution by analyzing only a small fraction of all possible purchase decisions.

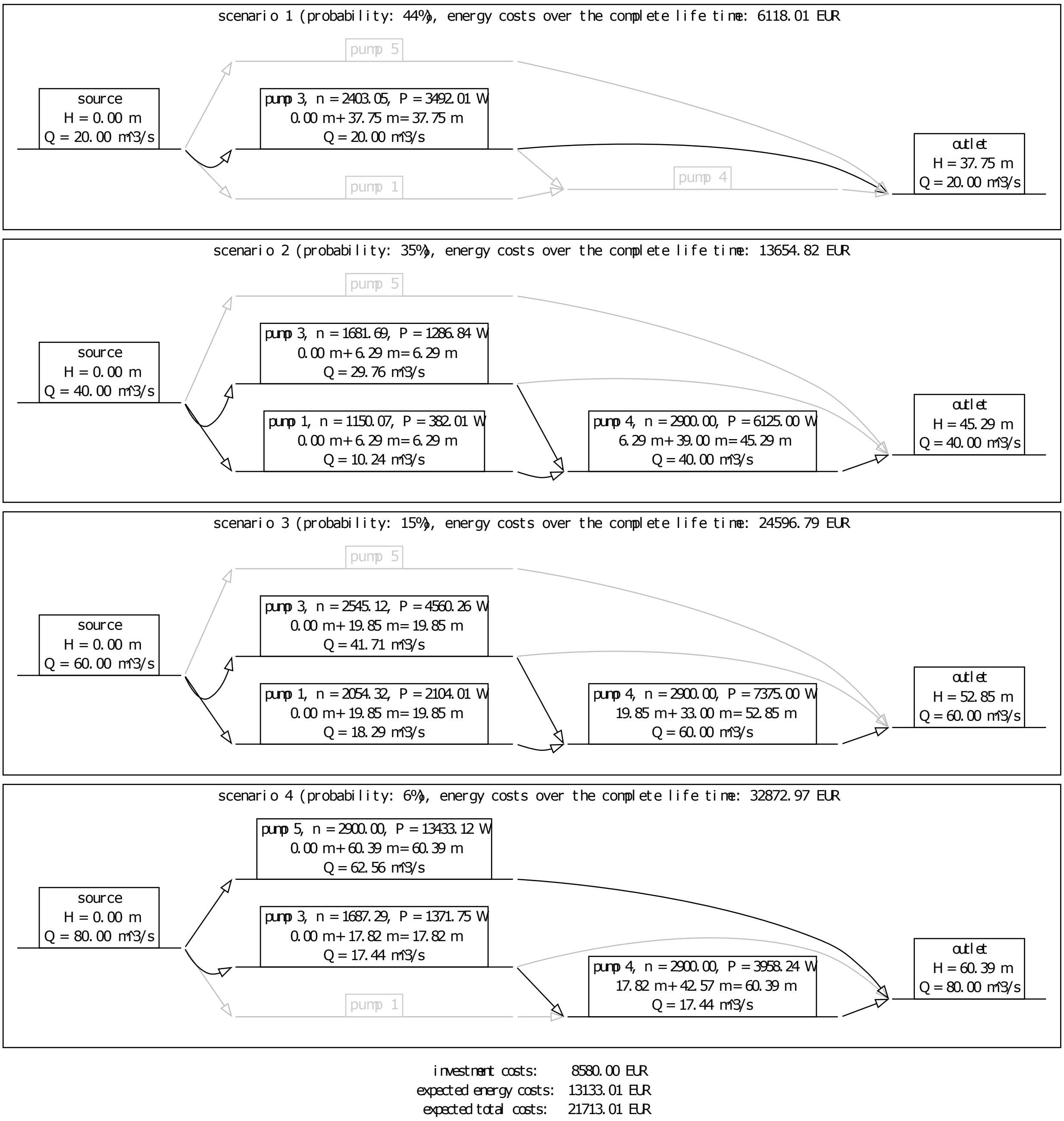

Figure 2: Example output of the optimization model for a typical HVAC load profile

Our algorithmic research interest is devoted to so called quantified linear programs [4] which allow us to encode the input instances without replication of variables for each scenario. It can be extracted out of the given model from above by only a few minor changes to the optimization model. This smaller coding allows automatic decomposition techniques as known from game tree search [2] and Benders decomposition [3].

The main idea behind a decomposition is to test various first stage decisions, i.e. purchase decisions in this case, and to generate upper and lower bounds on objective values such that the effort to find an optimal solution and to prove its optimality is kept as small as possible. The procedure reminds of a chess player who examines a given chess position thinking about his opportunities, considering the possible moves of his opponent, under consideration of his own options thereafter, and so on.

Summary and Outlook

In the past decades, more and more software tools have emerged to aid humans in developing pump systems. However, the initial topology design is still determined without software assistance, even though there seems to be the highest potential for cost savings in the early development stages. We have presented a novel approach to this problem based on discrete mathematical topology optimization. Based on a construction kit and a technical specification, it is possible to design a pump system from scratch. Moreover, our algorithms construct the verifiably most affordable of all suitable systems regarding its life cycle costs. Of course, the quality of the artificially designed pump system still depends on the cooperation between a customer and the human designer, because they have to provide a suitable construction kit and anticipate authentic load collectives.

We are going to continue our research in order to find and exceed the limits of our approach. It has to be found out how complex the systems can become such that optimal solutions can be algorithmically extracted in reasonable time. Moreover, it will be important to examine how far the extracted solutions fit to practice. It is our aim to physically build proposed systems in order to find out how well the optimization model and reality match. If there are deficiencies, the model must be extended and modified correspondingly.

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