Scaling Friction and Inertia Losses for the Performance Prediction of Turbomachines

Peter F. Pelz[1] and Michael Heß[1]

[1] Chair of Fluid Systems Technology
Technische Universität Darmstadt
Darmstadt, Germany
Phone: +49-6151-16-2153, FAX: +49-6151-16-2453, E-mail: peter.pelz@fst.tu-darmstadt.de

NOMENCLATURE

a	speed of sound
C	power coefficient
d	diameter
k	roughness height, relative roughness height
M	scale factor
Ma	Mach number
n	rotational shaft speed
P	power
p	static pressure
Re	Reynolds number
t	tip clearance height, relative tip clearance height
$\dot{V}$	volume flow
V	loss distribution factor
$Y=gH$	specific work

Greek symbols

α	exponent
δ	thickness
η	efficiency
ν	kinematic viscosity
κ	geometry scaling factor
λ	friction factor
ψ	pressure coefficient
ρ	density
φ	flow number

Subscripts

fluid	
f	friction loss
i	inertia loss
ideal	
l	loss
m	model machine
mech	mechanical
rs	real size machine
t	tip
v	viscous
w	relative velocity

INTRODUCTION

In practice measurements during the design process or for final inspections often cannot be carried out on full scale machines. Mostly it is too costly to build an own test rig for each machine, so test rigs with standardized diameters and measuring equipment are used. Sometimes the diameters of the real scale machines are so large that a measurement is utterly impossible. Examples are water turbines or fans used for cooling in power plants – these machines may have diameters of several meters and require a brake respectively drive power in the MW range. On the opposite machines with diameters of only a few millimeters do not allow the operation of established invasive measuring techniques – such as Pitot-tubes, five hole probes or hot wire anemometry – because of the significant influence of the probes on the flow field. Hence measurements to determine the performance are carried out on models which are down or up scaled. As was shown by Spurk (1992) a full similarity between model and prototype machine is in the context of turbomachines possible as long as the Mach number is small, i.e. the compressibility can be neglected.

SIMILARITY

To provide transferability of measurements on scaled models to the real size machines the similarity between model and prototype has to be considered. Scaling the geometric means scaling of all dimensions such as diameter d, blade length, chord length of rotor and guide vanes, surface roughness height k, tip clearance height t with the same scale factor $\kappa = d'/d$, where the model data here and in the following are indicated by a dash. Even so full geometric similarity is possible, the similarity in relative roughness k/d and relative tip clearance t/d are sometimes sacrificed.

Following the Bridgman postulate, *on the absolute significance of relative magnitudes* (1920) full similarity is reached when the relative magnitudes of a system, which are called dimensionless products today, are identical for the model and prototype system. One example for a geometric relative magnitude is the relative tip clearance t/d. One example for a kinematic relative magnitude or dimensionless product is the flow coefficient $\varphi = 4\dot{V}/\pi^2 nd^3$. In the first example the tip clearance is not measured in millimeter or inch, but in multiples of the impeller diameter. In the second example the volume flow rate is measured in multiples of the machine typical flow rate $\sim nd^3$. On the other hand the rotational speed is measured not in Hertz or revolutions per minute but in multiples of the inverse of a viscose diffusion time ν/d^2, which leads to the Reynolds number $Re = nd^2/\nu$.

Following the Buckingham Π -theorem (1914) the total specific work $Y = gH(\dot{V}, a, n, \nu, d, k, t)$ is equivalent to $\psi = \psi(\varphi, Ma, Re, k, t)$ where here and in the following the roughness and tip clearance are relative quantities: $k/d \to k$, $t/d \to t$. For the low Mach number case, i.e. nearly incompressible flow, $Ma = nd/a$ plays no role and the pressure coefficient

$\psi := 2gH / \pi^2 n^2 d^2$ is given by

$$\psi = \psi(\varphi, Re, k, t). \quad (1)$$

The dissipated energy per unit time P_l is a function of $P_l = P_l(P, \dot{V}, a, n, \nu, d, k, t)$ which is equivalent to $P_l / P = P_l(\varphi, Ma, Re, k, t) / P$. For a pump, fan or compressor the efficiency is defined as $\eta = 1 - P_l / P$ and the power coefficient is $C = \varphi\psi / \eta$, if P is mechanical power transferred by the shaft to the impeller. For a turbine the efficiency is $\eta = (1 + P_l / P)^{-1}$, and the power coefficient $C = \eta\varphi\psi$ if P denotes the mechanical power transferred from the impeller to the shaft. Again for small Mach number the efficiency in both cases can be written as

$$\eta = \eta(\varphi, Re, k, t). \quad (2)$$

Equations (1) and (2) fully describe the performance of a turbomachine. Hence, as long as the dimensionless arguments $\varphi, \mathrm{Re}, k, t$ are the same, model and prototype show the same pressure coefficient and efficiency. Again if the model is denoted by a dash the ratios, or scaling factors $\varphi' / \varphi = M_\varphi = M_{\dot{V}} / (M_d^3 M_n)$, $Re' / Re = M_{\mathrm{Re}}$, … have to be one. This leads to the system 4 of equations

$$\frac{M_{\dot{V}}}{M_d^3 M_n} = 1, \quad (3)$$

$$\frac{M_d^2 M_n}{M_\nu} = 1, \quad (4)$$

$$\frac{M_k}{M_d} = 1, \quad (5)$$

$$\frac{M_t}{M_d} = 1, \quad (6)$$

for seven scale factors, where three can be chosen arbitrarily. For the same fluid in model and prototype $M_\nu = 1$ holds. With $M_d = d'/d = \kappa$ as the geometrical scale factor this results in:

$$M_n = \kappa^{-2}, \quad (7)$$

$$M_{\dot{V}} = \kappa, \quad (8)$$

$$M_s = \kappa, \quad (9)$$

$$M_k = \kappa. \quad (10)$$

With those scale factors, the scale factor for the head change becomes

$$M_{gH} = \kappa^{-2}. \quad (11)$$

The scale factor for the mechanical power results in $M_P = M_\rho M_{gH} M_{\dot{V}} = \kappa^{-1}$. Hence the volume specific power of the machine would scales as

$$M_{P/d^3} = \kappa^{-4}. \quad (12)$$

Hence, in principle complete similarity is accessible. In practice several problems arise. Assuming a geometric scale factor of $\kappa = 1/10$, the rotational speed and the power centuple, whereas the volume flow would be a tenth. Estimating a usual turbomachine with an outer diameter of 5 m[1] and a rotational speed of 500 rpm (circumferential speed at tip 130 m/s), providing geometric similarity would require scaling the relative roughness and relative gap width to a tenth of the real size machine. It is possible to achieve such small dimensions, but it is at least very costly. Using Eq. (7) to (11) by assuming complete similarity would lead to a rotational speed of 50 000 rpm (tip speed 1310 m/s) respectively a volume specific power in the area of several GW/m^3 for the model machine which is impossible to achieve. The mentioned problem of unreachable complete similarity can only be avoided by giving up similarity of all dimensionless products. In this case another scale factor can be chosen free. Usually the similarity in Reynolds number and relative roughness is sacrificed and the pressure rise is kept constant. As a result the pressure coefficient and the efficiency is different in model and prototype machine. For example $\eta'(\varphi, Re') < \eta(\varphi, Re)$ and $\psi'(\varphi, Re') < \psi(\varphi, Re)$ for $Re' < Re$.

LOSSES

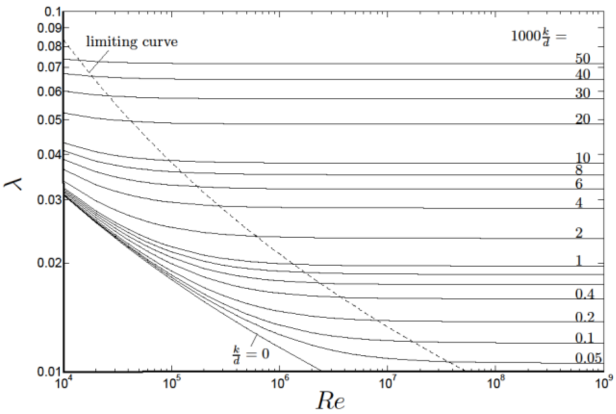

Fig. 1: Loss coefficient for pipes at different Reynolds numbers and relative wall roughness

Sacrificing the Reynolds number similarity means, that the similarity in the relative boundary layer thicknesses is sacrificed. Hence for $Re' < Re$ the relative boundary layer of the prototype is thinner in comparison to the model. Hence scaling the model to the prototype the relative *friction losses* are decreased which results in a higher efficiency. This is valid as long as the viscous sub layer $\delta_\nu = \nu / \sqrt{\tau_w / \rho}$ (Spurk, 2008) is thicker than the surface roughness k. The wall shear stress scales as $\tau_w \sim \lambda\rho\, n^2 d^2 (1 + \varphi^2)$ with the well known friction factor $\lambda = \lambda(Re_w, k/d)$ shown in Fig. 1.

$$\frac{k}{\delta_\nu} = \frac{k\sqrt{\tau_w / \rho}}{\nu} \sim \frac{k}{d}\sqrt{\lambda(1 + \varphi^2)} Re < 5 \quad (13)$$

the friction losses will be dependent on the Reynolds number. For the hydraulic smooth case the empirical Blasius law (Spurk, 2008)

$$\lambda \sim Re^{-1/4} \text{ for } k / \delta_\nu \le 5 \quad (14)$$

is an appropriate approach. For higher Reynolds number flow, when the viscous sub layer becomes thinner than the surface roughness, the friction losses will become independent on the

[1] whereas former mentioned water turbines or power plant fans could reach diameters of 10 m and more

Reynolds number. Hence for the hydraulic rough surface the friction law (Spurk, 2008)

$$\lambda = \left(2\lg\frac{d}{2k} + 1.74\right)^{-2} \text{ for } k/\delta_v \geq 70 \tag{13}$$

is appropriate. Since for the purpose of up scaling only the relative change in friction has to be considered, it is feasible to work with the above mentioned friction laws.

After this recall of the findings of Prandtl, von Kármán and Blasius one has to raise the question what to do with those classical results in the context of scaling. Since pressure coefficient and efficiency both depend on the Reynolds number, relative roughness and relative tip clearance so has the power coefficient $C(\mathrm{Re}, k, t) = \psi\varphi/\eta$ (here for the example of a compressor, fan or pump). Nevertheless for axial machines the power coefficient is independent on Reynolds number, relative roughness and tip clearance which was confirmed by experiments over a wide parameter range (Hess, Pelz, 2009). Hence $\eta'/\eta = \psi'/\psi$ holds for the same flow number for a fan, compressor or pump and $\eta'\psi' = \eta\psi$ holds for a turbine. Thus there is only one scaling method for pressure coefficient and efficiency. One approach to gain a scaling method is a physical analysis of the possible losses. They are treated as additive of Reynolds number or viscosity dependent, i.e. *friction losses* and Reynolds number independent or viscosity independent losses which are called *inertia losses:*

$$\psi_l \approx \psi_f(Re, \varphi, k) + \psi_i(\varphi, t), \tag{16}$$

Ackeret (Mühlemann, 1948) called the friction losses scalable losses and introduced the loss distribution factor

$$V(\varphi, Re, k, t) := \frac{\psi_f}{\psi_l} = 1 - \frac{\psi_i}{\psi_l}. \tag{17}$$

which was set by Ackeret to the fixed value of 0.5. But from (16) it becomes clear, that it is a rather crude assumption to fix the value to 0.5. It is the main task of this contribution, to gain a physical motivated loss distribution function, which will depend on the Reynolds and flow number.

Before doing so it is worthwhile to discuss the *inertia losses* even further. They can be split up in a loss part due to the flow between casing and tip and a part due to incidence losses:

$$\psi_i \approx \psi_t(\varphi, \mathrm{t}) + \psi_{i0}(\varphi). \tag{18}$$

The first part was first discussed by Albert Betz (1926) for axial machines. Following his thoughts the resistance is due to mixing and an induced resistance caused by the tip vortex. This induced resistance has to be an even function of the flow number φ or going further it has to be an even function of the ideal machine performance and it has to vanish for zero tip clearance. Hence the ansatz

$$\psi_s(\varphi, t) = b_t t \psi_{th}^2(\varphi) \tag{19}$$

is motivated by this assumption for small relative tip clearance. Even though we do not use (19) in the following it might be an interesting thought to scale the relative tip clearance. The theoretical machine characteristics is obtained by the measured pressure coefficient and efficiency η, ψ

$$\psi_{th} = \frac{\psi}{\eta} = \hat{\psi} - c\varphi, \tag{20}$$

where $b_t, \hat{\psi}, c$ are dimensionless machine constants. Figure 3 shows the validity of equation (20) for an axial fan. From the measurement data the constants $\hat{\psi}, c$ are easily determined. Following this discussion the tip losses are maximum for small flow number and the slope of the losses approach zero $d\psi_t/d\varphi \to 0$ for $\varphi \to 0$. The second part of the inertia loss $\psi_{i0}(\varphi)$ shows a minimum at the design point $d\psi_{i0}/d\varphi = 0$ at $\varphi = \varphi_0$ of the machine. For part and overload the incidence losses increase in an first assumption proportional to $(\varphi - \varphi_0)^2$.

FIRST MODEL FOR THE LOSS DISTRIBUTION FACTOR

To gain a loss distribution factor, which takes at least the dependence of the loss distribution with flow number into account, it is promising to study the shape of the efficiency curve $\eta = \eta(\varphi, Re)$ which can be written as:

$$\eta = 1 - \frac{\psi_l}{\psi_{th}} = 1 - \frac{\psi_l}{\hat{\psi} - c\varphi}. \tag{21}$$

For small flow number $\varphi < 1$ the friction model

$$\psi_f = \psi_f(\varphi, Re, k) \approx \psi_f(Re, k) \sim \lambda(\mathrm{Re}, k) \quad \text{for } \varphi < 1 \tag{22}$$

is a reasonable approximation. Hence (21) is for this approximation equivalent to

$$\eta = 1 - \frac{\psi_f(Re) + \psi_i(\varphi)}{\hat{\psi} - c\varphi}. \tag{23}$$

Partial differentiating with respect to φ leads to

$$-\left.\frac{\partial\eta}{\partial\varphi}\right|_{\mathrm{Re}} = \frac{\psi_{th}\, d\psi_i/d\varphi + c\psi_l}{\psi_{th}^2} = \frac{1}{\psi_{th}}\left[\frac{d\psi_i}{d\varphi} + c(1-\eta)\right]. \tag{24}$$

Rearranging and integrating from φ_{opt} where the efficiency is maximal to φ yields to

$$\psi_i(\varphi) - \psi_i(\varphi_{opt}) = -\int_{\varphi_{opt}}^{\varphi}\left[\psi_{th}\left.\frac{\partial\eta}{\partial\varphi}\right|_{\mathrm{Re}} + c(1-\eta)\right]d\varphi. \tag{25}$$

Integrating by parts using the relation $\psi = \psi_{\mathrm{th}}\eta$ results in

$$\psi_i(\varphi) - \psi_i(\varphi_{opt}) = -(\psi(\varphi) - \psi(\varphi_{opt})) - c(\varphi - \varphi_{opt}). \tag{26}$$

Hence by looking only at one characteristic curve (efficiency and pressure rise) at one rotational speed, i.e. Reynolds number it is possible to determine the loss factor in the form

$$\begin{aligned} V(\varphi) &:= \frac{\psi_f}{\psi_l} \\ &= 1 - \frac{\eta}{1-\eta}\frac{1}{\psi}\left[\psi_i(\varphi_{opt}) + \psi(\varphi_{opt}) - \psi(\varphi) - c(\varphi - \varphi_{opt})\right] \end{aligned} \tag{27}$$

Figure 2 show a significant improvement in the scaling of the efficiency by using the loss distribution factor (27) in comparison to a fixed value of 0.5 as Ackeret did. One major advantage of the result (27) is that there is no fitting parameter used. The one drawback should be mentioned concerning the above derivation.

Measurements show, that the best operation point of a turbomachine shows a shift to higher flow rate coefficients with increasing rotational speed, i.e. Reynolds number (see Fig. 2). This scaling effect would not be covered using Eq. (27). Hence it is worthwhile to discuss an alternative way to gain the ratio $V := \psi_f / \psi_l$.

SECOND MODEL FOR THE LOSS DISTRIBUTION FACTOR

The critical assumption done so far was that the friction losses are independent on the flow rate coefficient. Essential for the losses in the blade passage is the relative velocity, i.e. the difference between absolute and circumferential velocity: $\vec{w} = \vec{c} - \vec{u}$. Hence the square of the velocity magnitude is given by $w^2 = u^2(\varphi^2 + 1)$. Since the friction losses are due to boundary layer friction, they are of the form

$$\psi_f = b\left(\varphi^2 + 1\right) \lambda(wl/\nu, k/d) \approx b\left(\varphi^2 + 1\right) \lambda(Re, k/d), \qquad (28)$$

with the known limiting behaviour $\psi_f \sim (\varphi^2 + 1)\, Re^{-\alpha}$ for $k \leq 5\delta_\nu$ and $\psi_f \sim (\varphi^2 + 1)$ for $k >> 5\delta_\nu$. The dimensionless constant b is again a dimensionless geometry quantity describing the machine. Hence the ratio $V := \psi_f / \psi_l$ is now given by

$$V = \frac{\psi_f}{\psi_l} = b \frac{\eta}{1-\eta} \frac{\varphi^2 + 1}{\psi} \lambda(Re, k/d). \qquad (29)$$

Using the mentioned measurements by Hess and Pelz to determine the constant b leads to a value of approximately 11. Fig. 4 to Fig. 6 show scale-up for efficiency respectively pressure coefficient using Eq. (29).

GENERAL SCALE UP METHOD

We now compare model, denoted by a prime (‘) and prototype machine. From (16) and (21) it follows

$$\frac{1-\eta}{1-\eta'} = \frac{\psi_l}{\psi'_l} = \frac{\psi_f}{\psi'_f + \psi'_i} + \frac{\psi_i}{\psi'_f + \psi'_i}. \qquad (30)$$

With the factor $V' = \psi'_f / \psi'_l$ we end up with the scaling formula

$$\frac{1-\eta}{1-\eta'} = \frac{\psi_i}{\psi'_i} + V'\left(\frac{\lambda(Re, k/d)}{\lambda(Re', (k/d)')} - \frac{\psi_i}{\psi'_i}\right). \qquad (31)$$

For the special case where the relative tip clearance for both machines are the same $\psi_i / \psi'_i \equiv 1$ the result

$$\frac{1-\eta}{1-\eta'} = 1 + V'\left(\frac{\lambda(Re, k/d)}{\lambda(Re', (k/d)')} - 1\right) \qquad (32)$$

is obtained where V' is given by the fixed value of 0.5 (Ackeret) or now by either Eq. (27) or Eq. (29).

The absolute value of the friction factor λ in Eq. (32) is not required, since only the quotient of the friction factors is needed. Assuming similarity in the relative roughness Eq. (32) simplifies for the hydraulic smooth case to:

$$\frac{1-\eta}{1-\eta'} = 1 + V'\left[\left(\frac{Re}{Re'}\right)^{-\alpha} - 1\right]. \qquad (33)$$

Fig. 2 compares scale-up with Eq. (33) – using Eq. (27) to determine the factor V – with measurements on an axial turbomachine by Hess and Pelz (2009) and a common scale-up method by Ackeret (Mühlemann, 1948). The measurements were carried out with a relative tip clearance t/d of 0.1% and a relative roughness k/d of 36E-06, Re = 0.6E06 respectively 12E-06, Re = 6.5E06. The exponent α is set to 0.2.

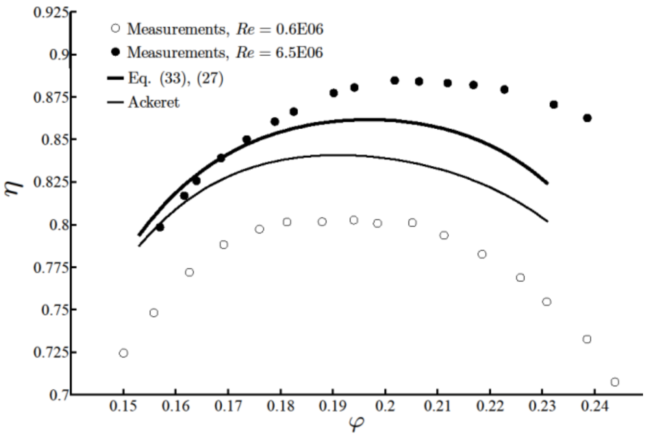

Fig. 2 Comparison of efficiency scale-up using Eq. (33) and (27) with measurements and the scale-up method by Ackeret

Although the calculated values are still smaller than the measured ones an increase of accuracy compared to Ackeret is obtained. Fig. 3 shows scale-up of the pressure coefficient where the same tendency is visible. It is assumed, that the power coefficient keeps constant with increasing Reynolds number (Hess and Pelz, 2009) and the pressure coefficient behaves in the form $\psi / \psi' = \eta / \eta'$.

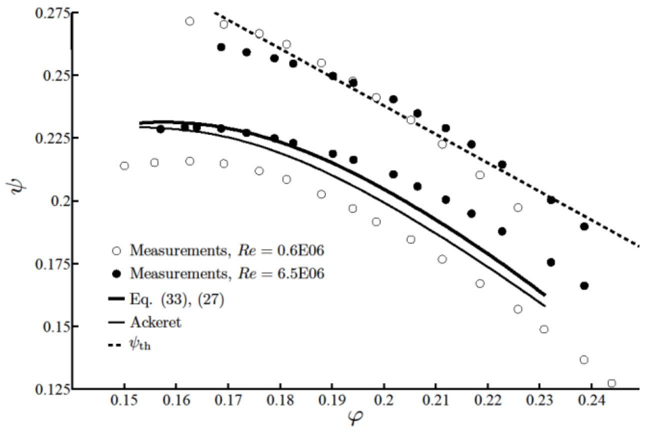

Fig. 3 Comparison of pressure coefficient scale-up with Eq. (33) and (27), measurements and the scale-up method by Ackeret

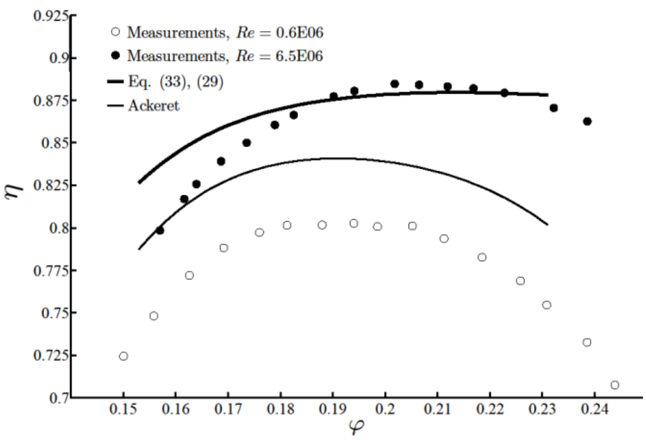

Fig. 4 Comparison of efficiency scale-up using Eq. (33) and (29)

with measurements and the scale-up method by Ackeret

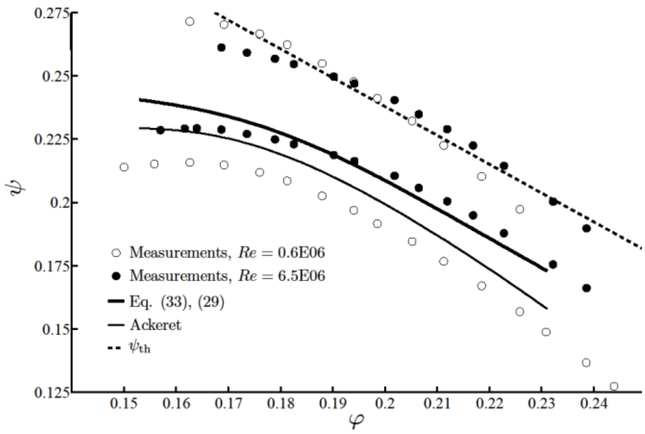

Fig. 5 Comparison of pressure coefficient scale-up with Eq. (33) and (29), measurements and the scale-up method by Ackeret

Even good results are reached in case of lower differences in Reynolds number shown in Fig. 6.

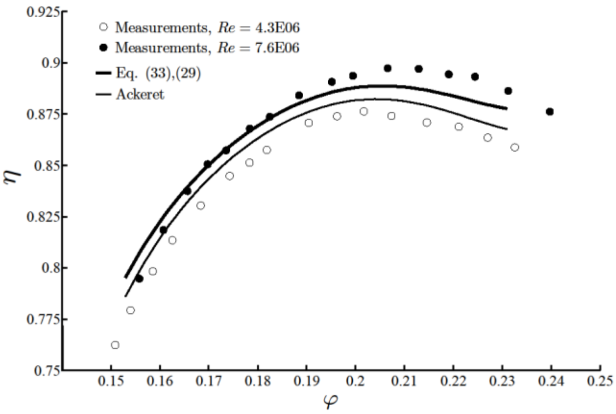

Fig. 6 Comparison of efficiency scale-up using Eq. (33) and (29) with measurements and the scale-up method by Ackeret

On the one hand a further increase of accuracy compared to method one for determining the factor *V* is visible. On the other hand measurements at different Reynolds numbers are needed for determining the dimensionless constant *b*.

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