ABSTRACT

Global warming is a climate phenomenon with world-wide ecological, economic and social impact which calls for strong measures in reducing automotive fuel consumption and thus CO₂ emissions. In this regard, turbocharging and the associated designing of the air path of the engine are key technologies in elaborating more efficient and downsized engines. Engine performance simulation or development, parameterization and testing of model-based air path control strategies require adequate performance maps characterizing the working behavior of turbochargers. The working behavior is typically identified on test rig which is expensive in terms of costs and time required. Hence, the objective of the research project “virtual Exhaust Gas Turbocharger” (vEGTC) is an alternative approach which considers a physical modeled vEGTC to allow a founded prediction of efficiency, pressure rise as well as pressure losses of an arbitrary turbocharger with known geometry. The model is conceived to use smallest possible number of geometry as well as material parameters. Thus, conventional expensive and time-consuming application processes can be countered and test rig as well as in vehicle measurements can be reduced. Furthermore, the vEGTC model enables the prediction of different turbocharger behavior caused by geometry variations.

Within this paper it is shown in which way the radial compressor as a representative modeling component can be described by zero-dimensional equations: in order to simulate the working behavior of the compressor the geometry, the thermodynamic state of the inlet-air and the turbocharger speed are assumed to be known. The loss mechanisms are devised using analytical and semi-empirical loss correlations. In order to validate the compressor efficiency the heat transfer from the turbine to the compressor is considered. Finally, the simulation output is compared to manufacturer maps of three different turbochargers pointing out the reliability of the model. Thus, a comprehensive validation of the vEGTC model is yielded. The object-oriented language Modelica is used for modeling and the simulations are provided by the Dymola solver.

INTRODUCTION

Driven by the rising emission and mileage requirements the digital control of all relevant engine actuators (e.g. fuel-system and injection, exhaust gas turbocharger with variable turbine geometry) with engine control units (ECUs) was introduced into the automotive market in the 90's. A characteristic feature of these ECUs is highly reusable control algorithms which can be adapted on different car as well as engine concepts by adjusting relevant software parameters. This car-specific parameter adaptation process is called ECU calibration.

A next significant development step in engine control was the introduction of model based control strategies, i.e. the calculation of appropriate actuator input signals by solving the relevant physical zero-dimensional steady state equations (e.g. pressure loss over throttle, turbo charger power, mass flows) in real-time. Model based control strategies have less complex parameter structures and thus need less calibration effort while keeping high control accuracy under different operation conditions.
The model based control strategies for turbochargers in automotive applications are based on the steady-state power balance and call for efficiency maps as the main physical parameter. Test rig and vehicle experiments and the subsequent offline analysis of the gained measurements is the standard way for calibrating these maps. An alternative calibration method is based on EGTC modeling and simulation. This model based calibration approach addresses the following main requirements:

- Experiment based calibration of turbocharger efficiency maps is time consuming and expensive.
- Test rig measurements reflect only a part of the working range of turbochargers. The aim is to determine the extended operation areas without extrapolation.
- Slight changes in the turbocharger geometry lead to complete new test campaigns. With a turbocharger model the expected changes of efficiency and pressure rise respectively caused by geometry changes can be examined.
- Modeling is understanding. ECU engineers gain insight into thermodynamics of turbochargers by modeling the relevant physical phenomena. This in turn is a key enabler for the development of new or improved existing control strategies.

The presented simulation approach called virtual Exhaust Gas Turbocharger is based on a physical turbocharger model in order to make computer-aided calibration and parameter adaptation possible. The model is able to predict efficiency, pressure rise as well as pressure losses of an arbitrary turbocharger with known geometry. Thus, model-based air path control strategies should yield a higher level of accuracy and reliability while testing costs are saved.

Here, the decision is knowingly made to work with physical modeling rather than CFD. Thus, the complexity of the model is significantly reduced which allows easier understanding of the connections between the parameters of the model. Additionally, geometry parameters can be varied easily while in case of CFD any small change in geometry requires the generation of a new computational mesh or at least a refinement of the mesh. The calculation of a working point by physical modeling is much faster than a CFD simulation. The computing time is an essential aspect regarding the real-time capability of the model in hardware in the loop simulations or on-board applications. So, physical modeling is a trade-off between accuracy and time costs. Therefore, the proposed model is intended for engine performance simulation or controls modeling but less appropriate for turbomachinery design.

**SIMULATION ENVIRONMENT**

*Modelica* is a modeling language which is primarily applied for component-oriented modeling of complex systems and simulation tasks. One particular feature of Modelica is the declarative description of the models. The equal sign is not an assignment operator but an equality operator in mathematical sense. So the equations have no predefined causality. This results in a system of equations which should be neither over-nor under-determined nor singular. Furthermore, Modelica supports object-oriented programing enhancing the modularity and reusability of the model. For simulation, the Modelica code is translated into objects which are then exercised by an Ordinary Differential Equation ODE or a Differential Algebraic Equation DAE solver.

In this work, the commercial tool *Dymola* is deployed as the simulation environment which is a combination of a user-friendly interface and a solver. Further it provides a graphical editor which allows for programing with visual expressions.

**MODEL STRUCTURE**

The zero-dimensional approach uses equations of gas dynamics, empirical and semi-empirical flow models as well as experimentally generated loss correlations available in literature (see (Spurk, 2008), (Shapiro, 1954), (Traupel, 1977), (Denton, 1993), (Pfleiderer, 2004), (Eck, 2002)). The zero-dimensionality means use of lumped parameters. The flow quantities are only known at the interfaces of the components, e.g. the pressure course inside the component is not determined.

![Figure 1. Cross section of a radial compressor](image)

For modeling, the radial compressor is taken apart in its individual components (see Figure 1). Each component is described by a system of equations (Stonjek, 2008). Thus, in a bottom-up strategy a library of components is generated, including inlet, rotor, radial diffuser and volute housing with outlet cone. To accomplish the overall system of the compressor, the components are composed and connected to each other, as shown in Figure 2. For simulation the thermodynamic state of the inlet air, the flow rate as well as the turbocharger rotational speed are assumed to be known.
Additionally, the geometry of the compressor is given. According to the main application field of the vEGTC, the model is conceived to use the smallest possible number of geometry parameters as well as material parameters. In this work the number of geometry parameters does not exceed 23. A list of geometry parameters is provided in Appendix A.

Figure 2. Compressor Components as partial models connected to an overall system

Considering the steady state flow each component is modeled by linking the balance equations for continuity and energy as well as the ideal gas law while the ideal change of state is assumed to be isentropic. Additionally, the power supplied to the flow in the rotor is indicated by Euler's turbine equation which is obtained from the balance of angular momentum.

\[
\rho(\bar{c} \cdot \bar{n})A = \text{const.}
\]

\[
h_{t,2} - h_{t,1} = w + q
\]

\[
w = c_{u,0}u_{0} - c_{u,i}u_{i}
\]

\[
pv^\gamma = \text{const.}
\]

\[
\frac{p}{\rho} = RT
\]

Further equations are added for each component taking losses into account. The losses are mostly described on the basis of approaches proposed in turbomachinery literature. Some of them are taken over directly or modified. It is to be noted that the application of these loss correlations involves uncertainties. Loss mechanisms are typically dependent on each other, though there is a breakdown of losses into profile and endwall losses as well as leakage losses in literature. Another uncertainty is the fact that the specified loss correlations in literature often apply for the design point and do not describe the off-design performance.

A useful loss definition is the energy or enthalpy loss which is equivalent to an entropy loss coefficient or a pressure loss coefficient in incompressible flow, as i.a. proposed by Denton (Denton, 1993). Referring to Figure 3 the enthalpy loss coefficient is described by:

\[
\zeta = \frac{h_{0} - h_{0,5}}{h_{t,1} - h_{1}}
\]

Since the denominator of Equation 6 represents the dynamic pressure the definition of loss results in a change of enthalpy which is proportional to the square of the flow velocity.

\[
\Delta h_{\text{loss}} = \zeta \frac{c_t^2}{2}
\]

In this work losses are considered as enthalpy losses which do not contribute to pressure rise. But they result in an enthalpy rise of the air. The overall enthalpy loss is determined additively in the model. Thus, Equation 2 can be rewritten as \(h_{t,2s} + \Delta h_{\text{loss}} = h_{t,1} = w + q\), where \(h_{t,2s}\) results from Equation 4. In the following the loss mechanisms are introduced which are considered in the compressor model.

\[
\rho(\bar{c} \cdot \bar{n})A = \text{const.}
\]

\[
h_{t,2} - h_{t,1} = w + q
\]

\[
w = c_{u,0}u_{0} - c_{u,i}u_{i}
\]

\[
pv^\gamma = \text{const.}
\]

\[
\frac{p}{\rho} = RT
\]

Figure 3. Enthalpy-entropy diagram for a diffuser; adiabatic compression process

FRICTION LOSS

With respect to the wall shear stress in a flow channel the friction loss is introduced

\[
\frac{dh_f}{dx} = \frac{\lambda}{2} \frac{c^2}{
\]

while a constant cross-section of the flow channel, an incompressible medium, a developed flow and steady state behavior are assumed. As most of the assumptions cannot be fulfilled in a real turbocharging process, this definition of the friction inherently constitutes an uncertainty. For the application on the turbocharger the friction factor \(\lambda\) and the
hydraulic diameter \( D_h \) are determined at the inlet and outlet of each component and are averaged linearly. However, in case of the flow velocity, the square of the velocity is averaged which represents the average kinetic energy.

The friction loss is considered in this form for the modeling of the components inlet and outlet cone. Further in the volute housing a modified calculation is performed by using a cylindrical coordinate system. Assuming a linear cross-sectional change the friction loss is defined as

\[
\Delta h_t = \frac{\sqrt{2}}{8} \pi^3 \lambda \frac{c_{D}^2 R^2 D^2}{\frac{4}{3} \sqrt{\phi}} D d\phi
\]

(9)

In the rotor and radial diffuser the friction loss models include further considerations and assumptions which are elaborated in the following.

**Rotor**

Equation 8 is also applied for the rotor from the assumption of blade channels which guide the flow into the radial diffuser. This is a rough estimate of the rotor friction loss only, since the twisted blade channel features compressible swirling flow. Here it is further needed to define an effective length of the blade channel the air passes through (due to the spatial course). For this purpose, it is allowed for a maximum and a minimum length while the effective length is assumed to be the average value.

Considering a point on the average streamline at the inlet of the blade channel and at the outlet respectively, the linking vector can be quoted as \( \vec{v} = (x \ y \ z) \). The minimum length of the blade channel can be introduced as the vector length \( L_{\text{min}} = |\vec{v}| \) while the maximum length is determined by the sum of the coordinates \( L_{\text{max}} = |x| + |y| + |z| \). It is to note that in this approach the rotor is treated as shrouded. That is, the housing wall loss is not determined separately, but is regarded as a part of the blade channel loss.

Further, Shapiro (Shapiro, 1954) indicates that the friction factor \( \lambda \) in the rotor is higher than in case of a fully developed pipe flow which is known from the Moody diagram. Thus, the approach inherently underestimates the rotor friction loss. Due to that, the blade channels are considered in full length regardless of splitter vanes in order to reduce this underestimation.

**Radial Diffuser**

The friction loss in the radial diffuser is referred to the loss of angular momentum. Eckert and Schnell (Eckert, 1980) specify the circumferential component of the velocity on an arbitrary radius of the radial diffuser to

\[
c_u = \frac{4f(D)}{D(f^2(D) - \frac{1}{2}^2)}
\]

(10)

with

\[
f(D) = e^{\frac{\lambda(D-D_i)}{2B_1}} \left( \frac{2}{c_{u,i}B_1} + \frac{1}{\sqrt{2}} + \frac{4}{(c_{u,i}B_1)^2} \right)
\]

(11)

\[
Z = \frac{0}{2\pi B_1}
\]

(12)

The circumferential component of the velocity in Equation 10 differs from the velocity calculated with the principle of angular momentum without consideration of loss (i.e. \( c_u = \frac{D_i}{D} c_{u,i} \)). This difference can be understood as kinetic energy dissipated by friction which leads to an enthalpy rise of the air.

**INCIDENCE LOSS**

A classical incidence model implies that the circumferential component of the relative velocity and therefore also a part of the kinetic energy is dissipated (NASA-Shock-Loss-Theory). Due to the abrupt change of direction at the rotor inlet, the difference between the circumferential components of the optimal inflow and the actual inflow in dependence on the working point leads to dissipation. The approach is based on empirical studies and can be founded physically referring to (Pfleiderer, 2004)

\[
\Delta h_{\text{inc}} = \frac{(w_{u,i} - w_{u,i,\text{opt}})^2}{2}
\]

(13)

\[
w_{u,i,\text{opt}} = \frac{\eta}{\rho_i \Delta_t \tan \beta_{i,\text{geo}}}
\]

(14)

Furthermore, the real circumferential component of the relative velocity \( w_{u,i} \) can be determined by means of the velocity triangle of the working point. Incidence is not considered for splitter vanes. The reason is that the flow is guided in correspondence with the curvature of the blade channel. Thus, the flow direction matches with the geometrical inflow angle of the splitter vanes. Additionally, it should be noted that the shock loss theory assumes shock-free flow at zero angle of incidence while there are also theories in literature introducing minimum loss at slightly negative angle of incidence.
TIP CLEARANCE LOSS

The loss induced by the flow around the blade between the pressure and suction side is considered using the following empirical correlation

\[ \Delta h_{\text{tip}} = \begin{cases} 0.6 \left( \frac{s}{B_{o,\text{Blade}}} - 0.013 \right) \frac{u_o^2}{2} & \text{if } \frac{s}{B_{o,\text{Blade}}} > 0.013, \\ 0 & \text{else} \end{cases} \]

(15)

which is introduced by Traupel (Traupel, 1977) and goes back to Dean (Dean, 1975). The consideration of the tip clearance loss is particularly proposed for large relative gap widths which is the case in automotive turbochargers. In Equation 15, \( B_{o,\text{Blade}} \) stands for the blade height at the outlet and is the average gap width while \( u_o \) is the circumferential speed at the outlet.

CARNOT'S SHOCK LOSS

In the radial compressor several components show diffuser behavior transforming the kinetic energy into pressure energy. In a diffuser the decrease of velocity together with pressure rise in flow direction can lead to boundary layer separation. Thus, the flow experiences a smaller effective cross-sectional increase which does not correspond to the actual channel geometry. As a result a smaller pressure rise is accomplished. This pressure deficit can be accounted for by using Carnot's shock loss formula for an abrupt change of cross-section that serves as an ideal poor diffuser (Spurk, 2008).

\[ \Delta h_{\text{carnot}} = \epsilon \left( c_{i} - c_{o} \right)^2 \]

(16)

In the transition from the rotor to the vaneless radial diffuser the obstruction of the blades is omitted abruptly which allows for a Carnot shock loss. In the radial diffuser itself, a Carnot shock loss is applied to the estimation of the decrease of the radial component of the velocity.

The flow enters the volute housing with a large tangential velocity component while the outflow velocity into the outlet cone is noticeably smaller which is explicitly determined by the equation of continuity. This dissipative deceleration indicates a loss referring to the Carnot shock loss.

The outlet cone represents a classical diffuser with an aperture angle of less than 10° which is imposed to a Carnot shock loss including an attenuation factor as proposed in (Van den Braembussche, 2006). (Weber, 1986). It is to notice that in all other cases mentioned above the attenuation factor is assumed to be 1.

The secondary flow losses which take place, inter alia, due to the redirection of the flow in the blade channel are generally described by partly dissipation of kinetic energy regarding both the absolute velocity at the inlet \( \left( \epsilon \frac{c_i^2}{2} \right) \) and the relative velocity in the rotor channel \( \left( \epsilon \frac{w_i^2 - w_o^2}{2} \right) \), see e.g. (Eck, 2002).

It is to note that explicit values for the attenuation factors \( \epsilon \) are not given in literature (rather intervals). In this work the effect of relative velocity is considered in form of a Carnot loss which as well describes a dissipation of the kinetic energy due to deceleration and is smaller than the difference of kinetic energy between output and input \( \Delta h_{\text{sec}} = \frac{\left( w_i - w_o \right)^2}{2} \). Hence, the attenuation factor is left out.

The absolute velocity is not considered for the description of the secondary flow losses due to insufficient effect on the simulated characteristics.

SLIP FACTOR

The limited number of blades effects the outflow angle which differs from the blade congruent flow angle at the rotor exit resulting in a smaller tangential component of the absolute velocity of the outflow \( c_{u,\text{eff}} = \mu c_{u} \). This results in a reduced pressure rise considering Euler's turbine equation. Therefore, the slip factor is accounted by

\[ \mu = \frac{1}{1 + \frac{H_{\text{eff}}}{H_{\text{eff}}^2 (1 - \psi/D_o)^3} \sin \beta_{o,\text{geo}}} \]

(17)

according to Eckert and Schnell (Eckert, 1980). Due to the Coriolis force and the consequential relative swirl (with a \(-2\Omega\) revolution to ensure \( \overline{\omega} = 0 \) in the absolute system) there is a difference in velocity between both sides of the blade channel. Furthermore, the flow involves friction so that the processes are affected by the boundary layer which develops differently for the different velocities on both sides of the blade channel. Thus, the shrinking of the tangential component of the absolute velocity at the exit is emphasized so that the effective channel width is set to \( H_{\text{eff}} = 0.8H \).

TRAILING EDGE LOSS

As a consequence of flow around the trailing edge of the blades, vortices can arise. The energy consumed by the vortices is identified as dissipation (Bammert, 1966).

\[ \Delta h_{\text{trail}} = \epsilon \frac{T_o}{t_o \sin \beta_o} \frac{w_o^2}{2} \]

(18)

Bammert and Fiedler (Bammert, 1966) and Denton (Denton, 1993) propose \( 0.15 < \epsilon < 0.2 \). \( t_o \) is the blade pitch at the exit while \( T_o \) stands for the blade thickness.
FURTHER LOSS MECHANISMS

At the rotor inlet, the inflow is obstructed by the hub face and the twisted blades which dissipates energy. In order to characterize this loss, the principle of linear momentum is considered in a very narrow domain of integration, a so-called control volume including the hub and the leading edge of the blade. The force acting on the hub is referred to the total pressure before the hub though it only applies for the stagnation point at the center of the hub. Considering the equations of continuity, energy and the ideal gas, a system of equations can be set for the control volume. The resulting change of enthalpy does not correspond to the isentropic change of state. The difference can be related to the square of the flow velocity representing a loss correlation.

Another loss consideration refers to the volute housing which cannot exploit the radial component of the velocity of the inflow. This results in kinetic energy dissipation which is introduced by Van den Braembussche (Van den Braembussche, 2006) as a meridional dump loss. The approach goes back to Japikse (Japikse, 1982) and Weber and Koronowski (Weber, 1986).

CORRECTION FACTOR

The loss models introduced above are proposed for modeling under compressible conditions though they are mostly generated for incompressible flow. From the point of view of mechanical similarity, the relative change of volume due to compressive forces should be taken into consideration. In this regard, the Mach number is the relevant factor which is highly variable in the compression process. Hence, the loss models can inherently not describe the compressor flow accurately so that there is at least a need for a correction factor. In this work the enthalpy losses are upgraded with the multiplicative factor \( \rho_o \). Considering a straight flow channel with constant cross section the Bernoulli equation introduces

\[
\Delta h_{\text{dump}} = \frac{c_{l,i}^2}{2}
\]

(19)

\[ p_{t,o} = p_i \left( \frac{p_{t,i}}{p_i} - \frac{\rho_o}{\rho_i} \Delta h_{\text{loss}} \right) \]

(22)

\[ p_{t,o} - p_{t,i} = -\rho_o \Delta h_{\text{loss}} \]

(23)

Considering the definition of speed of sound \( a^2 = \left( \frac{\partial p}{\partial \rho} \right) s \), the change of density can be rewritten as \( \Delta \rho / \rho_i \approx \frac{1}{2} M a^2 \). Thus, the correction factor is linked to the Mach number:

\[
\frac{\rho_o}{\rho_i} = 1 + \frac{\Delta \rho}{\rho_i} \approx 1 + \frac{1}{2} M a^2.
\]

(24)

The correction factor substantially improves the model quality, particularly for high circumferential speed.

HEAT TRANSFER

With the loss models so far the compressor performance can be described adiabatically. However, due to the temperature gradient between the turbine and the compressor and the resulting heat flow the compressor process is non-adiabatic. According to the first law of thermodynamics the specific amount of heat transferred to/from the compressor is considered in Equation 2. At the same time it is allowed for an enthalpy loss \( \Delta h_{\text{heat}} \) since the heat deteriorates the efficiency of the compressor. If the heat enters the compressor before the compression process, more work is required to compress the warmer gas which is justified by diverging isobars in an h-s-diagram, as e.g. shown in Figure 3. Otherwise, if the heat enters the compressor after the gas is already compressed, the compression process will not be affected directly. Then, the efficiency deterioration goes back to the temperature-based definition of the isentropic efficiency since the gas is additionally heated.

On the one hand heat is exchanged between compressor and turbine, on the other hand between compressor and environment. In order to estimate the heat transfer, Bohn et al. (Bohn, 2003) propose a formula using the dimensionless Nusselt number as a function of Reynolds number and turbine inlet temperature. The heat model also accounts for negative heat transfer, i.e. from the compressor to its environment. This can occur at high compression level since the air is heated up through compression. In this paper the heat transfer is modeled according to Bohn et al. while heat is assumed to enter the compressor after the compression process.

SIMULATION RESULTS

In this section simulation results of an automotive turbocharger are presented using key performance indicators.
of the turbomachinery, i.e. the dimensionless parameters power coefficient $\lambda = \frac{2\Delta h_t}{u_0^2}$ the flow coefficient $\phi = \frac{Q_{\text{red}}}{u_0 \cdot A_0}$.

In Figure 4 the power coefficient $\lambda$ is plotted versus flow coefficient $\phi$ for an adiabatic process. It can be seen that the power coefficient increases with the circumferential speed which is expected according to scale-up theory. Additionally, the curvature of the characteristics, especially for higher circumferential speeds, points out the pressure deterioration in consequence of the loss mechanisms. The lossless case of each characteristic would exhibit a straight course, following Euler's turbine equation. In this regard Figure 5 shows the progression of the rotor enthalpy losses against the flow coefficient while all quantities of enthalpy losses are normalized by $\frac{u_0^2}{2}$. The course of the overall enthalpy loss is mainly specified by the incidence which is essential in off-design working range due to quadratic dependency on the flow coefficient. Besides, tip clearance and friction losses constitute the main pressure deterioration. The Carnot shock loss shrinks with the flow coefficient which is due to a decreasing deceleration of the flow. The trailing edge loss is only noticeable close to choking. It should be noted that the loss models cannot all meet the real-world case accurately. Some losses may be overestimated or underestimated. However, the overall loss is assumed to be a balance without substantial deviation from the real-world scenario.

**VALIDATION AND DISCUSSION**

In order to estimate quality and reliability of the compressor model, a comprehensive validation is performed. For this purpose three different automotive turbochargers are used while manufacturer maps are applied as reference data. The manufacturer maps describe non-adiabatic compressor performance. General geometry parameters of the turbochargers are listed in Table 1.

![Figure 4. Dimensionless aerodynamic characteristics of a radial compressor](image)

![Figure 5. Normalized rotor enthalpy losses illustrated for a single circumferential speed](image)

Table 1. General geometry parameters of the radial compressors applied for validation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Turbocharger I</th>
<th>Turbocharger II</th>
<th>Turbocharger III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture ratio</td>
<td>64%</td>
<td>74%</td>
<td>75%</td>
</tr>
<tr>
<td>Blade outlet angle from circumferential direction</td>
<td>60°</td>
<td>69°</td>
<td>58°</td>
</tr>
<tr>
<td>Blade number</td>
<td>6+6</td>
<td>6+6</td>
<td>5+5</td>
</tr>
<tr>
<td>Rotor outlet diameter</td>
<td>66 mm</td>
<td>60 mm</td>
<td>38 mm</td>
</tr>
<tr>
<td>Gap</td>
<td>0.5 mm</td>
<td>0.36 mm</td>
<td>0.14 mm</td>
</tr>
<tr>
<td>Diameter ratio in radial diffuser</td>
<td>1.5</td>
<td>1.43</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Model results are displayed as total pressure ratio $\Pi_t = \frac{P_{t,2}}{P_{t,1}}$ and isentropic efficiency $\eta_s = \frac{h_{t,2,\text{is}} - h_{t,1}}{h_{t,2} - h_{t,1}}$ versus reduced volume flow $Q_{\text{red}} = Q \sqrt{\frac{\rho_{\text{ref}}}{\rho_{t,1}}} \frac{\sqrt{t_{r,1}}}{\sqrt{t_{r,2}}}$ at different circumferential speeds in a range of 20000 rpm to 100000 rpm for
turbocharger I, 50000 rpm to 185000 rpm for turbocharger II and 75000 rpm to 280000 rpm for turbocharger III.

In Figure 6, Figure 7, Figure 8, Figure 9, Figure 10, Figure 11 the simulation results are featured with markers while the manufacturer map is displayed with continuous characteristics. In order to demonstrate the range of validation a consistent scale on the diagram axes is used so that the different working ranges of the applied turbochargers can be recognized. For each simulated turbocharger it can be stated that the model is able to predict the total pressure ratio reliably in the entire working range of the compressor depicted in the manufacturer map (see Figure 6, Figure 8 and Figure 10). It should be noted that the model can also identify working points at circumferential speeds below the minimum speed indicated in the manufacturer map. Thus, it is possible to extend the manufacturer map without using extrapolation methods. The indication of surge is not considered here since the compressor surge is dependent upon upstream and downstream components and bends. Hence, indication of surge demands information about test rig geometry. However, a prediction of instability can be achieved using the diffusion factor according to (Rodgers, 1978) which serves as a measure for the rotor load. In literature critical values of 0.6 to 0.8 are proposed for the diffusion factor. In this work critical values of 0.7, 0.65 and 0.6 are identified for turbocharger I to III which are labeled with circles in the diagrams. The critical value can be linked with the size of the turbocharger considering that turbocharger I has the largest rotor and turbocharger III the smallest one. Further, Choking is considered with regard to $Ma = 1$.

For each turbocharger the simulated isentropic efficiency is compared with the respective manufacturer map. For a better overview only four selected characteristics are plotted (see Figure 7, Figure 9 and Figure 11). Figure 7 introduces the results for turbocharger I which show good agreement between simulation and reference data at higher circumferential speeds. The characteristic $0.2u_{\text{max}}$ at exhibits maximum difference. The simulation results for turbocharger II have a lower accuracy; the efficiency is overestimated in the entire working range. With growing circumferential speed the difference in the magnitude of efficiency decreases at the cost of accuracy regarding the location of the maxima (see Figure 9). Agreement with reference data can also be recorded in Figure 11 for turbocharger III which is the smallest turbocharger considered for validation. It should be noted that the overall characteristic of the efficiency map can be described for all applied turbochargers, i.e. the non-adiabatic isentropic efficiency is expected to increase with the circumferential speed and then drop towards maximum speed.
In the following some uncertainties referring to simulation and validation are discussed. The geometry parameters entering the compressor model are exposed to uncertainties since they are measured and collected manually due to lack of CAD data. In order to minimize these uncertainties, a stereoscopic survey of the geometric parameters is carried out (see Appendix A). Furthermore, the procedure with consideration of manufacturer maps as reference data involves uncertainties since there is no exact information about the measuring technique as well as measuring points on the test rigs, a typical scenario for ECU engineers. That is, the quantities pressure, temperature and flow rate are not measured directly at the inlet or outlet of the compressor. Test rigs have given pipe geometries, so that turbochargers can only be mounted using individual fittings. However, the measuring points are located in the test rig pipes. As a result of this, the system boundaries are automatically extended (including duct losses). This issue has to be taken into account when comparing simulation and test rig maps, however the effects are assumed to be moderate.

The discrepancy between the simulated isentropic efficiency and the reference data can be related to the general enthalpy loss model on the one hand, see Equation 7. Besides the magnitude of efficiency, the location of the optima of efficiency characteristics is determined by the different loss mechanisms, in particular by the incidence loss. Incidence is caused by manufacturing tolerances, part/over load operation, inhomogeneous inlet flow and wall boundary layer though the incidence loss model only accounts for part/over load operation while assuming a homogeneous inlet flow, regardless of the development of a wall boundary layer and its form. Further, the shock loss theory assumes shock-free flow at zero angle of incidence. In literature there are also theories which introduce minimum loss at slightly negative angle of incidence. Hence, the shock loss theory might inherently lead to shifting efficiency maximum towards surge line with positive angles of attack. Further, the location of the optimum is also affected by other loss mechanisms shifting the optimum from the shock-free flow rate, see Figure 5. The simplified loss models, e.g. friction which enters each component, can lead to a deficient displacement of the optimum. Additionally, the magnitude of shifting of the optimum shows dependency on the turbocharger circumferential speed. Further, the difference in slope of the efficiency characteristics in part load can be referred to inlet recirculation. Due to insufficient models in literature, recirculation is, however, disregarded in favor of better prediction of total pressure ratio. In this connection, the interaction between recirculation and incidence is important which is rarely addressed in literature. On the other hand the model for the heat transfer from the turbine to the compressor and from the compressor to the environment can introduce uncertainties. Considering the close correspondence of the simulated total pressure ratio to the reference data as well as the more severe discrepancy of the efficiency characteristics at low circumferential speed the heat model is supposed to be another significant uncertainty deteriorating the simulation results for the compressor efficiency (see also Appendix B).
The isentropic efficiency depicted by the manufacturer map is representative for turbocharger performance with a constant turbine inlet temperature (800°C is a common value). However, the turbine inlet temperature i.e. the exhaust gas temperature significantly changes with the engine load. In a real-world scenario, turbine inlet temperatures of 300°C are also possible. As a consequence of lower turbine inlet temperatures the heat transfer to the compressor decreases and the isentropic efficiency increases accordingly. This issue is addressed by the simulation results in Figure 12 showing the range of isentropic efficiency exemplarily for a selected low circumferential speed (difference up to 11 percentage points). At high circumferential speeds, the heat transfer dependent deterioration of the efficiency decreases since the ratio of temperature rise due to heat transfer to temperature rise due to compression sinks. It should be noted that the pressure rise does not change in this observation. This refers to the fact that heat is assumed to enter the compressor after the compression process, but also the amount of heat transferred to the compressor is small compared to the work input (see also Appendix C).

A comprehensive analysis of heat on turbochargers is in progress in order to be able to identify the respective relations more reliably and will be presented in future publication. For this purpose the compressor model depicted above provides the adiabatic compressor performance. The aim is to investigate if the heat transfer handicaps the compression process or the deterioration of efficiency is only an artifact which is supposed to be a result of the temperature based definition of the isentropic efficiency. In literature it is often assumed that heat basically enters the compressor in the volute housing (where the flow is maximum exposed to hot surfaces) so that the temperature rise due to heat transfer occurs after the main compression process. This supports the hypothesis the deterioration of efficiency being an artifact. In fact, the heat transfer heats up the compressed air. Considering the downstream charge air cooler, the real efficiency of the compressor should be more relevant (i.e. the adiabatic efficiency). However, the effect of heat still needs to be dealt with e.g. regarding Mach-number and/or the ratio of static pressure and dynamic pressure in the total pressure at compressor outlet (see also Appendix C).

For the investigation, the work output of the turbine should further be taken into account which includes modeling and analysis of the turbine process. These investigations are essential with regard to ECU calibration and development of control strategies and will be presented in future publications.
CONCLUSION

In this paper a physical modeling approach is introduced in order to identify the steady state performance of a radial compressor (with regard to pressure rise, pressure losses as well as isentropic efficiency). The zero-dimensional approach needs a small number of geometry parameters and material parameters. Simplicity is a demand regarding computation costs. For simulation the geometry, the thermodynamic state of the inlet air and the turbocharger speed are assumed to be known. Different loss mechanisms are devised using analytical and semi-empirical loss correlations, as well as a model for the exchange of heat. The compressor model can be applied to different automotive turbochargers without any individual adaptation of loss correlations (i.e. without parameter variation) which is a requirement placed on the model. From the point of view of validation simulation results are compared with measured manufacturer maps. The validation procedure with manufacturer maps is relevant regarding ECU calibration which is generally carried out on the basis of manufacturer maps. The results of total pressure ratio exhibit large consensus with the reference data. The simulated isentropic efficiency shows also good agreement, however a lower level of confidence is achieved. This is referred to the simple loss models like incidence on the one hand and the insufficient heat model on the other hand. Further, it has been shown that the isentropic efficiency increases with decreasing turbine inlet temperature while the pressure ratio does not change. Considering the deterioration of efficiency due to heat being an artifact, the adiabatic efficiency should be indicated as the real efficiency of the compressor. This issue revalues the simulation results of the compressor model at low speed. The compressor model originally determines the adiabatic efficiency which is corrected for a non-adiabatic process using a heat model in order to achieve comparability with manufacturer maps. As a consequence, manufacturer efficiency maps should not be used as reference data for ECU calibration at low speed. This hypothesis will be substantiated with adequate experimental data in future publication.

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DEFINITIONS/ABBREVIATIONS

Nomenclature

\( q \) Specific heat

\( Q \) Volume flow rate

\( R \) Ideal gas constant

\( s \) Specific entropy

\( S \) Average gap

\( t \) Blade pitch

\( T \) Temperature / Blade thickness

\( v \) Specific volume

\( w \) Relative velocity / Specific work

\( Z \) Constant

Greek symbols

\( \beta \) Relative angle

\( \epsilon \) Attenuation factor

\( \zeta \) Loss coefficient

\( \eta \) Isentropic efficiency

\( \gamma \) Isentropic exponent
\( \lambda \)  
Friction factor / Power coefficient

\( \mu \)  
Slip factor

\( \rho \)  
Density

\( \Pi \)  
Pressure ratio

\( \phi \)  
Angle

\( \varphi \)  
Flow coefficient

\( \Omega \)  
Rotational speed

**Subscripts**

+  
Normalized value

eff  
Effective

geo  
Geometric

i  
Inlet of a component

o  
Outlet of a component

opt  
Optimal

r  
Radial component

red  
Reduced

ref  
Reference condition

s  
Isentropic state

t  
Total value

u  
Circumferential component
APPENDIX

A - STEREOSCOPIC SURVEY OF GEOMETRY

Exemplary results of the stereoscopic survey of the turbocharger geometry are illustrated in the following. (Thesing, 1999) delivers more detailed insight into the approach. Lack of CAD-data is a typical scenario for calibration engineers. However, accurate geometry information is necessary for simulation and can be provided stereoscopically. Suppliers of optical systems provide tools which enable to insert centric, parallel or perpendicular lines and surfaces to determine different dimensions as shown in Figure 13 for the Characteristic Diameter of the volute at the outlet (referring to mean streamline and rotor center). Further, Figure 14 exhibits exemplarily the inlet blade angle of the rotor.

![Figure 13. Stereoscopic survey of the compressor volute housing](image)

| Table 2. Geometry parameters applied in the model |
|----------------------|--------------------------------------------|
| **Inlet**            | Cross-section at the inlet               |
|                      | Length of the inlet                      |
|                      | Relative roughness                       |
| **Rotor**            | Tip diameter                              |
|                      | Hub diameter                              |
|                      | Outlet diameter                           |
|                      | Blade height at the outlet               |
|                      | Blade number                              |
|                      | Axial chord                               |
|                      | Blade inlet angle                         |
|                      | Blade outlet angle                        |
|                      | Blade thickness                           |
|                      | Relative gap                              |
|                      | Radial redirection angle of the mean streamline |
|                      | Relative roughness                        |
| **Radial diffuser**  | Outlet diameter                           |
|                      | Width                                      |
|                      | Relative roughness                        |
| **Volute**           | Cross-section at the volute tongue        |
|                      | Characteristic Diameter of the volute at the outlet (referring to mean streamline and rotor center) |
| **Outlet**           | Cross-section at the outlet               |
|                      | Length of the outlet                      |
|                      | Relative roughness                        |

In the following a list of geometry parameters are provided which are applied in the model:
B - HEAT MODEL UNCERTAINTY

The used heat model does not yield sufficient results for all turbocharger types. The heat model improves the efficiency results for turbochargers I and III in a certain extent. The difference at low speed remains. In case of turbocharger II the heat model falsifies the efficiency characteristics almost in the entire speed range. Figure 15 shows the isentropic efficiency of turbocharger II considering an adiabatic process. Compared to Figure 9 the adiabatic characteristics here show better agreement to the reference data. In Figure 9 the efficiencies are of a higher level. The reason is that the heat model outputs negative heat (i.e. heat transfer from the compressor to the environment) which decreases the air temperature at compressor outlet. This occurs already at $0.44u_{\text{max}}$. As a result of this the efficiency increases. Hence, the heat model cannot sufficiently describe the heat transfer on turbocharger II so that the simulated adiabatic efficiency better corresponds to the non-adiabatic reference data, yet.

Figure 15. Isentropic efficiency on turbocharger II for selected circumferential speeds considering an adiabatic process; simulated (marker) and reference (continuous)
C - HEAT AND EXERGY

Heat is energy. Considering Equation 2, the question arises if the heat transferred to the compressor can also carry out useful work. The second law of thermodynamics, however, introduces that heat always inserts entropy into the system. Though, a fractional amount of heat increases the working capacity of the gas, i.e. exergy. For a preliminary estimate, the exergy of the heat is evaluated as the maximum useful work possible.

\[ e_{x_q} = (1 - \frac{T_{\text{amb}}}{T_*})q \]  
(25)

By definition only a part of the heat can be transformed into exergy which corresponds to the fractional amount \( 1 - \frac{T_{\text{atm}}}{T_*} \). Hereby \( T_{\text{atm}} \) is the ambient temperature while \( T_* \) is the temperature of the heat reservoir. For a rough calculation \( T_* \) is set to turbine inlet temperature. Thus, exergy is overestimated since the more proper source of heat would be the bearing housing or the compressor back face. As a result, it is assumed that the exergy is at least overestimated over 30%. Moreover, exergy loss i.e. entropy generation due to non-ideal process is not considered which again leads to overestimation.

In Figure 16 the exergy is normalized by \( u_0^2 \) and plotted against reduced volume flow. The exergy ratio is higher at low speed where the work input is small.

The conclusion is that heat increases the working capacity of the air in/after the compression process. The evaluated exergy ratio does not exceed 10%. It is object of inquiry what the increased working capacity, especially at low speed, means to the compressor which is a work machine (contrary to the turbine). Effects on the Mach-number and/or the ratio of static and dynamic pressure in the total pressure at compressor outlet are possible.

Figure 16. Exergy estimate on the compressor due to transferred heat

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