Experimental and theoretical studies of the displacement of a hydrodynamic supported idle spindle of a three-spindle screw pump

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Abstract  
The bending and displacement of a hydrodynamic supported idle screw of a 3-spindle screw pump is measured by a set of inductive sensors. The experimental results are compared with a coupled fluid and structure model. The idle screw is modeled as a Bernoulli beam interacting with a hydrodynamic lubrication film.

Introduction  
Screw pumps are kinematic pulsation free positive displacement pumps. The hydrodynamic supported sidewise idle spindles are driven by engagement of the center drive spindle (Figure 1). In unpropitious operation points of low rotating speed, low viscosity and high pressure difference contact between the idle spindle and the casing may occur. State of the art, is to detect this operation limit experimentally. The aim of the present research is to improve the physical understanding of the problem and to give a theoretical approach of this operation limit.

Figure 1: 3-spindle screw pump, Leistritz Pumpen GmbH
**Experimental setup**

Former experimental studies led to the conclusion that the movement of the idle screw can’t be understood as a rigid body movement. Bending aspects have to be taken into account [3]. Therefore a set of four inductive distance sensors was placed along the idle spindle axis. The axial distance of the sensors is equal to the pitch of the idle screw. The sensor head is embedded in a magnetic inert sealing elastomer plug, which is shaped to the casing contour. The sensor and the plug are hold in position by a brass screw (Figure 2).

![Axial sensor positions](image)

**Figure 2: Axial sensor positions**

For horizontal and vertical displacement measurement the system is applied in two perpendicular measurement planes. (Figure 3).

![Coordinate systems, view from suction side](image)

**Figure 3: Coordinate systems, view from suction side**

Due to the fact that an absolute changeless calibration of the center of the screw to the center of the housing is impossible, the origin of coordinates is defined by an initial point of operation, which has to be reproduced every measurement series. Experience shows that the measurement results are reproducible.
Hydrodynamic model

The pressure and shear load, \( p \) and \( \tau \), on the idle screw is modeled by 2-dimensional lubrication theory, given by Reynolds equation

\[
\nabla \cdot \left( \frac{h^3}{\mu} \nabla p \right) = 6 \nabla \cdot (h U_1 + h U_2) + 12 \frac{dh}{dt} \tag{1}
\]

and shear equation

\[
\tau = \frac{\mu}{h} (U_2 - U_1) - \frac{h}{2} \nabla p. \tag{2}
\]

with gap height \( h \), dynamic viscosity \( \mu \), spindle wall speed \( U_1 \), opposite wall speed \( U_2 \), time \( t \), and surface normal \( n \), leading to the 3-dimensioinal stress vector on the spindle surface

\[
t = -p n + \tau. \tag{3}
\]

Figure 4 shows the pressure deviation of a standard operation point with a dynamic viscosity \( \mu \) of 5 mPas, a rotating speed of 3000 rpm and a pressure difference \( \Delta p \) of 39 bar, on the left side on the 2D-projection of the geometry, on the right side on the spindle surface. Leakage between drive and idle spindle is not taken into account, which is reasonable for having rotor dynamics in focus and not volumetric efficiency of the pump [1] [2] [4].

Figure 4: pressure deviation in standard operation point

We take advantage of the linearity of Reynolds and shear equation in pressure \( p \), shear stress \( \tau \) and various speed terms. The influence of pressure difference, rotating speed and lateral spindle displacement can be handled separately.

The homogenous solution of Reynolds equation can be scaled by pressure difference and suction pressure \( p_0 \).

\[
p_1 = p_1 \Delta p = 1, \varphi, \xi_1, \xi_2 \Delta p + p_0. \tag{4}
\]
Unfortunately it is a nonlinear function of turn angle $\varphi$ and displacement $\xi_1$ and $\xi_2$, due to the non-linear influence of gap height $h$. Figure 5 shows $p_1$ separately.

Figure 5: 1$^{\text{st}}$ fundamental solution, influence of pressure difference

The most important inhomogeneous solution results from angular speed $\Omega$ and viscosity $\mu$.

$$ p_2 = p_2 \mu \Omega = 1, \varphi, \xi_1, \xi_2 \mu \Omega. \quad (5) $$

In Figure 6 the idle spindles are deviated in $\xi_1$-direction. The typical pressure deviation of the journal bearing effect can be seen.

Figure 6: 2$^{\text{nd}}$ fundamental solution, influence of rotating speed
Further fundamental solutions are given by displacement speeds $\xi_1$ and $\xi_2$. Due to our measurement results, transient aspects are less important for the understanding of the deviation problem, so we disclaim presenting them here.

**Figure 7**: Line load diagrams for $\varphi = 0^\circ$ and $\varphi = 90^\circ$ and simplified for bending line model

**Bending line model**

From the simulation results line load functions $q$ can be generated. Figure 7 shows line load diagrams of the idle spindle pressure solution for different turn angles. Although the deviation of the line load is different due to different chamber positions, the resulting radial force in $\xi_2$-direction from the screw geometry section of the spindle is the same. For that component exists an analytical solution from Hamelberg [5] which is approved by 3D-CFD-simulations. Due to geometric inaccuracy the simulated forces from 2D-lubrication theory are a bit too small. The radial force from screw geometry in $\xi_1$-direction is zero in sum. The line load peak in $\xi_1$-direction on the discharge side of the spindle is conditioned from the design of the sealing system (see also Figures 1, 4, 5). The bending line model works with simplified line load functions.

The bending line is given by the Bernoulli beam equation in coordinate direction $i$

$$E_1\xi_i^{\prime\prime\prime} = q_i,$$

and its boundary conditions at both spindle ends

$$E_1\xi_i^{\prime\prime\prime}\big|_{\text{end}} = E_1\xi_i^{\prime\prime\prime}\big|_{\text{start}} = 0.$$  

(7)

The line load is given by pressure difference line load $q_{ip}$ and rotating speed line load $q_{ij}$ representing the journal bearing effect, given.

$$q_i = q_{ip} + q_{ij}(\xi_j, \mu, \Omega).$$

(8)

The problem was solved numerically and iterative.

**Results**

Due to the fact that a changeless calibration is not possible with the measurement system, the displacement is referred to an operation point of 2 bar, 1500 rpm and an oil temperature of 30°C, that leads to a dynamic viscosity of 5 mPas. For three test series, the result is plotted in Figure 8. 8a shows a series that was done with a constant rotating speed of 1500 rpm and various pressures. In the $\xi_{1M}$-direction the idle spindle moves towards the casing wall with rising pressure. In $\xi_{2M}$-direction a cardanic displacement in combination with a strong bending influence can be seen. The simulation results in Figure 8b reproduce the
measurement results qualitatively. Another measurement series was done at 1000 rpm. The behavior is nearly the same. Those measurements were done only till 40 bar with regard to operation limits of the pump within this fluidic hydraulic oil. A third measurement series was done at 30 bar and 1500 rpm. By warming the oil the viscosity declined. Here as well the basic behavior could be reproduced by simulations [6].

**Figure 8a:** 1500 rpm, 30°C measured

**Figure 8b:** 1500 rpm, 5 mPas simulated

**Figure 8c:** 1000 rpm, 30°C measured

**Figure 8d:** 1000 rpm, 5 mPas simulated

**Figure 8e:** 1500 rpm, 30 bar measured

**Figure 8f:** 1500 rpm, 30 bar simulated
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List of Symbols
\[ \begin{align*}
p & \text{ pressure} & \Omega & \text{ angular speed} \\
\Delta p & \text{ pressure difference} & \xi & \text{ displacement} \\
p_0 & \text{ suction pressure} & \xi_1 & \text{ horizontal displacement} \\
h & \text{ gap height} & \xi_2 & \text{ vertical displacement} \\
\mu & \text{ dynamic viscosity} & E & \text{ elastic modulus} \\
U_1 & \text{ spindle wall speed} & I_1 & \text{ geometrical moment of inertia} \\
U_2 & \text{ opposite wall speed} & q_l & \text{ line load} \\
t & \text{ time} & q_1 & \text{ horizontal line load} \\
\tau & \text{ 2D shear vector} & q_2 & \text{ vertical line load} \\
t & \text{ stress vector} & q_{ip} & \text{ pressure indicated line load} \\
n & \text{ spindle surface normal} & q_{lj} & \text{ journal bearing line load} \\
\varphi & \text{ turn angle} & & \\
\end{align*} \]

References


